

## OBSERVATION OF LOW-FREQUENCY MIXED-STATE RESONANCES IN PURE NIOBIUM\*

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In this Letter we present some recent observations of low-frequency resonances in the mixed state<sup>1</sup> of pure niobium. These low- $Q$  resonances have a line shape similar to low- $Q$  helicon resonances in a normal metal and resemble the resonances observed in the intermediate state of a pure type-I superconductor.<sup>2</sup> A minimum ac exciting-field amplitude is necessary to observe them. Furthermore, these mixed-state resonances merge into the normal-state helicon resonance at  $H_{c2}$ . In the mixed state, the damping of the resonance is determined by a flux-flow resistivity similar to the one found empirically.<sup>3</sup> There are significant differences between the mixed-state resonances reported here and the low- $Q$  resonances observed in the intermediate state of a type-I superconductor.<sup>2</sup>

Our observations are to be contrasted with the prediction<sup>4</sup> that circularly polarized electromagnetic modes could be excited in an array of Abrikosov<sup>5</sup> vortices (flux vortices). These excitations were predicted to be sharp ( $Q \sim 10^3$ ) and to occur at frequencies about 1 Mc/sec or lower. Excitations such as these would be similar to very-high-field helicon excitations in an extremely pure, nonmagneto-resistive metal (for example, 69-grade indium at  $2 \times 10^5$  G), and hence they should be readily observable. Experiments investigating the manner in which flux enters type-II superconducting alloys which have reversible magnetizations are consistent with no high- $Q$  heliconlike propagation in the mixed state of the superconductors.<sup>6</sup>

As pointed out in Ref. 6, these observations are intimately connected with the existence of the Hall effect in a type-II superconductor. Since this work, the Hall effect has been observed in the mixed and intermediate states<sup>7,8</sup> of superconductors. In the mixed state,  $\omega_c\tau$ , the tangent of the Hall angle, is seldom more than twice the normal-state value at the upper critical field, not nearly as large as expected from the arguments contained within Ref. 4. We estimate that the experiments in Ref. 6 could not see effects of a Hall field if  $\omega_c\tau$  were less than unity; that is, the sensitivity was not high enough to see ac Hall effects in type-II superconductors in which the dc Hall effect

has been observed.<sup>7,8</sup>

We have used a standard crossed-coil technique to look for heliconlike effects in the mixed state of two high-purity single-crystal niobium cylinders. A static magnetic field,  $H_0$  is applied perpendicular to the axis of the cylinder. The static magnetic field, an exciting magnetic field  $h_0 \sin\omega t$ , and the axis of a pick-up coil are mutually orthogonal. The pick-up voltage was detected phase-sensitively at constant static field as a function of the frequency of the exciting magnetic field. Our sensitivity is sufficient to see helicon resonances characterized by  $\omega \sim 10^2 \text{ sec}^{-1}$  and  $\omega_c\tau = 0.01$ .<sup>9</sup> This method is the same as that used to observe low- $Q$  resonances in the intermediate state.<sup>2</sup>

The first specimen studied was 6 mm in diameter and had a residual resistance ratio of about 600. We observed very small-amplitude resonances just below  $H_{c2}$ . These weak resonances were strongly dependent upon the static magnetic field and the amplitude of the exciting field.<sup>10</sup>

A second specimen, Nb-2, with a diameter of 2.5 mm and a residual resistance ratio of about 1600, has been studied recently. We find definite low- $Q$  mixed-state resonances which are similar in shape to the normal-state helicon resonance.<sup>9</sup> A minimum exciting magnetic field of a few gauss is required to excite these resonances.

At all temperatures between 1.5 and 4.2°K and at all fields below  $H_{c2}$  we find a threshold value,  $h_t$ , for the exciting field. Below  $h_t$ , no resonance is observable. Well above  $h_t$  the resonant frequency is only slightly dependent upon the amplitude of the exciting field. There is a transition region where the resonant frequency depends strongly upon the amplitude of the exciting field. This behavior is understandable if the resonance is due to the motion of flux vortices, since a certain critical current is necessary to unpin them. This could account for the difference in behavior of our two specimens. Nb-2 is purer and, probably more important, it is annealed better than the first specimen. We now believe that we could not establish a good flux-flow condition in the first specimen.<sup>9</sup> Except for static fields close to  $H_p$  (see

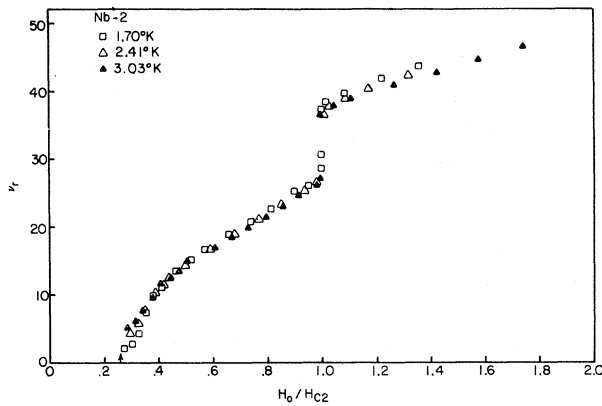


FIG. 1. The dependence of  $\nu_r$  upon  $H_0/H_{C2}(t)$  below 4.2°K. The resonant frequency is temperature dependent for static fields up to about  $H_{C3}$ , the field at which surface superconductivity vanishes. The exciting field is 250 G peak to peak. The arrow indicates the penetration field,  $H_p$ .

Fig. 1), a 40-G peak-to-peak exciting field is sufficient to give data nearly independent of the amplitude of the exciting field. Close to  $H_p$ , much larger fields are required.

From the experimental data we determined the resonant frequency, width, and maximum amplitude. In Fig. 1 we show the dependence of the resonant frequency,  $\nu_r$ , upon  $H_0/H_{C2}(t)$ , where  $H_{C2}(t)$  is the upper critical field at the reduced temperature  $t = T/T_c$ . In the mixed state, data for all temperatures investigated lie on a single curve. An exciting field amplitude of 250 G is required to obtain an amplitude-independent resonant frequency near  $H_p$ . From previous measurements of the magnetization of niobium cylinders in a longitudinal field,<sup>11,12</sup> we have determined the magnetization of our specimen in a transverse field. In this manner we determine the magnetic induction,  $B$ , and we find that  $\nu_r$  is nearly linear in  $B$ . The field at which the resonant frequency increases sharply is identified with  $H_{C2}$  and agrees very well with measurements of  $H_{C2}(t)$  in pure niobium.<sup>11,12</sup>

The amplitude of the absorptive component of the pick-up voltage at the resonant frequency (absorptive signal) is proportional to the tangent of the Hall angle for a normal-state helicon resonance.<sup>13</sup> In Fig. 2 the dependence of the mixed-state absorptive signal on the static field,  $H_0$ , is shown. There is a marked, qualitative difference in behavior of the absorptive signal as the exciting-field amplitude decreases. For large exciting amplitudes, the

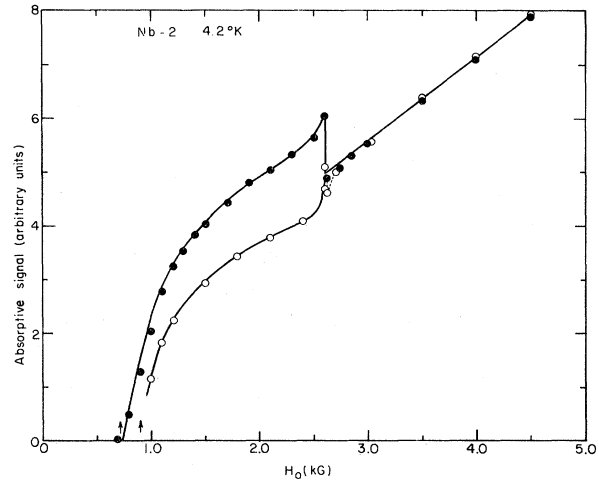


FIG. 2. The dependence of the absorptive signal on the applied field,  $H_0$ . Open circles represent data for a 40-G peak-to-peak exciting field and the solid circles represent data for 250-G peak-to-peak exciting field. The penetration fields,  $H_p$  (shown by the arrows), are slightly different for these two exciting amplitudes. The qualitative dependence of the absorptive signal is independent of temperature below 4.2°K.

absorptive signal decreases sharply at  $H_{C2}$ , while for low amplitudes the absorptive signal increases at  $H_{C2}$ . The behavior at large exciting fields is much different from the observed behavior of the Hall angle in pure Nb near  $H_{C2}$ .<sup>8</sup> However, the fact that our absorptive signal depends upon the exciting field is consistent with the observation that the Hall angle in the mixed state depends upon the transport current. The data shown in Figs. 1 and 2 are independent of the magnetic history of the specimen.

The  $Q$  of the mixed-state resonance is independent of field and temperature. Its value is about 0.5, the same as in the normal state at  $H_{C2}$ . The normal-state helicon resonance has a field-dependent  $Q$  only above about 15 kG.

Our method of pick-up is only sensitive to circularly polarized excitations within the specimen. For a normal metal,<sup>13</sup> the resonant frequency, signal amplitude, and  $Q$  are related to the Hall coefficient and magnetoresistance of the core. For a superconductor, the measurable quantities will give an effective macroscopic Hall coefficient and an effective macroscopic magnetoresistance.

The helicon dispersion relation for propagation along the magnetic field is given by

$$\nu = (2\pi)^{-1} \rho k^2 (1 + u^2)^{1/2}, \quad (1)$$

where  $u = RB/\rho$  and  $\rho$ ,  $R$ , and  $B$  are the resistivity, Hall coefficient, and magnetic induction, respectively. Experiments on cylinders of normal metals of known Hall coefficient and magnetoresistance show that, for the fundamental resonance,  $k \approx 4/(\text{cylinder diameter})$ . For a given  $u$ , there is little difference between the shapes of a resonance in a cylinder and one in a flat plate. The sharpness of the resonance is  $Q = \frac{1}{2}(1+u^2)^{1/2}$  and can therefore be used to determine  $u$  directly. In practice, however, one needs  $u^2 > 1$  ( $Q > 0.7$ ) for this procedure to be useful. We measure  $Q = 0.5 \pm 0.1$  at all fields in the mixed state and from this conclude that in the mixed state  $u < 0.5$ . From the known  $R$  and  $\rho$  we calculate  $u = 0.34$  just above  $H_{c2}$ .

The flux-flow concept has been useful in describing the dissipative motion of flux vortices, so it is interesting to see what it would give when used as the resistivity in Eq. (1). Since the flow resistivity is  $\rho_f = \rho_n B/H_{c2}(t)$ , where  $\rho_n$  is the normal-state resistivity, we see that  $\nu$  should depend upon  $B/H_{c2}(t)$ . In the temperature region investigated, most of the temperature dependence is contained in  $H_{c2}(t)$  so  $\nu$  should have a temperature dependence given by  $H_0/H_{c2}(t)$ , in accord with experiment. This model also gives  $\nu$  proportional to  $B$ , which agrees with our experiments at the highest exciting fields. This flow resistivity does not give a sharp change in the resonant frequency at  $H_{c2}$ .

In the mixed state, we expect each flux vortex to contribute an equal amount to the pick-up voltage. Using this and the flux-flow resistivity, we find that the absorptive signal should be proportional to  $B$ . Our experiments give a small linear region at low magnetic induction for the largest exciting fields used.

The line shape in these experiments is the same as that found in the intermediate state of indium<sup>2</sup> but the dependence of the resonance parameters upon the external variables  $H$  and  $T$  is somewhat different. In the intermediate state, the damping was observed to decrease by as much as a factor of 3 from the normal-state value at  $H_c$ . The mixed state showed no decrease in damping below  $H_{c2}$ . A decrease of 50% from the normal-state helicon damping could have been resolved. In the low-field region of the intermediate state, the resonant frequency was found to be independent of the static field, while in the mixed state, the reso-

nant frequency increases continuously with the static field.

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<sup>9</sup>Preliminary data on the mixed state of a Pb-In alloy carrying a transport current indicates similar low- $Q$  resonances characterized by  $\omega \sim 10^5 \text{ sec}^{-1}$  and  $\omega_c \tau \sim 5 \times 10^{-3}$ . Note that no transport current is used in the experiments on Nb reported in this Letter.

<sup>10</sup>We have searched unsuccessfully for low-amplitude, high- $Q$  resonances in the mixed state of niobium. All fields between zero and  $H_{c3}$  were covered for various frequencies between 10 cps and 15 Mc/sec.

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