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PI-MESONIC ATOMS*

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We have measured pi-mesonic x-ray energies for 70 elements, by using a lithium-drifted germanium detector, and for aluminum by using a bent-crystal spectrometer. We observe differences between some x-ray energies and the energies predicted by the Klein-Gordon equation, and we interpret these differences as being due to vacuum polarization and the presence of the nucleus. The vacuum-polarization correction to the x-ray energies is well known, and we have used the work of Mickelwait and Corben to calculate this effect.[1] The presence of the nucleus induces both an energy shift and a broadening of the x-ray line because the pion interacts strongly with the nucleons in the nucleus. Furthermore, the nucleus has a finite size which produces a departure from the point approximation of the Coulomb interaction. These effects have not been precisely calculated, and in this paper we present our results for the nuclear energy level shift.

The nuclear effect in pi-mesonic x-ray energies has been discussed by Ericson,[2] who suggests that the level shift can be understood in terms of a Kisslinger potential[3] that contains a gradient operator. The gradient operator is introduced because of the strong p-wave force in the pion-nucleon interaction. Therefore, we have calculated the potential shift using perturbation theory and an optical-model potential of the form

$$V(r)=V_0\rho(r)+V_1\nabla\cdot\rho(r)\nabla, \qquad (1)$$

where V_0 and V_1 are constants to be determined from experiment, ∇ is the gradient operator, and

$$\rho(r)=\{1+\exp[(r-c)/z_1]\}^{-1} \qquad (2)$$

is the Saxon-Woods potential function for the nucleus. We use $c=1.08A^{1/3}$ F and $z_1=0.545$ F as found by Anderson, Johnson, and Hincks from the muonic x-ray data.[4] The muonic x-ray parameters describe the Coulomb distribution of the nucleus, and this distribution is not expected to be the same as that seen by the pion because the pion interacts strongly with the nucleus. Lacking better information, we have used the muonic x-ray parameters as a first approximation. Then the energy-level shift for a level with principal quantum number n and orbital angular momentum l is

$$\Delta E_{nl}=V_0\int_0^\infty\rho(r)|\psi_{nl}|^2d\tau+V_1\int_0^\infty\rho(r)|\nabla\psi_{nl}|^2d\tau, \qquad (3)$$

where we have used nonrelativistic wave functions for ψ_{nl}.

Mesonic x-ray energies for a series of elements were measured with a germanium detector. The accuracy of these measurements is limited by the electronics of the system used to measure pulse heights from the germanium. Gain shifts were minimized by using a digital gain stabilizer,[5] and the energy response was calibrated with radioactive sources at inter-

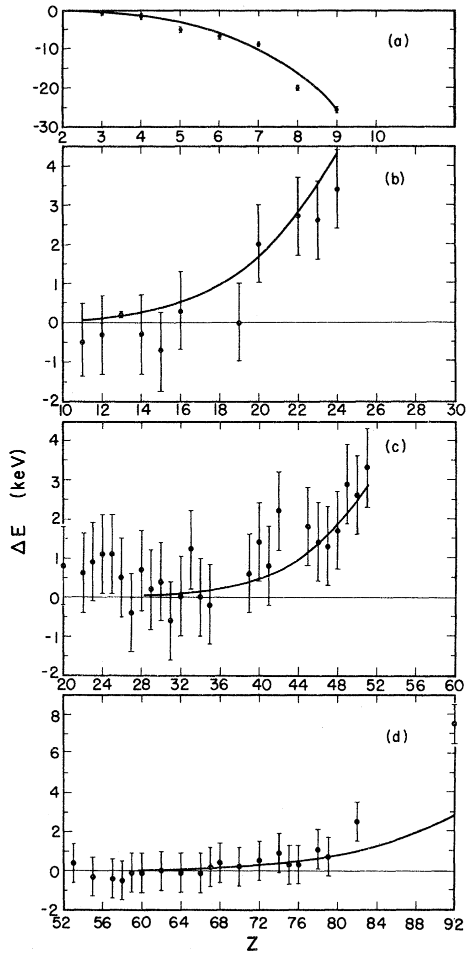

FIG. 1. Mesonic x-ray level shifts as a function of Z_1 (atomic number) for the (a) $2p \rightarrow 1s$, (b) $3d \rightarrow 2p$, (c) $4f \rightarrow 3d$, and (d) $5g \rightarrow 4f$ x rays.

vals of about 100 keV. The energy measurements were taken for periods of about 15 min, and the energy response of the system was calibrated before and after each run. We therefore have confidence in our measurements to an accuracy of ±1.0 keV. We are now making more measurements of these energies, after which a detailed description of the experiment will be presented. The aluminum measurement with a bent-crystal spectrometer has been reported earlier.[6]

The data are presented in Fig. 1, which shows the difference in energy between the measured x-ray energies and the value predicted by the Klein-Gordon equation, including reduced-mass and vacuum-polarization corrections. The data for the $2p \rightarrow 1s$ transition were taken from the work of West,[7] and the energies for this transition have been corrected for the Coulomb shift due to the finite size of the nucleus; this correction is negligible for the other transitions. A negative energy in Fig. 1 indicates that the measured energy level is higher than the calculated value, and therefore the x-ray energy is lower than predicted. The curves in Fig. 1 are computed from Eq. (3) after V_0 and V_1 have been varied to obtain a good fit to the data. Practically all of the shift in the x-ray energy is due to the 1s level for the $2p \rightarrow 1s$ lines, and similarly for higher transitions it is due to the level with smaller l. This is theoretically very plausible and confirmed experimentally by the fact that the difference between the observed energy of the $3p \rightarrow 1s$ line and that predicted according to the Klein-Gordon equation is the same as for the $2p \rightarrow 1s$ line.

We assume that the energy difference plotted in the figure is due to the nuclear shift, and we observe that the sign of the $2p \rightarrow 1s$ x-ray shift is different from that of other x-ray transitions. The gradient of the 1s wave function near the origin is small, therefore the 1s energy shift is dominated by the first term in Eq. (3) (the local potential). However, the gradients of the $2p$, $3d$, and $4f$ wave functions are large, and the energy shifts for these levels are dominated by the second term in Eq. (3) (the nonlocal potential). The negative shift for the $2p \rightarrow 1s$ energy implies that the local potential is repulsive, while the positive shift for the $3d \rightarrow 2p$, $4f \rightarrow 3d$, and $5g \rightarrow 4f$ energies implies that the nonlocal potential is attractive. Our best values for these potentials are $V_0 = -8.1$ MeV and $V_1 = 120$ MeV F^2. The potentials do not give good agreement with the $5g \rightarrow 4f$ uranium x ray, but this could be a result of our using the wrong value for the nuclear radius.

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ONSET OF LONG-RANGE ORDER IN A CRITICAL SOLUTION OF MACROMOLECULES*

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Diffusion broadening has been observed in the inelastic scattering of monochromatic light off a binary fluid, cyclohexane-polystyrene, near its critical point for mixing. The broadening a few tenths of a degree above the critical temperature is an order of magnitude less than in a binary mixture of small molecules (cyclohexane-aniline)[1] and two orders of magnitude less than in a pure fluid (SF$_6$).[2] The dependence of the broadening on scattering angle agrees with that found for the other fluids,[1, 2] and with expectation[3]; but its increase with rising temperature is less rapid than for these fluids, being less than linear in ΔT.

The method used was substantially that of Benedek.[4, 2] A 12-mm diameter cylindrical sample of cyclohexane containing 6.5% by volume National Bureau of Standards standard 705 polystyrene, with narrow molecular-weight distribution,[5] was irradiated with monochromatic light at 6328 Å from a He:Ne laser. Light scattered at an angle θ with respect to the incident direction was detected at a photomultiplier, where interference between the different frequency components in the scattered light produced a fluctuating photoemission. The noise spectrum of the amplified photocurrent, with dc component suppressed, was analyzed into its frequency components using a spectrum analyzer (Nelson-Ross type 011) with a resolution of 10 Hz.

Photographs of typical spectra are displayed in Fig. 1. Each photograph shows the digitized average of the noise amplitudes obtained in about 100 sweeps through the spectrum, each of 8 sec duration. An ND-800 Enhancetron

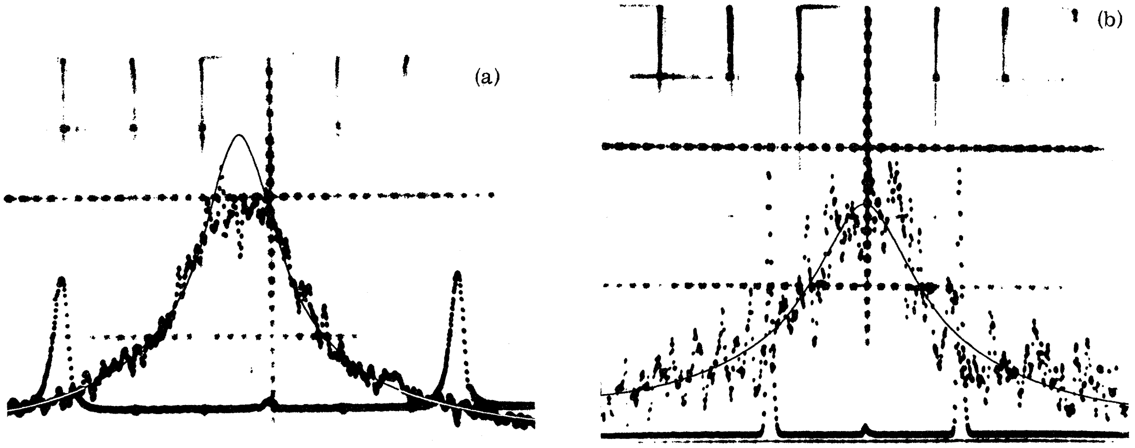

FIG. 1. Oscilloscope traces of digitized and time-averaged spectra of noise currents produced by the inelastically scattered light. (a) $\theta = 40°$, $T - T_S = 0.3°C$, marker pips at ±150 Hz. (b) $\theta = 80°$, $T - T_S = 4.0°C$, marker pips at ±1000 Hz. The solid curves are square-root Lorentzians fitted to the data. Suppression of low-frequency components and some drift of the heterodyne frequency of the spectrum analyzer account for the depression in the center of the narrower spectrum [part (a)]. Spectra obtained at $\theta = 20°$ and $\Delta T < 2°C$ are about as clean as that in (a), though narrower for small ΔT and therefore more susceptible to drift.