FURTHER DISCUSSION OF A FUNDAMENTAL THEOREM IN QUANTUM OPTICS

I. R. Senitzky

Institute for Exploratory Research, U. S. Army Electronics Command, Fort Monmouth, New Jersey

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A theorem was recently proposed[1] which simplifies the concepts and formalism of quantum optics. Subsequently, a "disproof" of the theorem appeared.[2] The present Letter has a three-fold purpose: (1) to expand the argument on which the proof of the theorem is based; (2) to show that the "disproof" is based on a misinterpretation; (3) to illustrate the theorem with a simple but instructive example.

The theorem under discussion states that "all sources on which the effect of the 'detector' is negligible may be treated as classical sources in the interaction under consideration," and is followed by the corollary that "the field acting on a quantum-mechanical system—when the sources are of the above type—consists of the superposition of a classical field and the 'vacuum' field." As explained in I, the word "detector" in the theorem stands for all systems the interaction of which with the field is of interest, and includes the dissipation mechanism; the "vacuum" field, in the presence of dissipation, is that due to the fluctuations of the dissipation mechanism.[3,4] The theorem is proved by considering a single mode of frequency ω and dissipation constant $\beta(\ll\omega)$, expressing the coordinate and momentum of the radiation oscillator by[5]

$$q(t)=\int_{-\infty}^{t}dt_1[\alpha S(t_1)+\gamma D(t_1)+F(t_1)]\times\exp[-\tfrac{1}{2}\beta(t-t_1)]\cos\omega(t-t_1)\equiv q_S(t)+q_D(t)+q_F(t), \quad (1)$$

$$p(t)=-\int_{-\infty}^{t}dt_1[\alpha S(t_1)+\gamma D(t_1)+F(t_1)]\times\exp[-\tfrac{1}{2}\beta(t-t_1)]\sin\omega(t-t_1)\equiv p_S(t)+p_D(t)+p_F(t), \quad (2)$$

and showing that the commutators of q_S and p_S with all variables involved in the interaction under consideration are negligible, so that q_S and p_S are effectively c-numbers. It is argued that $q_D(t_1)$ and $q_F(t_1)$ must commute with $q_S(t_2)$ for all t_1 and t_2; otherwise a measurement on the detector or dissipation mechanism will disturb the source, which is contrary to the hypothesis. Furthermore, q is measured by measuring certain dynamical variables of the detector; by similar argument, therefore, $q(t_1)$ commutes with $q_S(t_2)$. Similar reasoning applies to p_S, and also shows that $q_S(t_1)$ commutes with $p_S(t_2)$. Thus, q_S and p_S commute with all pertinent variables.

We now fill in some of the details of the above argument and make it more precise. The implicit mathematical hypothesis is the statement that $[S(t_1),F(t_2)]$ and $[S(t_1),D(t_2)]$ are negligible for all t_1 and t_2. Equations (1) and (2) show that this will assure the negligibility of the commutators of q_S or p_S with q_F, q_D, p_F, and p_D; the remainder of the argument is straightforward. Now, for $t=t'$, the commutator of two variables $[A(t), B(t')]$ is a measure of the disturbance of one—either one—by a (ideal) measurement of the other. For $t<t'$, it is a measure of the disturbance of $B(t')$ by a measurement of $A(t)$.[6] For $t_2\leq t_1$, the two commutators above are a measure of the disturbance produced in the source by a measurement on the "detector"; thus, if this disturbance is negligible the commutators are negligible. If one assumes that the commutators are of the same order of magnitude for $t_2>t_1$ as for $t_2\leq t_1$, then the statement that the effect of the "detector" on the source is negligible is equivalent to the above mathematical hypothesis. Such an assumption is obviously true if the negligibility of the commutators (for $t_2\leq t_1$) is due to weak coupling between detector and source, and is implicitly made in I.[7]

It is possible to reformulate the theorem so that reference to the detector is eliminated. Consider the field resolved into modes in such a manner that the weak coupling between source and "detector" is due to weak coupling between the source and those (few) modes which significantly affect the "detector," these modes being referred to as the "detector field."[8] The theorem may now be stated as follows: <u>If the effect of the detector field on the source is negligible, the contribution of the source to the detector field may be described classically.</u> The sources to which the theorem applies may radiate strongly into the entire field, that is, into modes not coupled significantly to the detector, but only slightly into the detector field. If there are no other modes into which the source radiates, the negligibility requirement implies that the source is coupled to the detector field only for a time short compared to the lifetime of its excited state, before being affected by other influences.

The essence of the theorem is that sufficient decoupling (in the above sense) makes any source classical as far as its contribution to the detector field is concerned. It might be argued that decoupling makes the radiation energy from the source vanish to the same order (in coupling constant) as some of the commutators, and that, consequently, the "classicalness" is illusory. It should be noted, however, that commutators small compared to those of the "vacuum" field are insignificant, and play no role in experiments (where minimum uncertainties are introduced by the "vacuum" field), while energies small compared to those of the "vacuum" field can be measured (since the nonthermal "vacuum"-field energy cannot be absorbed, in contrast to the energy radiated by the source), and frequently occur in optical experiments.

The "disproof" of Glassgold and Holliday consists of applying the theorem to a case in which no measurements may be made until the source is decoupled from the field. Since the detector cannot then affect the source, they argue, the theorem implies that the contribution of any source becomes classical as soon as it is decoupled from the field, and this involves "the incorrect assumption that, once two systems no longer interact, a measurement involving one system cannot influence the statistical properties of the other system."[2] It is not difficult to see that this argument contains a misinterpretation of the phrase "effect of the 'detector.'" The theorem actually requires that the source be insensitive to the "detector" (or to measurements) at any—or all—time. This follows, firstly, from the fact that the "detector" includes the loss mechanism, which exists at all times and determines the properties of the mode under consideration, and, secondly, from the fact that the meaning of q and p is associated with measurements made by the detector; the detector must therefore be available for such measurements whenever q and p are defined. (One could insert in the theorem the parenthetical expression "at any time" after the phrase "effect of the 'detector,'" but it is really superfluous.) The theorem is proved only for the case of weak coupling between source and detector field, while the case considered in the "disproof" may have arbitrarily strong coupling (prior to the decoupling).[9]

The present formulation of the theorem is qualitative; negligibility is not defined numerically and no prescription is given for describing the source classically. A simple example will, therefore, prove instructive.[10] We consider a mode of the detector field weakly coupled to a two-level electric-dipole system (TLS) in resonance with the mode. The time under consideration (the time during which the coupling lasts, uninterrupted by the influence of other systems) is sufficiently short so that the expectation value of the energy radiated by the TLS is a small fraction of its total energy. Perturbation theory is obviously justified, and we ignore interaction terms of higher order than the second. We are not concerned with q_D and p_D in the present instance, and write

$$q = q_F + q_S, \quad p = p_F + p_S. \tag{3}$$

$S(t)$ is now the electric dipole moment (in dimensionless units) of the TLS and, in zeroth order, is given (the notation is such that $E_1 < E_2$) by

$$S_{12}^{(0)} = \exp(-i\omega t), \quad S_{21}^{(0)} = S_{12}^{(0)*}, \quad S_{11}^{(0)} = S_{22}^{(0)} = 0.$$

Expressions for q_S and p_S up to second order are available.[11] The first-order expressions are obtained from Eqs. (1) and (2) by replacing $S(t_1)$ with $S^{(0)}(t_1)$ and changing the lower integration limit to zero, and the second-order expressions are given by

$$q_S^{(2)} = 2(\alpha^2/\beta) I \int_0^t dt_1\, p_F(t_1) \times (1 - e^{-\frac{1}{2}\beta(t-t_1)}) \sin\omega(t-t_1), \tag{4}$$

$$p_S^{(2)} = 2(\alpha^2/\beta) I \int_0^t dt_1\, p_F(t_1) \times (1 - e^{-\frac{1}{2}\beta(t-t_1)}) \cos\omega(t-t_1), \tag{5}$$

where $I_{11} = -1$, $I_{22} = 1$, $I_{12} = I_{21} = 0$, and $t = 0$ is the time at which the coupling between TLS and the mode begins. The expectation value of the energy lost in time t by the TLS to the mode is

$$2(\alpha^2/\beta^2)\hbar\omega |a_2|^2 [\beta t + 2(e^{-\frac{1}{2}\beta t} - 1)], \tag{6}$$

where $|a_2|^2$ is the probability of finding the TLS in the upper state, and the temperature of the dissipation mechanism has been taken to be zero, for simplicity.

Up to second order, we have

$$[q_S(t), p_S(t)] = -2i(\alpha^2/\beta^2)I(1-e^{-\frac{1}{2}\beta t})^2, \quad (7)$$

$$[q_S(t), p_F(t)] = [q_F(t), p_S(t)]$$
$$= i(\alpha^2/\beta^2)I(1-e^{-\frac{1}{2}\beta t})^2. \quad (8)$$

It is seen that the ratio of these commutators to that of the "vacuum" field $\{[q_F(t), p_F(t)] = i\}$ is of no greater order of magnitude than that of the expectation value of the fraction of the total energy lost by the TLS to the mode. Here we have a measure of the negligibility of the commutators.

We ask next, how is the source to be described by a c-number? Consider the expressions

$$q' = q_F + q_c, \quad p' = p_F + p_c, \quad (9)$$

where q_c and p_c are c-numbers of first order, and construct the characteristic functions

$$f(\mu, \nu) \equiv \langle \exp i(\mu q' + \nu p')\rangle, \quad (10)$$

$$\varphi(\mu, \nu) \equiv \langle \exp i(\mu q + \nu p)\rangle, \quad (11)$$

where q and p are given by Eqs. (3). The results of a calculation up to second order[10] [and approximations based on $(\beta/\omega) \ll 1$] show that these two functions are identical provided

$$\langle q_c\rangle = \langle q^{(1)}\rangle, \quad \langle p_c\rangle = \langle p^{(1)}\rangle, \quad (12)$$

$$\langle q_c^{\,2}\rangle = \langle q^{(1)2}\rangle + 2\langle q_F q^{(2)}\rangle, \quad (13a)$$

$$\langle p_c^{\,2}\rangle = \langle p^{(1)2}\rangle + 2\langle p_F p^{(2)}\rangle. \quad (13b)$$

Thus, if q_c and p_c are described statistically according to Eqs. (12) and (13), we obtain $f = \varphi$; furthermore, the required expressions for $\langle q_c^{\,2}\rangle - \langle q_c\rangle^2$ and $\langle p_c^{\,2}\rangle - \langle p_c\rangle^2$ are each equal to (averaged over a half-cycle)

$$2(\alpha^2/\beta^2)|a_2|^4(1-e^{-\frac{1}{2}\beta t})^2, \quad (14)$$

so that a statistical description is meaningful. Since the characteristic function generates all the moments of q and p (either directly or by suitable rearrangement of the order), and since $[q', p'] = [q, p]$, we see that the statistical properties of q', p' are the same as those of q, p. In particular, the expectation values of the amplitude and energy are the same. We have found, therefore, an appropriate c-number description of the TLS to the field. Although the nonclassical aspect of the quantum-mechanical source (noncommutativity of q_S and p_S with all pertinent variables) is made negligible by sufficient decoupling, the statistical aspect obviously remains. It should also be noted that q', p' describe a fully quantum-mechanical radiation oscillator.[12]

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[1]I. R. Senitzky, Phys. Rev. Letters 15, 233 (1965), hereafter referred to as I.

[2]A. E. Glassgold and Dennis Holliday, Phys. Rev. Letters 15, 741 (1965).

[3]I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124, 642 (1961).

[4]Julian Schwinger, J. Math. Phys. 2, 407 (1961).

[5]The notation is the same as that of I, and the present discussion should be associated with that found there, for greater clarity. Briefly, q and p are dimensionless, satisfying $[q, p] = i$, and S, D, and F are the variables through which the source, detector, and dissipation mechanism [the last two are treated separately in Eqs. (1) and (2)] couple to the mode. F is independent of q and p; it is a stochastic quantum mechanical variable related to β and the temperature, and is discussed in detail in Refs. 3 and 4.

[6]See, for instance, discussion of commutators of field strengths at different times by W. Heitler, Quantum Theory of Radiation, (Oxford University Press, London, 1954), 3rd ed., Chap. II, Sec. 9.

[7]Without this assumption, one needs the additional requirement (in the nonmathematical statement of the hypothesis) that a (ideal) measurement on the source does not significantly disturb the detector, a requirement usually well met in optics, where, as pointed out in I, source and detector are weakly coupled.

[8]For example, the field of a "single-mode" laser may be resolved, approximately, into two modes, one inside the laser cavity and the other outside the cavity. Only the outside mode is coupled to the detector, and this mode is weakly coupled to the laser. In other words, the modes are chosen so that weakness of coupling is concentrated at the source.

[9]Note that even if it were permissible to couple the detector to the field only after the source has been decoupled, the mathematical hypothesis would still not be satisfied by the counter example, in general. While $[S(t_1), D(t_2)]$ would vanish for $t_2 \leq t_1$, this commutator would not necessarily be negligible for $t_2 > t_1$, if no restriction is imposed on the coupling between source and field while they interact.

[10]Only the results are presented here. A more comprehensive discussion will be published elsewhere.

[11]I. R. Senitzky, Phys. Rev. 115, 227 (1959). The notation of this reference is related to the present (dimensionless) notation as follows:

$$Q(\omega/4\pi c^2\hbar)^{1/2} \equiv q, \quad P(4\pi c^2/\hbar\omega)^{1/2} \equiv p, \quad \tilde{\gamma} u(4\pi\omega/\hbar)^{1/2} \equiv \alpha.$$

[12]Some of the conceptual "difficulties" cited in Ref. 2 are due to a lack of recognition of this fact. The present theory is fully consistent with the quantum theory of radiation.

2^+ MESONS AND HIGH-ENERGY SCATTERING

Bipin R. Desai*
Department of Physics, University of California, Riverside, California

and

Peter G. O. Freund
Department of Physics and Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
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Recent experimental evidence indicates the existence of nine $J^P=2^+$ mesons: $f(1250)$, $f'(1520)$, $A_2(1310)$, and $K^*(1430)$.[1] The approximate validity of the mass formulas

$$f'+A_2=2K^*,\qquad f=A_2, \tag{1}$$

(where particle symbols stand for particle masses squared) makes it a simple possibility[2] to classify these nine 2^+ mesons in a reducible SU(3) nonet, much like in the case of the familiar vector-meson nonet. Thus, for instance, f and f' are not pure singlet and octet members, respectively, but are linear combinations of the octet and singlet members. One can write the 3×3 matrix of the mesons as

$$T_9=\begin{pmatrix} f/\sqrt{2}+A_2^0/\sqrt{2} & A_2^+ & K^{*+} \\ A_2^- & f/\sqrt{2}-A_2^0/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -f' \end{pmatrix}. \tag{2}$$

We now consider the effect of the 2^+ nonet exchange in the reactions involving pseudoscalar mesons (M) and baryons (B). The M-nonet coupling is pure D type given by[2]

$$\sqrt{2}d\,\mathrm{Tr}(\{M,\bar{M}\}T_9), \tag{3}$$

while the B-nonet coupling can be D, F, and S (singlet) type,

$$\sqrt{2}D\,\mathrm{Tr}(\{B,\bar{B}\}T_9)+\sqrt{2}F\,\mathrm{Tr}([B,\bar{B}]T_9)+\sqrt{2}S\,\mathrm{Tr}(T_9)\,\mathrm{Tr}(B\bar{B}). \tag{4}$$

Consider an elastic-scattering process $A+B \to A+B$, where B stands for baryon and A for baryon or meson (M). Let the sum of the particle and antiparticle total cross sections be represented by

$$\Sigma_{AB}=\sigma_t(AB)+\sigma_t(\bar{A}B). \tag{5}$$

The vector-meson-exchange contribution cancels out in this sum. At high energies, where the scalar- and pseudoscalar-meson exchange is presumably negligible, the 2^+-meson exchange will dominate the sum. In what follows, we assume the 2^+ mesons to lie on Regge trajectories.

The question we would like to answer is whether the 2^+ nonet given by (2) explains the experimental behavior of Σ_{AB} or whether additional Regge trajectories are needed. We will assume that the coupling constants or, more precisely, the relative values of the coupling constants are given by exact SU(3) while masses take on their physical values. This is, of course, a crucial assumption. There are indications from other sources, however, that such an assumption should give reasonably accurate results. For example, for the difference between particle and antiparticle total cross sections, where only vector mesons contribute at high energies, the vector-meson nonet with exact SU(3) coupling gives good results.[3] Among 2^+ mesons, it is also known that the exact SU(3) prediction for A_2 exchange gives close agreement with experiments. This has been confirmed by the recent data on $\pi^-+p\to\eta+n$ where only A_2 exchange is possible. The experimental behavior of this reaction was correctly predicted on the basis of Kp scattering and SU(3) invariance.[4] Furthermore, it is well known that at high energy the experimental evidence $d\sigma(\pi^+ +p\to K^+ +\Sigma^+)/d\sigma(\pi^+ +p\to\pi^+ +p)\ll 1$ at zero momentum transfer implies[5]

$$\Sigma_{\pi p}/\Sigma_{Kp}=1. \tag{6}$$

The experimental data around 20 BeV/c give[6]

$$\Sigma_{\pi p}/\Sigma_{Kp}=1.2, \tag{7}$$