[1] L. O'Raifertaigh, Phys. Rev. Letters 14, 575 (1965).
[2] P. Roman and C. J. Koh, Nuovo Cimento 39, 1015 (1965).
[3] S. Coleman, Phys. Rev. 138, B1262 (1965).
[4] S. Weinberg, Phys. Rev. 139, B597 (1965). There are two ways out of the difficulties suggested by Weinberg. One is to assume that the number of particles, and multiplets (or particle names as he calls them), is infinite. This is an attractive alternative. It is as if the first few levels of the hydrogen atom had been discovered. It would seem disappointing and rather unpleasant if the energy spectrum were found to stop at some finite and rather arbitrary principal quantum number. The second way out is to assume that at least some members of his group G (which is the group acting on the internal and spin indices) do not commute with the momenta. This seems quite acceptable, and in fact necessary if we are dealing with a group having the Poincaré and internal groups as subgroups (which we will call a combined group). It would seem strange to have, except in some approximation, a group acting on the spin indices but not on the coordinates. Such a group should be distinguished from the afrementioned combined group, which appears far more natural.
[5] N. I. Akhiezer and I. M. Glazman, Theory of Linear Operators in Hilbert Space (Frederick Ungar, New York, 1961), Vol. I, p. 91.
[6] A. Bohm, International Centre for Theoretical Physics Report No. ICTP 64/9 (unpublished).
[7] Akhiezer and Glazman, Ref. 5, p. 88.
[8] Another reason for preferring Lie groups to Lie algebras is that we know that space is invariant under a group. It would be rather difficult to imagine a theory of combined invariance with part of the invariance under a group and part under an algebra.
[9] That no multiplets appear here is due to the simplicity of the group. In more complicated examples, it is possible to pick inhomogeneous operators that commute with all operators of subgroups and thus allow the existence of multiplets.

DECREASING $I=0$, $J=0$ $\pi\pi$ PHASE SHIFT AND REGGE GHOSTS*

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The persistent forward asymmetry[1] in the decay of the peripherally generated neutral ρ suggests that the $I=0$, $J=0$ $\pi\pi$ phase shift is near an odd multiple of $\pi/2$ at energies close to the ρ mass (770 MeV). On the other hand, most experiments have shown no peaks other than the ρ ($I=1$, $J=1$) and the f ($I=0$, $J=2$) in the $\pi\pi$ cross section,[2] so (if this absence of peaks is confirmed) it may be concluded that the $I=0$, $J=0$ phase shift varies slowly through an interval $\approx\pi/2$ in going from threshold to the region of the ρ.[3] It is difficult to construct a dynamical model that would lead to an increasing phase shift (with increasing energy) of this character—although such behavior might at first sight be associated with an extremely broad resonance. The trouble is that the responsible pole of the S matrix would have to be so far from the physical region (to avoid producing a peak) that there would remain no reason for it to dominate the amplitude and generate a large phase shift. On the other hand, there is no difficulty in achieving a decreasing phase shift that changes by a large total increment without producing peaks. In fact, peaks never result from decreasing phase shifts, as was pointed out long ago by Wigner.[4] This Letter explores various aspects of the possibility that the $I=0$, $J=0$ $\pi\pi$ phase shift is a decreasing function of energy. The establishment of such behavior would be a vital development in the physics of strong interactions. There would then exist persuasive evidence for the "ghost states" that play such a curious and controversial role in Regge-pole theory.

Assuming that the forward and backward angular peaks observed in all reactions at very high energies are dominated by Regge poles,[5] one is led to the conclusion that the "top-ranking" trajectories have the quantum numbers of the vacuum. This conclusion is reinforced by bootstrap dynamics, in which the strongest attractive forces appear for the vacuum quantum numbers,[6] but the puzzling feature emerges that the $J=0$ intersections of the two leading trajectories in this class seem likely to occur at negative values of energy squared. That is, the Pomeranchuk (P) trajectory is already crossing $J=1$ at zero energy, while the Pomeranchuk primed (P') trajectory here is in the neighborhood of $J=0.7$.[7] Unless these trajectories approach limits greater than zero as the squared energy approaches minus infinity, they must therefore cut $J=0$ in an unphysical energy region.

Certain dynamical models do indeed lead to trajectory asymptotes above zero,[8] but the observed behavior of pp elastic scattering at large momentum transfers[9] makes the existence of such limits seem unnatural. In any event, let us proceed here on the assumption that at least the P' trajectory crosses $J=0$ at a finite negative value of squared energy. The assumption of a roughly linear trajectory behavior and the association of the $J=0$ intersection of the P' with the f'(1500 MeV) particle[2] leads to an estimate of the square of the "ghost mass" as $-1.2\ \mathrm{GeV}^2$. Were we instead to make the association with the f(1250 MeV) particle, the estimate would be $-0.8\ \mathrm{GeV}^2$, not significantly different for the purposes at hand. We leave open the question of where (if at all) the P trajectory cuts $J=0$; a single ghost is sufficient for the argument here.

Let us now recall Mandelstam's generalization of the Levinson theorem, which equates the number of bound states to the negative change (in units of π) between threshold and infinity in the sum of the eigenphase shifts.[10] In other words, bound states tend to produce decreasing phase shifts. Since there is an infinite number of eigenphase shifts, however, one must make a supplementary assumption in order to achieve useful conclusions. We believe that the following assumption, based both on models and on experience,[11] is reasonable: If the bound state closest to the lowest threshold is substantially coupled to the lowest channel, then the first eigenphase shift is likely to decrease by an appreciable fraction of π when the energy is increased above threshold by an amount comparable to the binding energy —provided that no resonances occur in this interval. This is no more, really, than the familiar assertion that the pole nearest to a particular region tends to dominate that region. Thus, if the P' ghost were a true bound state, it would be quite natural to have the leading $I=0$, $J=0$ eigenphase shift decrease by a large amount in going from the $\pi\pi$ threshold to an energy squared $m_\rho^2 \approx 0.6\ \mathrm{GeV}^2$. Since only the $\pi\pi$ channel is appreciably open in this interval, there is no practical distinction between the $\pi\pi$ phase shift and the leading eigenphase shift.

It is necessary, of course, to remember that in fact there is no pole at the ghost mass, and in this connection one must examine both how the Levinson theorem is proved and what, precisely, is meant by a ghost. To begin, one must realize that Gell-Mann's conjecture[12] —explaining the absence of $J=0$ particles on the Pomeranchuk trajectory through a dynamical dominance of high-spin channels unable to possess zero total angular momentum—does not correspond to the existence of ghosts in the sense employed here. The Gell-Mann phenomenon occurs already in nonrelativistic potential scattering of spinning particles and is unrelated to the peculiarly relativistic requirements of crossing. It would lead to "ordinary" behavior for the $I=0$, $J=0$ $\pi\pi$ phase shift, corresponding to the absence in low-spin channels of forces sufficiently attractive to produce bound states. On the other hand, estimates of the forces in such channels show them to be more attractive than in the $I=1$, $J=1$ $\pi\pi$ configuration where the ρ appears prominently.[6] Dynamically speaking, therefore, it would be unsurprising to find one or more low-lying $I=0$, $J=0$ states that communicate strongly with the $\pi\pi$ channel. The Regge-trajectory arguments (already explained) suggest that the binding is so powerful at least one of these states will appear at negative squared energy.

Simple bootstrap models based on zero-spin channels bear out the foregoing picture; when the N/D approach is employed in an approximation that yields a ρ of correct mass, zeros are found at negative squared energy in the determinant of the $I=0$, $J=0$ D matrix. At the same time it must be admitted that these models are defective in failing to guarantee unitarity in crossed reactions—a constraint that will not tolerate a pole inside the latter physical regions. Crossed unitarity, in other words, requires that if the determinant of the D matrix vanishes at negative squared energy then every element of the N matrix must vanish at this same energy. Although such a requirement is not met by existing models, it must prevail even if a large share of the net attraction occurs in zero-spin channels like $\pi\pi$. (Note that in the Gell-Mann mechanism the determinant of the physical $J=0$ D matrix does not vanish. It is only the determinant of the larger dimensional D matrix, including the high-spin channels for which $J=0$ is physically impossible, that vanishes for Gell-Mann.)

It can be proved for nonrelativistic potential scattering that (for physical J) elements of the N matrix are incapable of all vanishing simultaneously at a zero of det$D$.[13] No relativistic proof of such a theorem has appeared, however; on the contrary, we have argued that if det$D$ actually has a zero at negative squared energy, all elements of N must vanish at the same point. Hopefully, when models are constructed which are unitary in both direct and crossed reactions, one will discover the mechanism to produce this apparent miracle.[14]

Having established what we mean by a ghost, let us proceed to the Levinson theorem. The most general proofs of the Levinson theorem employ the matrix N/D decomposition of the partial-wave amplitude, assuming that neither N nor D has poles and associating bound states with zeros on the physical sheet of the determinant of the D matrix. The theorem, on the other hand, makes no reference to zeros of N; it relates the net decrease in the phase shifts directly to the zeros of det$D$.[10] Thus Regge ghosts are to be counted exactly like physical bound states in using the Levinson theorem.

If the $I=0$, $J=0$ $\pi\pi$ phase shift turns out after all to be an increasing function (as energy increases) at low energies, the existence of Regge ghosts is not disproved. The explanation could be that there exists one or more low-energy $\pi\pi$ resonances in addition to the ghosts. If, however, the $\pi\pi$ phase shift decreases by a large amount, it will be hard to find a more natural explanation than a ghost. (The forces, remember, are strongly attractive.)

We mention, in closing, one argument beyond that of our opening paragraph in favor of a decreasing $I=0$, $J=0$ phase shift. The latter would mean a negative threshold value of the $I=0$, $J=0$ amplitude and thus presumably a negative value at the energy $(\frac{4}{3})^{1/2}m_\pi$, slightly below threshold. At this latter point, crossing symmetry dictates that the $I=2$ and $I=0$ amplitudes have the same sign (standing in the ratio 2:5),[15] and a negative sign for the $I=2$, $J=0$ amplitude has long seemed likely. The theoretical reason is that the $I=2$ forces seem predominantly repulsive, while experimental support for a negative sign has been given by the direction of the asymmetry in the decay of the charged ρ produced in peripheral collisions.[16]

We are aware that a number of indirect methods have been used in attempts to deduce the $\pi\pi$ $I=0$ scattering length, and that positive values for this parameter have been inferred more often than negative.[17] Most such methods have tacitly assumed an absence of ghosts, however, and would have to be re-examined if ghosts exist.[18] We urge that experimenters give the highest priority to this matter of sign in experiments such as $K\to e+\nu+2\pi$,[19] where an unambiguous conclusion is possible. A negative $I=0$ scattering length would constitute a discovery of major import.

I am indebted to Dr. David Atkinson for discussion of a number of the points considered here.

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[2]A. Rosenfeld, University of California Lawrence Radiation Laboratory Report No. UCRL-16462, 1965 (unpublished).

[3]A linear dependence on momentum over the entire interval would imply a scattering length $|a_0| \approx 0.6 m_\pi^{-1}$.

[4]E. P. Wigner, Phys. Rev. 98, 145 (1955).

[5]For a fairly complete recent survey of the experimental situation, see R. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

[6]G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. 126, 1202 (1962); D. Neville, Phys. Rev. Letters 13, 118 (1964); N. Masuda, Progr. Theoret. Phys. (Kyoto) 33, 864 (1965).

[7]The first estimate of the P' zero-energy intercept was by K. Igi, Phys. Rev. 130, 820 (1963). Igi found $J \approx 0.5$, but this number has been revised upward (on the basis of more recent data) by Joseph Scanio, Lawrence Radiation Laboratory (private communication).

[8]G. F. Chew and C. E. Jones, Phys. Rev. 135, B208 (1964); D. B. Teplitz and V. L. Teplitz, Phys. Rev. 137, B142 (1965).

[9]G. Cocconi et al., Phys. Rev. 138, B165 (1965).

[10]S. Mandelstam, in Proceedings of the 12th Solvay Congress (Interscience Publishers, Inc., New York, 1961), p. 247. See also S. Frautschi, Regge Poles and S-Matrix Theory (W. A. Benjamin, Inc., New York, 1963), p. 34. I am indebted to David Gross for an advance look at the manuscript of his paper concerned with the multichannel N/D derivation of the Levinson theorem.

[11]A good example is the triplet S state of the np channel, which communicates with the deuteron.

[12]M. Gell-Mann, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 539.

[13]I am indebted to R. Sawyer for this observation.

[14]Conceivably there will turn out to be a relation between this mechanism and the long-range repulsion

from P and P' exchange [G. F. Chew, Phys. Rev. 140, B1427 (1965)] that tends to reduce the numerator function near the zeros of D.

[15]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[16]D. Carmony and R. Van de Walle, Phys. Rev. Letters 8, 73 (1962). See also Ref. 1.

[17]For example, see J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. 128, 1881 (1962).

[18]Phenomenological "effective-range" formulas for the $I=0$ phase shift are employed which are usually not sufficiently flexible to represent ghosts. Recently a negative $I=0$ scattering length was inferred by H. Rothe, Phys. Rev. 140, B1421 (1965), by a method that tolerates ghosts.

[19]R. Birge et al., Phys. Rev. 139, B1600 (1965).

REMARK CONCERNING THE BASIC SU(3) TRIPLETS

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The replacement of the quarks or aces, proposed by Gell-Mann and Zweig as the basic triplets for the SU(3) representation of baryons, by two different, integer-charge triplets, has been discussed by Van Hove and others.[1] As pointed out by him, this implies the appearance of a new quantum number in addition to spin, isospin, hypercharge, and baryon number in the description of baryons, which in his representation of the triplets appears in an additional term to the Gell-Mann–Nishijima charge formula, thereby motivating the name supercharge.

The first triplet T, in the notation of Van Hove, is obtained by adding $\frac{1}{3}$ to the charges of the three quark states and $\frac{2}{3}$ to the baryon number, while the other triplet $\overline{\theta}$ is obtained by subtracting $\frac{2}{3}$ from the charges and $\frac{4}{3}$ from the baryon number, thereby leaving the isospin and hypercharge assignment unchanged. Thus the charge formula is written as

$$Q=I_3+\frac{Y}{2}+\frac{D}{3}, \tag{1}$$

where I_3 and Y are the third isospin component and the hypercharge, respectively, while D has the value +1 for T and -2 for $\overline{\theta}$.

Now the first triplet may just as well be described by the same assignment of quantum numbers as was used by Sakata in his pioneer work,[2] where he tried to regard the three baryons p, n, and Λ as a basic triplet for all baryons and mesons. This means adding $\frac{2}{3}$ to the hypercharge of the quark, thereby removing the term $D/3$ from (1). In a similar way the triplet $\overline{\theta}$ is related to the triplet of leptons (ν, e^-, μ^-) whose analogy with the Sakata triplet was emphasized by Gamba, Marshak, and Okubo,[3] and myself.[4] It may be described by subtracting $\frac{4}{3}$ from the quark hypercharge, which again makes the term $D/3$ disappear. This is identical with the description proposed by Marshak et al., namely, to write the charge formula as

$$Q=(N_B-N_L+S)/2+I_3, \tag{2}$$

where N_B is the ordinary baryon number and N_L the corresponding lepton number, +1 for a lepton and -1 for an antilepton. Introducing a fermion number N, which is +1 for a baryon and -1 for a lepton, (2) takes the usual Gell-Mann–Nishijima form. Thereby S is the same for both triplets and the quark (defined as $Y-N$, i.e., $-\frac{1}{3}$ for the latter), namely $0, 0, -1$ in the order p, n, Λ, etc. If, instead of maintaining the Y assignment of the quark, the S assignment had been used, the quantum numbers of the two triplets T and $\overline{\theta}$ would automatically have been described in the same way as the Sakata and the lepton triplet, respectively, thereby making the D term superfluous. However, the new quantum number remains, as has been strongly emphasized by Van Hove.[5] As is easily seen, it is now replaced by the lepton number N_L. In fact, in a compound consisting of n_1 triplets T, $\bar{n}_1$ triplets $\overline{T}$, n_2 triplets $\overline{\theta}$, and $\bar{n}_2$ triplets θ, we have

$$N_B=n_1-\bar{n}_1, \quad N_L=n_2-\bar{n}_2,$$

$$D=n_1-\bar{n}_1-2(n_2-\bar{n}_2). \tag{3}$$

Hence,

$$D=N_B-2, \quad N_L=N-N_L,$$