REGGE-POLE ANALYSIS OF $\pi^+ + n \rightarrow \omega + p$*

M. Barmawi†

The Enrico Fermi Institute for Nuclear Studies and The Department of Physics,
The University of Chicago, Chicago, Illinois

(Received 22 December 1965)

Recent measurements[1] of the decay density matrices for ω mesons produced in the reaction $\pi^+ + n \rightarrow \omega + p$ show that the matrix element ρ_{00} is significantly different from zero. Simple Regge-pole models with ρ exchange alone give $\rho_{00} \equiv 0$, and therefore they cannot explain these experiments, even though they may give reasonable results for most other aspects of vector-meson production.[2] Therefore, these experiments indicate that the exchange of many particles, or of specific other resonances, is not negligible.[3] It is the purpose of this note to show that a Regge-pole model can well explain all aspects of ω production, provided an axial-vector meson trajectory is included, which gives rise to a resonance with $J^{PG} = 1^{++}$, C_n^-. This resonance corresponds to the proposed B meson.[4]

We note that the ω production can also be described by the phenomenological absorption model.[5] However, it is of great interest to find out whether the Regge-pole theory[6] can explain inelastic reactions in spite of its difficulties in the case of elastic amplitudes, where the crossed channel has the quantum numbers of the vacuum. The results of this paper, together with the related calculation for πp charge-exchange scattering,[7] seem to be very encouraging, at least as far as the lower energies which have been considered so far are concerned.

The expression for the Regge-pole contribution to the helicity amplitude is given by[2,8]

$$\langle \lambda_v | F_R(s,t) | \lambda, \bar{\lambda}_2 \rangle = -(2\alpha+1)\pi\alpha'(0) \frac{e^{-i\pi\beta(s)} \pm 1}{2\sin\pi\beta(s)} \times c_{\lambda_v,\lambda} \left(\frac{4p_s k_s}{2M(m_\pi m_\omega)^{1/2}} \right)^{\beta(s)} d_{\lambda_v\lambda}^{\alpha}(z), \tag{1}$$

where $\lambda = \lambda_1 - \lambda_2$; $\beta(s) = \alpha(s) - \max(|\lambda|, |\lambda_v|)$, p_S and k_S = initial and final momenta, and z = cosine of the angle between the momenta in the c.m. system of the channel $p + \bar{n} \rightarrow \pi^+ + \omega$.

In our model, we assume that the residues of the Regge poles in the l plane are constants and that the trajectories are approximately parallel straight lines[2,7] with the slope $\alpha' = 0.64$ $(\text{GeV})^{-2}$ for all trajectories and $\alpha_\rho(0) = 0.621$, $\alpha_B(0) = 0.047$ $[\alpha_\rho(m_\rho^2) = 1,\ \alpha_B(m_B^2) = 1]$. This value of α' has been chosen to give reasonable results for $\pi^- + p \rightarrow \pi^0 + n$ and for vector-meson production.[2] The residues are included in the kinematical factors $c_{\lambda_v\lambda}$ as given in Table I and they are determined using the fact that Eq. (1) reduces to the helicity components of the Born term for $s \rightarrow m_{\text{ex}}^2$ (m_{ex} = mass of the exchanged particle). In this way the selection rules are taken into account in the determination of coefficients $c_{\lambda_v\lambda}$.

From Eq. (1), the momentum-transfer distribution and the decay density matrices can be calculated using the expressions[9]

$$\frac{d\sigma}{ds} = \frac{M^2}{16\pi p_t t} \frac{1}{2} \sum_{\lambda_v \lambda_1 \bar{\lambda}_2} |\langle \lambda_v | F_R(s,t) | \lambda_1, \bar{\lambda}_2 \rangle|^2, \tag{2}$$

$$\rho_{mm'} = N_0 \sum_{\lambda_1 \bar{\lambda}_2} \langle m | F_R(s,t) | \lambda_1 \bar{\lambda}_2 \rangle \times \langle \lambda_1 \bar{\lambda}_2 | F_R(s,t) | m' \rangle, \tag{3}$$

where N_0 is a normalization constant.

In order to determine $c_{\lambda_v\lambda}$, we have to consider the coupling of the B meson with the nucleon and with the mesons ω and π. The general form for the axial-vector current produced by the nucleons is given by

$$j_\mu^A = -i\bar{\psi}[\gamma_5 q_\mu g_1 + \gamma_5 \gamma_\mu g_2 + \gamma_5 \sigma_{\mu\nu} q_\nu g_3]\psi. \tag{4}$$

Table I. Kinematical factors for B exchange. For the corresponding table for ρ exchange, see Ref. 2.

λ_v λ	0	+1
+1	$+g_3\left(\frac{1}{m_0}\right)\frac{p_s S^{1/2}}{3M}$	0
0	$-\frac{1}{3M} g_3 \left(\frac{1}{m_0}\right)\left[a p_s S^{1/2} \frac{k_{0s}(\omega)}{m_\omega} + \frac{b}{m_\omega^2 m_B} k_s^2 S\right]$	0
−1	$-g_3 \frac{1}{m_0} \frac{p_s S^{1/2}}{3M}$	0

The G parity of the first two terms is negative, while the G parity of the last term is positive. From G parity conservation, it follows that the coupling between B mesons and nucleons is given by

$$-ig_3\bar{\psi}\gamma_5\sigma_{\mu\nu}q_\nu\vec{t}\psi\cdot\vec{B}_\mu .$$

The $B\omega\pi$ coupling is of the form[10]

$$(1/m_0)\varphi B_\mu\omega_\nu[a\delta_{\mu\nu}+(b/m_\omega m_B)k_\mu(\omega)k_\nu(B)],$$

where the first term corresponds to s-wave and the second one to the d-wave coupling.

Once the ratio b/a is fixed, the dimensionless coupling constant a can be determined from the decay width and the mass of the B meson, which are 125 and 1220 MeV, respectively. The ρ coupling constants are taken to be $f_{\omega\rho\pi}{}^2/4\pi=10.5$,[11] and $g_V{}^2/4\pi=2.54$.[12] The ratio g_Tg_V can be obtained from the pole fit of the isovector form factors, and we use[2] $g_T/g_V=2.98$.

For the values $b/a=9.0$ and $g_3=5.8$, the momentum transfer distribution and the decay density matrices of ω at 3.25 GeV/c are calculated. The result for momentum-transfer distribution is shown in Fig. 1, where it is compared with the data.[13] The histogram is normalized to the calculated cross section for $s>-0.8$ (GeV)^2, which is 0.27 mb. The estimated total cross section is 0.2 mb.[5] The result of the calculation of the density matrices is shown in Fig. 2(a), where it is compared with the data of Ref. 1. In the same figure we plot $\rho_{1,-1}$ for pure ρ exchange ($\rho_{00}=0$ in this case). We observe that the effect of the B trajectory is to increase ρ_{00} and to decrease $\rho_{1,-1}$ in the right direction. The momentum-transfer distribution and the decay density matrices at 1.7 GeV/c are also calculated. The result of the calculation of the density matrix is shown in Fig. 2(b), where it is compared with the corresponding data.[14]

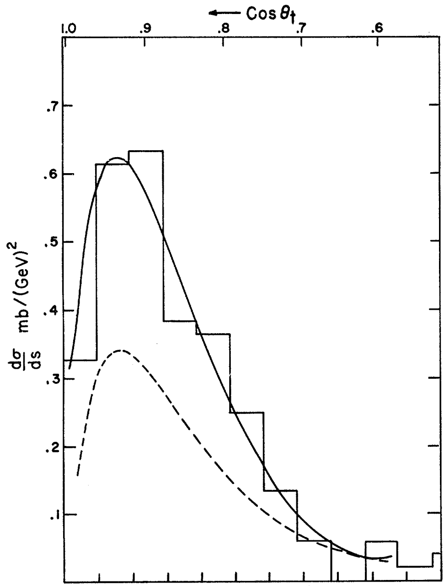

FIG. 1. Momentum-transfer distribution $d\sigma/ds$ at 3.25 GeV/c. The solid curve is the result of the calculation with $b/a=q$ and $g_3=5.8$. Dashed curve shows the momentum transfer distribution for pure ρ exchange. θ_t is the angle between initial and final momenta in the c.m. system of $\pi^++n\rightarrow\omega+p$.

The total cross section, estimated from the experimental data at 3.25 GeV/c, is ~0.2 mb. The calculated cross section, for $s>-0.8$ (GeV)^2, is ~0.27 mb (only 9% of the total events is outside this range). At 1.7 GeV/c for $\cos\theta>0.7$, the calculated cross section is 0.46 mb, while experimentally in the same interval it is 0.40 mb.[14] Here we observe the right trend of the variation of the cross section with the energy of the incident π^+.

In Fig. 2 we see that ρ_{00} has a zero near $s=-0.85$ (GeV)^2, which corresponds to $\alpha_B=-\frac{1}{2}$. This minimum is a characteristic of our simple model. The location of the minimum at $s=-0.85$ (GeV)^2 is a consequence of the assumption that the trajectory is a straight line.

For pure s-wave coupling at the meson vertex, it is found that ρ_{00} is large near the forward direction and decreases very rapidly [<0.1 at $s\sim-0.2$ (GeV)^2]. This is inconsistent with the data. Therefore pure s-wave coupling at the meson vertex seems to be unlikely. On the other hand, pure d-wave coupling remains consistent with the data in Fig. 1, except that the peak is shifted to $s=-0.12$ (GeV)^2. The elements ρ_{00} and $\rho_{1,-1}$ are the same within 10% as those for $b/a=9$ and $g_3=5.8$.

The decay density matrices are not very sensitive to the ratio b/a for $a/b<15\%$. On the other hand, if g_3 is increased, the momentum transfer distribution $d\sigma/ds$ and the function ρ_{00} increase and $\rho_{1,-1}$ decreases. We can also increase ρ_{00} by decreasing g_T/g_V; then $\rho_{1,-1}$ will remain about the same and $d\sigma/ds$ becomes slightly smaller. We have not attempted to make a best fit with the present data.

From the present analysis using the avail-

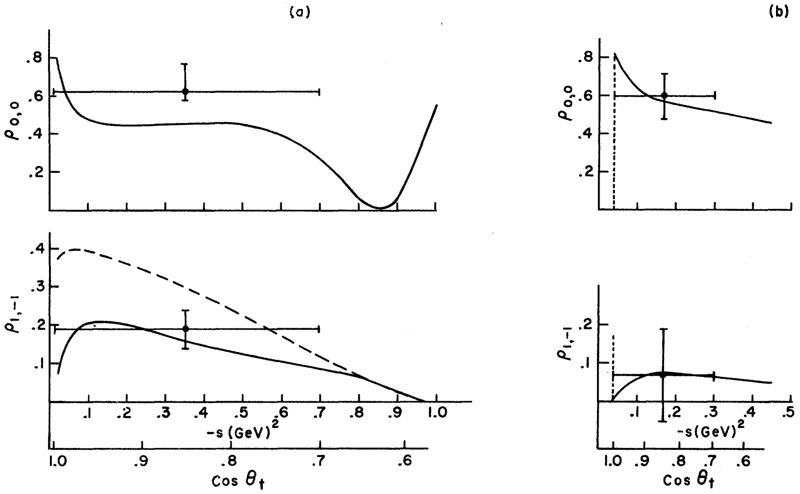

FIG. 2. Decay density matrices: (a) at 3.25 GeV/c, (b) at 1.7 GeV/c. Reρ_{10} is not shown. Our calculation yields Re$\rho_{10} \sim 0.004$ at 3.25 GeV/c and Re$\rho_{10} \sim 0.002$ at 1.7 GeV/c. The latter is consistent with the experimental value at 1.7 GeV/c.[14]

able data, we may conclude that, by including the B trajectory in our model, the production and decay of ω in $\pi^+ + n \to \omega + p$ can be described quite reasonably. Concerning the parameters of the B meson, we obtain the following general restrictions: (1) The B meson has an $\omega\pi$ decay mode with $a/b \leq 15\%$. (2) The coupling constant of the B meson to the nucleon is on the order of $g_3^2/4\pi \sim 2.9$.

The author wishes to express his gratitude to Professor Reinhard Oehme for the encouragement, stimulating guidance and discussions. He is also grateful to Professor Norman Gelfand for providing him the data and to Roland Köberle for interesting discussions.

---

*Work supported in part by the U. S. Atomic Energy Commission.

†Indonesian Fellow, on leave of absence from Bandung Institute of Technology, Bandung, Indonesia.

[1] H. O. Cohn, W. M. Bugg, G. T. Condo, Phys. Letters 15, 344 (1965).

[2] M. Barmawi, Phys. Rev. 142, 1088 (1966).

[3] The author is deeply indebted to Professor R. Oehme for emphasizing this point, which leads to the present approach.

[4] G. Goldhaber, S. Goldhaber, J. A. Kadyk, B. C. Shen, Phys. Rev. Letters 15, 118 (1965); G. Goldhaber in Proceedings of the Second Coral Gables Conference on Symmetry-Principles at High Energies, edited by B. Kurşonuğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman, San Francisco, California, 1965).

[5] J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. 139, B428 (1965).

[6] R. Oehme in Strong Interactions in High Energy Physics, edited by R. G. Moorhouse (Oliver Boyd, Edinburgh, Scotland, 1964), p. 129; this paper contains further references.

[7] R. K. Logan, Phys. Rev. Letters 14, 414 (1965).

[8] F. Calogero, S. M. Charap, and E. J. Squires, Ann. Phys. (N. Y.) 25, 325 (1964).

[9] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

[10] S. M. Berman and S. D. Drell, Phys. Rev. Letters 11, 220 (1963).

[11] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[12] A. Scotti and D. J. Wong in Nuclear Structure, Proceedings of the International Conference at Stanford University, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1964), p. 298.

[13] W. M. Bugg, H. O. Cohn, G. T. Condo, N. Gelfand, and G. Lütjens, private communication from N. Gelfand.

[14] T. C. Bacon, W. J. Fickinger, D. G. Hill, H. W. K. Hopkins, D. K. Robinson, and E. O. Salant, Bull. Am. Phys. Soc. 10, 66 (1965), as quoted in Ref. 5.