fects may be easily masked even by small sample strain.

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DEAD-TIME CORRECTIONS TO PHOTON COUNTING DISTRIBUTIONS

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Considerable interest is currently being focused on the study and measurement of photon counting distributions.[1-5] With most experimental techniques,[6] such a measured distribution is affected by the existence in the observing system of a dead time τ after each registration of the arrival of a photon, during which no subsequent photon arrival can be detected. We report here the first detailed observation and analysis of these dead-time effects on a photon counting distribution.

The modifications made to the actual counting probability distribution of photons arriving at a detector by dead-time effects in the observing system have been studied by De Lotto, Manfredi, and Principio.[7] Their results can be used to show that for a nonparalyzable system,[7] an arrival distribution of random Poisson form is modified in such a way that the probability of n counts being registered in a sampling time $T \gg \tau$ is given to second order in τ/T by

$$p(n,T) = (\bar{n}_0^{\,n}/n!)\exp(-\bar{n}_0)\{1 + n(\bar{n}_0 - n + 1)\tau/T - [(\bar{n}_0+1)n - (\bar{n}_0^2 + 2\bar{n}_0 + 3)n^2 + (2\bar{n}_0+3)n^3 - n^4]\tau^2/2T^2\}, \qquad (1)$$

where $\bar{n}_0$ is the mean number of photons arriving in a period T, reduced by the quantum efficiency of the detector. Associated with this modified distribution is the experimentally convenient, derived quantity[1]

$$F(n) = (n+1)p(n+1,T)/p(n,T) = \bar{n}_0\{1 + \bar{n}_0\tau/T + \bar{n}_0(\bar{n}_0 - 1)\tau^2/2T^2] - n[1 + (4\bar{n}_0 - 1)\tau/4T]2\tau/T + n^2\tau^2/2T^2\}, \qquad (2)$$

which to first order in τ/T is seen to be linear with slope $-2\bar{n}_0\tau/T$. This negative slope is to be expected insofar as the existence of a dead time has the opposite effect on a Poisson distribution to photon degeneracy effects which cause photon bunching[8] and lead in first order to a positive slope for $F(n)$.[9]

Light from a tungsten lamp has completely negligible degeneracy and can be regarded as forming a beam of photons whose arrival probability distribution at a detector is Poisson. The counting distribution obtained from such a beam was measured using a photomultiplier

Table I. The observed counting distribution with measured and corrected values of $p(n,T)$.

n	No. of counts	Measured $p(n,T)$	Corrected $p(n,T)$
0	38 948 002	0.266 072[a]	0.266 145[a]
1	52 861 890	0.361 124	0.352 300
2	34 517 405	0.235 805	0.233 172
3	14 455 237	0.098 750	0.102 984
4	4 364 942	0.029 819	0.034 167
5	1 013 884	0.006 926	0.009 096
6	187 312	0.001 280	0.002 012
7	28 646	0.000 196	0.000 385
8	3672	0.000 025	0.000 065
9	398	0.000 003	0.000 010
10	25	0.000 000	0.000 001
11	2	0.000 000	0.000 000

[a]The measured and corrected values of $p(O,T)$ do not differ due to any dead-time effects. The difference arises from the fact that the measured value is obtained from the number of counts over a finite range of n and the corrected value is defined to be $\exp(-\bar{n}_0)$.

and the recording system previously described.[1] The sampling time used was $T=10^{-6}$ sec and the nominal dead time was $\tau=2\times10^{-8}$ sec. The observed distribution given in Table I is based on 1.46×10^8 samples with a mean count per sample of 1.292, and this is sufficient to determine $p(n,T)$ with an accuracy of 0.02% for $n=0$, decreasing to 2% for $n=8$. The experimentally determined values of $F(n)$ over this range are shown in Fig. 1; they are seen to define a curve which is essentially linear with a slope of roughly $-2\bar{n}_0\tau/T=-0.052$.

Although the photomultiplier tube was run under conditions designed to minimize afterpulsing effects,[10] small residual effects were undoubtedly still present. Since the values of $F(0)$ and $F(1)$ would be least affected by these residual effects, they were taken to define the dependence of $F(n)$ on n. The values $\bar{n}_0=1.324$ and $\tau/T=0.019\,08$ were determined by fitting expression (2) to the values of $F(0)$ and $F(1)$. These values were then used to correct the observed distribution for dead-time effects. The corrected distribution is given in Table I and the corrected values of $F(n)$ plotted in Fig. 1 from which it is clear that the major deviation of the counting distribution from its true Poisson form, characterized by the horizontal straight line, has been removed. The remaining small deviations are attributed to after-pulsing effects which need only be present to the extent of about 1 part in 10^4 to produce deviations of this magnitude.

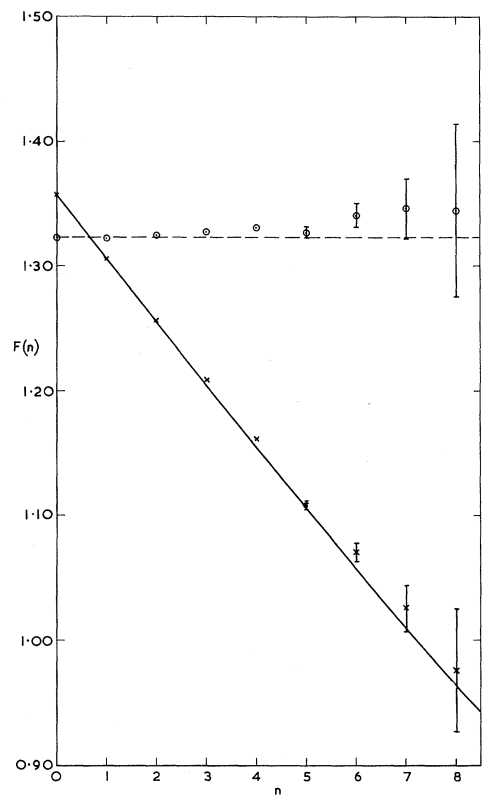

FIG. 1. The experimentally measured (crosses) and dead-time corrected (circled dots) values of $F(n)$ with errors shown when appreciable. The dashed line represents a Poisson distribution for a mean of 1.324 counts per microsecond; the full line represents this same distribution modified by the effects of a dead time of 19.08 nsec.

The characters of the measured and corrected distributions are also exhibited in Table II where their factorial moments[11] of all orders up to the seventh are compared with those of Poisson distributions with means of 1.292 and 1.324, respectively. The large deviation of the measured distribution from the Poisson form is evident, as is the success with which the deviation can be removed by interpreting it as arising from dead-time effects.

Dead-time corrections are readily made to any distribution measured with a sampling time much shorter than the inverse linewidth of the radiation. For then, in the absence of dead-time effects, we have[8]

$$p(n,t)=\int_0^\infty \frac{(rT)^n}{n!}e^{-rT}p(r)dr, \tag{3}$$

where $p(r)$ is the probability that, at the time of sampling, counts are registered at a (constant) mean rate r. With a finite dead time in the observing system each component Poisson $[(rT)^n/n!]\exp(-rT)$ in (3) is modified as is given in (1) to second order in τ/T, so that

$$p(n,T)=\int_0^\infty \frac{(rT)^n}{n!} \times e^{-rT}\{1+n(rT-n+1)\tau/T-[(rT+1)n-(r^2T^2+2rT+3)n^2+(2rT+3)n^3-n^4]\tau^2/2T^2\}p(r)dr. \tag{4}$$

For Gaussian light $p(r)=r_0^{-1}\exp(-r/r_0)$, r_0 being the mean counting rate in the absence of dead-time effects, and (4) gives

$$p(n,T)=\frac{\bar{n}_0^{\,n}}{(1+\bar{n}_0)^{n+1}}\left\{1-n\left[(n-1)-(n+1)\frac{\bar{n}_0}{1+\bar{n}_0}\right]\tau/T +n\left[(n-1)^3-(2n^2-2n+1)(n+1)\frac{\bar{n}_0}{1+\bar{n}_0}+n(n+1)(n+2)\left(\frac{\bar{n}_0}{1+\bar{n}_0}\right)^2\right]\tau^2/2T^2\right\}, \tag{5}$$

where $\bar{n}_0=r_0T$. This expression has been evaluated with parameters characteristic of the system reported by Freed and Haus,[3] viz., $\bar{n}_0=14$, $T=10^{-5}$ sec, $\tau=1.4\times10^{-8}$ sec. The result is shown in Fig. 2 along with the corresponding curve for zero dead time on which they essentially base their comparison between theory and experiment.[12] It is clear that there are major differences between the two curves amounting to as much as a factor of 2 at $n=100$. Unfortunately, the Freed and Haus data are not accurate enough to resolve between the two

Table II. The kth factorial moments of the measured and corrected distributions relative to those of Poisson distributions with means of 1.292 and 1.324, respectively.

k	Measured distribution	Corrected distribution
1	1.000	1.001
2	0.963	1.003
3	0.894	1.006
4	0.797	1.011
5	0.683	1.015
6	0.560	1.014
7	0.435	0.987

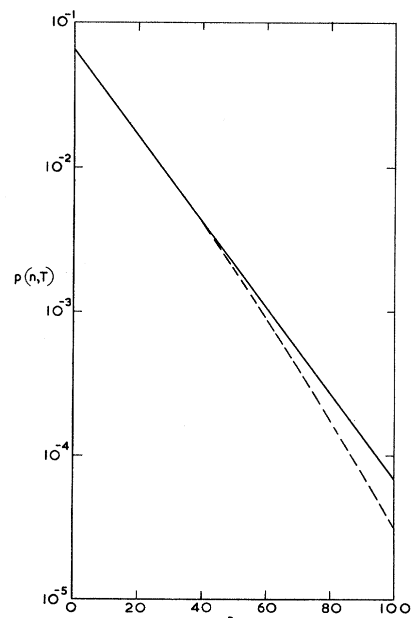

FIG. 2. The distributions $\bar{n}_0^{\,n}(1+\bar{n}_0)^{-n-1}$ with $\bar{n}_0=14$ (full line) and the same distribution corrected according to (3) for a dead time of 14 nsec and sampling time of 10 μsec (dashed line).

lines which are equally good fits to their experimental points.

Although dead-time effects modify quantities like the mean and variance of a counting distribution by small amounts of the order of $\bar{n}_0\tau/T$ with $\tau \ll T$, their effect on the distribution as a whole can be quite profound. Loosely speaking, the modification to the nth point in the distribution depends not simply on the value of τ/T but on that of $n\tau/T$. When detailed features of the distribution are being investigated, it is therefore essential to correct for dead-time effects.

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MEASUREMENTS OF NEUTRON FORM FACTORS*

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We have investigated the electromagnetic neutron form factors at momentum transfers of 5.5 F^{-2}, 10.0 F^{-2}, and 14.5 F^{-2} by measuring the differential cross sections for quasi-elastically scattered electrons by deuterium in coincidence with a scattered nucleon in the reaction $e+d\to e+p+n$.

Measurements were made of the ratio of the differential cross section $d^3\sigma/d\Omega_n d\Omega_e dE_e$ to the differential cross section $d^3\sigma/d\Omega_p d\Omega_e dE_e$ for two different electron scattering angles for each value of the momentum transfer q^2.

Durand[1] has shown that this ratio of differential cross sections from deuterium is nearly equal to the ratio of the differential cross sections for free electron-neutron and electron-proton scattering. This result is relatively insensitive to assumptions made about the wave function of the deuteron. An earlier experiment[2] of this type for $q^2 = 4.9$ F^{-2} has been reported.

The experiment was done using the internal electron beam of the Cornell electron synchrotron. In the $q^2 = 10.0$ F^{-2} experiment, the beam was incident upon a thin deuterated-polyethylene target. In the $q^2 = 5.5$ F^{-2} and the $q^2 = 14.5$ F^{-2} experiments, a liquid deuterium target was used. This target had 0.25-mil aluminum walls and was cooled with liquid helium.

The scattered electrons were analyzed in a vertically focusing quadrupole magnet. The electrons were detected by a momentum-defining scintillation counter in coincidence with a second scintillation counter and a totally absorbing lead glass Cherenkov counter.

The scattered nucleons were detected in coincidence with the scattered electron. The neutrons were detected by their interactions in a plastic scintillation counter 24 in. in diameter and 18 in. thick. Three to five inches of lead were placed between the neutron counter and the target. In all cases the neutron counter was centered around the direction corresponding to scattering from a free neutron and it subtended a cone of half-angle 3.8°, 3.7°, and 5.3°