ULTRASONIC KJELDAAS-EDGE EXPERIMENT IN POTASSIUM*

R. L. Thomas
Department of Physics and Department of Metallurgical Engineering, Wayne State University, Detroit, Michigan

and

H. V. Bohm
Department of Physics, Wayne State University, Detroit, Michigan
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This paper reports the results of an ultrasonic shear-wave experiment proposed by Alig, Quinn, and Rodriguez,[1] to distinguish between a "quasi-free" electron Fermi surface and a surface modified from this configuration by spin-density wave states (SDW) in accordance with Overhauser's[2] calculations that the electronic ground states of potassium, rubidium, and cesium may be SDW states. With a magnetic field aligned parallel to the ultrasound propagation direction (which configuration meets Overhauser's suggestion[3] that the SDW in potassium may orient itself parallel to a sufficiently strong dc magnetic field), one expects to observe the Kjeldaas absorption edge.[4] For the free-electron model of potassium, the absorption-edge field, B_e, is given by

$$B_e = qc\hbar k_F/e, \tag{1}$$

where $k_F = (3\pi^2 n)^{1/3}$, is the radius of curvature of the Fermi sphere, with n being the electron concentration. The ultrasonic wave number, q, is given by $q = 2\pi/\lambda = \omega/v_s$, where λ is the sound wavelength, ω the sound angular frequency, and v_s the (shear) sound velocity. It is important to note that for the free-electron case, B_e is independent of the effective mass of the electron. Implicit in Eq. (1) is the condition $ql \gg 1$ where l is the electron mean free path. In our experiments, the value of ql was always less than 25, and hence a detailed solution of the equation of motion of the positive ions within the metal in the presence of a magnetic field was necessary in order to predict B_e theoretically. Alig, Quinn, and Rodriguez[5] have extended the previous calculations[1] to values of ql appropriate to our experiments in a single-crystal potassium sample.

The experimental values of B_e reported here for various values of q are consistent with theoretically calculated free-electron values rather than the SDW values. Measurements were made of the relative ultrasonic attenuation coefficient as a function of magnetic field for both fast and slow shear modes propagated along the ⟨110⟩ axis. The results are presented in Table I. Both Alig, Quinn, and Rodriguez[5] and the present authors have used shear sound velocities given by Marquardt and Trivisonno,[6] 1.78×10^5 cm/sec and 0.646×10^5 cm/sec for fast and slow shear modes, respectively.[7]

The sample,[8] approximately 5 mm thick for the initial runs, and approximately 3.5 mm thick for the higher frequency runs, was prepared with flat and parallel faces in the (110) plane. Using criteria found elsewhere,[9,10] transverse magnetoacoustic runs on this sample yield a value of ql of approximately 10 at 50 Mc/sec. Standard ultrasonic pulse techniques in the frequency range 10-110 Mc/sec at 4.2°K were employed, using AC-cut, 10-Mc/sec fundamental frequency, $\frac{1}{4}$ in. diam quartz transducers. Both pulse-echo and transmission measurements were made, the latter at higher frequen-

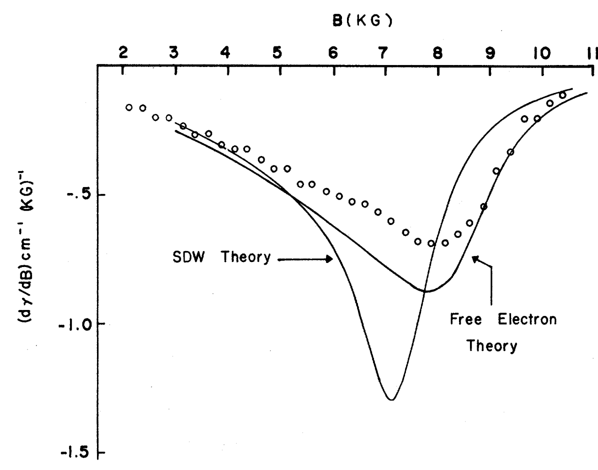

FIG. 1. The derivative of γ, the ultrasonic (power) attenuation coefficient, with respect to B versus B, the magnetic induction (measured in kilogauss). The solid curves represent free-electron and SDW calculations for $\omega/2\pi = 50$ Mc/sec and $ql = 10$. The experimental points, plotted in the same units, were measured with $\omega/2\pi = 50.6$ Mc/sec and $ql \approx 8$-10.

Table I. Kjeldaas-edge positions.

Experimental			Theoretical[a]			
$\omega/2\pi$ (Mc/sec)	Approximate experimental ql	B_e (kG)	$\omega/2\pi$ (Mc/sec)	Assumed ql	Free-electron model B_e (kG)	SDW B_e (kG)
10	1.5-2	1.4	10	2	1.2	1.17
10	1.5-2	1.4				
10[b]	4.5-5.5	4.1	10[b]	5.52	4.2	3.9
29.3	5-6	4.4	30	6	4.6	4.25
29.3	5-6	4.3				
49.7	8-10	7.6	50	10	7.85	7.1
50.6	8-10	7.8				
50.6	8-10	7.5				
49.7	8-10	7.5				
49.7	8-10	7.7				
70.6	11-14	11.2	70	14	11.2	10.0
70.6	11-14	11.4				
70.6	11-14	11.3				
90.3	14-18	14.6	90	18	14.6	12.8
110.7	18-22	18.5	110	22	18.0	15.75
109.7	18-22	18.2				
110.7	18-22	18.4				
110.5	18-22	18.2				
110.5	18-22	18.4				

[a]Calculations of Alig, Quinn, and Rodriguez, Ref. 5.
[b]Slow shear mode.

cies. Relative attenuation was recorded continuously as a function of magnetic field, and was calibrated stepwise in relative dB/cm with a precision attenuator and a signal generator operating at the same frequency and subjected to the same electronic circuitry.

Figure 1 shows the results of a 50-Mc/sec run (the run having the best signal-to-noise ratio of these data) in differentiated form, together with the corresponding SDW and free-electron theoretical curves. The experimental curve was obtained numerically from the γ-vs-B curve after adjustment for the receiver calibration. An important feature of the curve shown is its pronounced asymmetric shape; the magnitude of the maximum (negative) $d\gamma/dB$ is somewhat less than that predicted by the free-electron model and considerably different from the SDW prediction. It should be noted that the latter observation holds for the data taken at other, particularly higher, frequencies. This suggests that our approximate experimental evaluation of sample purity (i.e., ql) may be somewhat high. Making a downward correction in l, however, would only have the effect of displacing the calculated free-electron peak in Fig. 1 closer to the SDW curve (which is comparatively insensitive to l).

The over-all experimental uncertainty of these measurements is estimated to be less than 5%. Sample misorientation from the ⟨110⟩ axis, certainly less than 4° in these experiments, contributes corrections[11] totaling at most 2%. Faraday rotation effects[4] are negligible, since the shear-wave velocity degeneracy is removed by propagating along the ⟨110⟩ direction.

There appears to be no evidence in these measurements to show the existence of an SDW in potassium along ⟨110⟩. It should be recognized that the magnetic fields which were employed may not be sufficiently strong to reorient an SDW from a preferred crystallographic direction, possibly the ⟨123⟩ direction, along which the helicon-magnetoresistance measurements of Penz[12] show a marked difference from other crystallographic directions. From this and other experiments[13,14] it is clear that further ultrasonic work at higher frequencies and in other crystallographic directions, particularly the ⟨123⟩ direction, is indicated. Although we were careful to avoid introducing strains into our sample, it must be noted that SDW effects may be easily masked even by small sample strain.

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DEAD-TIME CORRECTIONS TO PHOTON COUNTING DISTRIBUTIONS

F. A. Johnson, R. Jones, T. P. McLean, and E. R. Pike

Royal Radar Establishment, Malvern, England

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Considerable interest is currently being focused on the study and measurement of photon counting distributions.[1-5] With most experimental techniques,[6] such a measured distribution is affected by the existence in the observing system of a dead time τ after each registration of the arrival of a photon, during which no subsequent photon arrival can be detected. We report here the first detailed observation and analysis of these dead-time effects on a photon counting distribution.

The modifications made to the actual counting probability distribution of photons arriving at a detector by dead-time effects in the observing system have been studied by De Lotto, Manfredi, and Principio.[7] Their results can be used to show that for a nonparalyzable system,[7] an arrival distribution of random Poisson form is modified in such a way that the probability of n counts being registered in a sampling time $T \gg \tau$ is given to second order in τ/T by

$$p(n,T) = (\bar{n}_0^{\,n}/n!)\exp(-\bar{n}_0)\{1+n(\bar{n}_0-n+1)\tau/T-[(\bar{n}_0+1)n-(\bar{n}_0^{\,2}+2\bar{n}_0+3)n^2+(2\bar{n}_0+3)n^3-n^4]\tau^2/2T^2\}, \quad (1)$$

where $\bar{n}_0$ is the mean number of photons arriving in a period T, reduced by the quantum efficiency of the detector. Associated with this modified distribution is the experimentally convenient, derived quantity[1]

$$F(n) = (n+1)p(n+1,T)/p(n,T) = \bar{n}_0\{1+\bar{n}_0\tau/T+\bar{n}_0(\bar{n}_0-1)\tau^2/2T^2]-n[1+(4\bar{n}_0-1)\tau/4T]2\tau/T+n^2\tau^2/2T^2\}, \quad (2)$$

which to first order in τ/T is seen to be linear with slope $-2\bar{n}_0\tau/T$. This negative slope is to be expected insofar as the existence of a dead time has the opposite effect on a Poisson distribution to photon degeneracy effects which cause photon bunching[8] and lead in first order to a positive slope for $F(n)$.[9]

Light from a tungsten lamp has completely negligible degeneracy and can be regarded as forming a beam of photons whose arrival probability distribution at a detector is Poisson. The counting distribution obtained from such a beam was measured using a photomultiplier