of the polarization in $\overline{K}$-N elastic scattering at high energy.

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[1]In this paper we consider a resonant state N^* with a mass larger than 1.7 GeV, because our method cannot be applied to the elastic scattering at low energy in which the angular distribution has no pronounced peak in the forward direction.

[2]When $2\Gamma_\pi < \Gamma$, the phase shift δ for the $J = l + \frac{1}{2}$ $(J = l - \frac{1}{2})$ state is nearly equal to zero as will be stated below. Therefore the conclusions are not available for this case.

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[7]In this case, the differential cross sections are slightly different from those given by Perl and Corey. Because the forward peak is due to constructive interference among the scattering amplitudes for many states, a resonance N^* has not a large effect on the forward peak, but on large-angle scattering. So far as elastic scattering in the region of small θ is concerned, the estimated values of $a\,[=2k\,\mathrm{Im}f(\theta)]$ are fairly reliable and, by our estimation, we can obtain the significant results for the polarization $P(\theta)$ in the region.

[8]The numerical values of $P(\theta)$ illustrated in Figs. 1 and 2 should not be taken so seriously because of the rough approximation in our treatment.

DETERMINATION OF ρ_0-NUCLEON TOTAL CROSS SECTION FROM COHERENT PHOTOPRODUCTION*

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Recent data[1] on the small-angle coherent photoproduction of ρ_0 mesons from nuclei,

$$\gamma + (A, Z) \to \rho_0 + (A, Z),$$

shows an energy and momentum dependence very similar to that observed in elastic pion and proton diffraction scattering. The similarity of small-angle ρ_0 photoproduction with scattering of pions and protons is to be expected,[2] since the ρ_0 has quantum numbers in common with the photon.

It has also been observed[1] that ρ_0 photoproduction at 0° and 4.4 BeV/c depends on the atomic number A of the target approximately as $A^{5/3}$. This means that the nucleus is neither completely opaque (which would lead to a dependence $\propto A^{4/3}$) nor completely transparent (which would be $\propto A^2$) to the ρ_0 mesons. Since we have neither extreme case, it should be possible to determine the mean free path of the ρ_0 in nuclear matter by analyzing this A dependence, and, combining this with a knowledge of the nuclear density, deduce the total ρ_0-nucleon cross section at this energy. We can then compare the resulting cross section with the predictions of various symmetry schemes, and, by studying this process at higher energies, trace the possible approach of the ρ_0-nucleon cross section to an asymptotic value.

The calculations presented here are based on the simplified eikonal approximation[3] which has proved useful in the analysis of high-energy scattering in terms of nuclear potentials constructed from known individual nucleon scattering parameters. In particular, in order to estimate the accuracy of this method, we apply it first to the calculation of the total pion-nucleon and proton-nucleon cross sections by matching the A dependence of pion-nucleus and proton-nucleus total cross sections.

If the nucleus is represented as a purely absorbing medium of density $\rho(r)$, then, in the simplified eikonal approximation, the total cross section can be expressed[3] by an integral over b, the impact parameter:

$$\sigma_T = 4\pi \int_0^\infty b db \{1 - \exp[-\bar{\sigma} \int_0^\infty \rho(z, b) dz]\}, \qquad (1)$$

where $\bar{\sigma}$ is the total cross section of the projectile on a free nucleon, averaged over protons and neutrons, and $r \equiv (b^2 + z^2)^{1/2}$.

In a simple hard-sphere model, the uniform density is ρ for $r < r_0 A^{1/3} = R$ and zero everywhere else; so Eq. (1) can be integrated directly to give

$$\sigma_T = 4\pi \left\{ \frac{R^2}{2} + e^{-\rho\bar{\sigma}R} \left(\frac{R}{\rho\bar{\sigma}} + \frac{1}{\rho^2\bar{\sigma}^2} \right) - \frac{1}{\rho^2\bar{\sigma}^2} \right\}. \qquad (2)$$

A more realistic model of the density is the so-called "modified Gaussian" distribution, which has the form

$$\rho(r) = \frac{\rho}{1 + \exp[(r^2 - c^2)/B]}, \qquad (3)$$

where

$$B = (c/2.2)s \qquad (4)$$

and s is the usual nuclear skin depth.[4] In this case, the total cross section can be written

$$\sigma_T = 4\pi \int_0^\infty b db \times \left[1 - \exp\left\{ \rho\bar{\sigma} \left(\frac{\pi}{B} \right)^{1/2} \varphi \left(\tfrac{1}{2}, -\exp\left(\frac{c^2 - b^2}{B} \right) \right) \right\} \right]. \qquad (5)$$

where

$$\varphi(x, y) = \int_0^\infty z^x dz / (e^z + y)$$

is the Lerch zeta function.[5]

We can now ask the question, "What value of $\bar{\sigma}$ must we insert into Eq. (1) to match the experimental dependence of σ_T on A?"

The theoretical curves and experimental points[6] for elastic pion scattering at 3 BeV/c are shown in Fig. 1. We note two important points. First, the value of $\bar{\sigma}$ is given by

$$\bar{\sigma}_{\pi N} = 25 \pm 10 \text{ mb} \qquad (6)$$

which agrees, within wide error limits, with the free pion-nucleon cross section.[7] Secondly, the results are independent of the type of nuclear surface used, since both the hard-sphere and modified Gaussian models give the same result.

In Fig. 2 we investigate the same problem for proton scattering at 19.3 BeV/c.[8] Here we find that

$$\bar{\sigma}_{NN} = 45 \pm 5 \text{ mb} \qquad (7)$$

which, again, is in good agreement with the free-particle cross section.[7]

An important feature of this result, as seen in Fig. 2 and Eq. (7), is that it is possible to make a much more accurate determination of $\bar{\sigma}$ for the proton case than it was for the pion

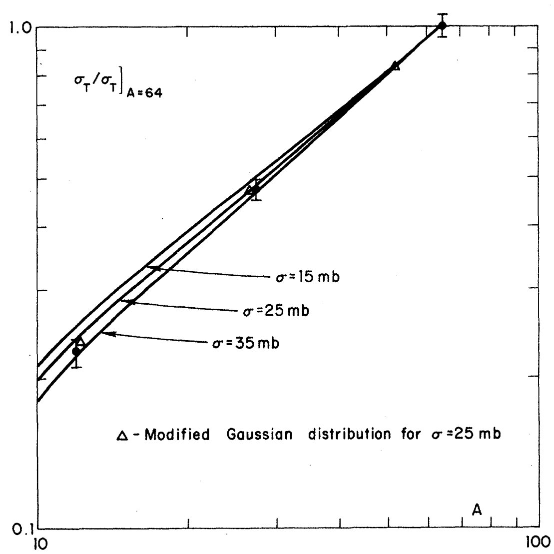

FIG. 1. $\pi + A \to \pi + A$ at 3 BeV/c. The theoretical curves are taken from the hard-sphere model and normalized to $A = 64$. Data from Ref. 6.

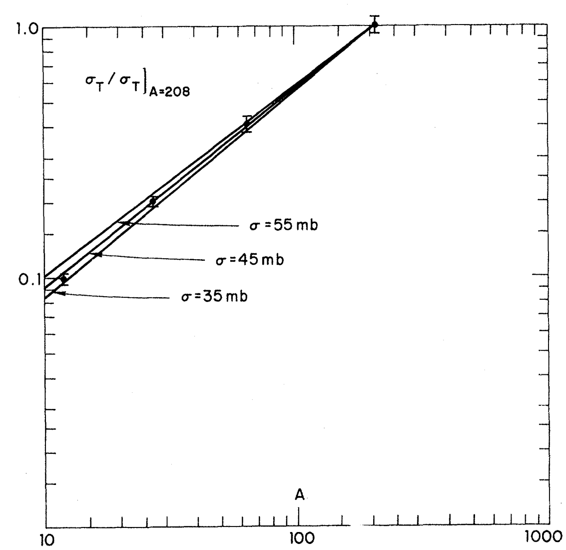

FIG. 2. $p + A \to p + A$ at 19.3 BeV/c. The theoretical curves are taken from the modified Gaussian model and normalized to $A = 208$. Data from Ref. 8.

case because the data for the former exist over a much wider range of A.

It should be noted that for a heavy nucleus like lead, with about 40 more neutrons than protons, the use of the size parameters from electron scattering (which give the charge radius) is not justified. In calculating the result for lead, we have said that the mass radius exceeds the charge radius by 0.5 F, as indicated in Ref. 8.

As a final check of this approximation method, we have compared the magnitude of the total cross section for lead as predicted by Eq. (1) with the experimental value of

$$\sigma_T = 3.29 \pm 0.1 \text{ b}. \tag{8}$$

The theoretical value, taking into account the above-mentioned difference between the mass and charge radius, is

$$\sigma_T = 3.0 \text{ b}. \tag{9}$$

and is low by $9 \pm 3\%$.

Part of this difference can be attributed to the fact that we neglected the real part of the p-p scattering amplitude in deriving Eq. (1). If we denote the ratio of the real to imaginary part of the amplitude by α, then the analogue of Eq. (2) is, to order α^2,

$$\sigma_T = 4\pi\left\{\frac{R^2}{2} + e^{-\rho\bar{\sigma}R}\left[\frac{1}{\rho\bar{\sigma}} + \frac{1}{\rho^2\bar{\sigma}^2}\right] - \frac{1}{\rho^2\bar{\sigma}^2}\right\} + 2\pi\alpha^2\left\{\frac{6}{\rho^2\bar{\sigma}^2} - e^{-\rho\bar{\sigma}R}\left[\rho\bar{\sigma}R^3 + 3R^2 + \frac{6R}{\rho\bar{\sigma}} + \frac{6}{\rho^2\bar{\sigma}^2}\right]\right\}. \tag{10}$$

If we take $\alpha \simeq \frac{1}{4}$,[7] the real part of the amplitude enhances the cross section by about 3%. This brings the difference between Eqs. (8) and (9) to about 6%, which is within the accuracy of our results.

Bolstered by this success, we move on to the ρ_0 coherent production process and attempt to obtain a value for the ρ_0-nucleon cross section. For this process we are interested in computing only the reaction cross section. We must also take account of the fact that the momentum transfer in an individual collision inside the nucleus is not zero, but in fact has a minimum value

$$\Delta_{\min} = m_\rho^{\,2}/2K \tag{11}$$

which, for the data in Ref. 1, is $\Delta_{\min} = 62.2$ MeV $= 0.316$ F^{-1}.

The forward amplitude for the photoproduction process may be written[3]

$$f_T = 2\pi\langle f\rangle\int_0^\infty bdb\int_{-\infty}^\infty dz\,\exp\{-\bar{\sigma}\int_Z^\infty \rho(y,b)dy\}\rho(r)e^{i\Delta z}, \tag{12}$$

where $\langle f\rangle e^{i\Delta z}$ is the forward amplitude for ρ_0 photoproduction by an individual nucleon at coordinate $r = (z, b)$ and where we have converted the sum over scattering centers to an integral over the nuclear volume.[9] For the hard-sphere model, Eq. (12) can be integrated to give

$$f_T = \frac{2\pi\langle f\rangle\rho_0}{\rho\bar{\sigma} + i\Delta}\left\{e^{i\Delta R}\left[\frac{R}{i\Delta} + \frac{1}{\Delta^2}\right] - \frac{1}{\Delta^2} - \frac{1}{(2\rho\bar{\sigma} + i\Delta)^2} + e^{-(2\rho\bar{\sigma} + i\Delta)R}\left[\frac{R}{2\rho\bar{\sigma} + i\Delta} + \frac{1}{(2\rho\bar{\sigma} + i\Delta)^2}\right]\right\}. \tag{13}$$

In Fig. 3 we plot the forward differential cross section obtained by taking the absolute magnitude squared of the amplitude in Eq. (12), together with the experimental results of Ref. 1. We deduce from this graph that the total ρ_0-nucleon cross section is

$$33 \lesssim \bar{\sigma}_{\rho N} \lesssim 47 \text{ mb}. \tag{14}$$

An accurate measurement of ρ_0 photoproduc-

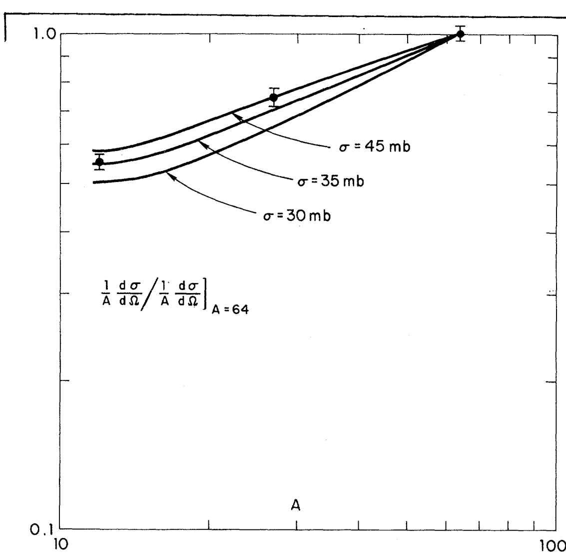

FIG. 3. $\gamma + A \rightarrow \rho_0 + A$ at 4.4 BeV/c. The data points are taken from Ref. 1. The theoretical curves are taken from the hard-sphere model and normalized to $A = 64$.

tion on lead would greatly increase the accuracy of this estimate. Such a measurement would be of interest in view of the importance of knowing $\bar{\sigma}_{\rho N}$ for comparison with the prediction of some symmetry schemes. In particular, the total cross section is determined from the forward scattering amplitude, and so is a direct test of $SU(6)_W$ predictions.[10] In the high-energy limit (presumably dominated by Pomerachuk exchange), the helicity of the ρ_0 does not change in the scattering, and the equality

$$\sigma_{\rho N} = \sigma_{\pi N} \tag{15}$$

should hold. Therefore, in investigating the production of nonstrange resonances (as, for example, the φ and ω) at higher energies, one should strive to include as wide a range of A values as possible.

Finally, we note that the cross section in Eq. (14) corresponds to an absorption mean free path

$$\lambda\!\!\!^{-}_{\text{abs}} \equiv 1/\rho\bar{\sigma} \simeq 1.2\times 10^{-13} \text{ cm}$$

which is an order of magnitude smaller than the decay path $\approx 12\times 10^{-13}$ cm for 4.4-BeV/c ρ^0 mesons. This justifies our neglect of the finite ρ_0 lifetime in these calculations, and the accuracy of this approximation improves with increasing energies since the decay path grows in proportion to the ρ_0 energy. In addition, the investigation of coherent photoproduction at higher energies has the advantage that the minimum momentum transfer given by Eq. (11) becomes small enough to be neglected so that the forward amplitude can be determined analytically in terms of more realistic nuclear density distributions.

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[9]One cannot extrapolate to zero momentum transfer on the basis of an electron-scattering form factor of the type $F(q^2) = 1-(q^2a^2/6)$. Such a procedure corresponds to an amplitude of the type

$$f_T = f(0) < e^{i\vec{\Delta}\cdot\vec{r}},$$

and we see, by comparison with Eq. (12), that this extrapolation procedure neglects the absorption of the ρ_0 in the nucleus. This is clearly not justified in the case of any strongly interacting particle.

[10]H. Harari, private communication.