ISOTOPE EFFECT IN Ca^{40}, Ca^{44}, AND Ca^{48} CHARGE DISTRIBUTIONS FROM 250-MeV ELASTIC ELECTRON SCATTERING*

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Results are presented on the charge-density difference between Ca^{40} and Ca^{48}. They show that pronounced changes are made in the charge density, that is, in the proton distribution, by the addition of the eight neutrons. The following features, which the earlier investigation of the Ca^{40}-Ca^{44} difference[1] suggested, are now apparent. The charge radius, i.e., the over-all size, increases by an amount comparable to the change in $A^{1/3}$. The charge density at the edge of the nucleus (beyond about 4 F) does not increase in the manner suggested by such an over-all dilatation, and it is even less in Ca^{48} than it is in Ca^{40}. Comments on the possible origin of this effect, and of other relevant scattering experiments, are made later in this report.

Experimental and computational procedures are very similar to those of Ref. 1. Briefly, the experimental method is that of elastic electron scattering from isotopically enriched foils of Ca^{40} and Ca^{48}. The electron beam was supplied by the Mark III linear accelerator. The isotopic purities of the targets were 99.97% (Ca^{40}) and 97.22% (Ca^{48}). The contamination in the latter target was due mainly to Ca^{40} and was corrected for. The thicknesses of the targets were 529 mg/cm^2 (Ca^{40}) and 487 mg/cm^2 (Ca^{48}).

The absence of low-lying levels in both isotopes permits us to use poorer energy resolution than in the Ca^{40}-Ca^{44} experiment,[1] and thus a larger slice of the incident accelerator beam. The measurements of the cross-section ratios, expressed in the form

$$D(\theta)=[\sigma_{40}(\theta)-\sigma_{48}(\theta)]/[\sigma_{40}(\theta)+\sigma_{48}(\theta)], \qquad (1)$$

can more easily be continued to larger angles. The added information so obtained about the ratio of the differential cross sections contributes to the greater detail with which we can obtain the quantity

$$Q(r)=4\pi r^2[\rho_{40}(r)-\rho_{48}(r)], \qquad (2)$$

compared to the Ca^{40}-Ca^{44} work. As with that work, we assume a given functional form for the Ca^{40} charge density, with parameter values determined previously.[1] The same form is assumed to apply to Ca^{48}, with altered parameter values.[2] These altered values are determined by making a least-squares fit to the experimentally measured quantity D, Eq. (1). By varying the values assumed for Ca^{40}, we conclude that the fit is insensitive to them and that what is being determined is the difference in parameter values between Ca^{40} and Ca^{48}. Two different functional forms, expressed by

$$\rho(r)=\rho_0[1+wr^2/c^2]\{\exp[(r^n-c^n)/z^n]+1\}^{-1} \qquad (3)$$

with $n=1$ (Fermi) and $n=2$ (modified Gaussian), are considered. The added flexibility of the parameter w is needed to achieve an acceptable fit to the data for the case $n=1$, and has been included in both cases for completeness. The main features of the predicted $Q(r)$, the quantity which we feel has most significance in our results, are to a large extent the same for the two shapes $n=1$ and $n=2$, i.e., they are largely independent of the functional form chosen for $\rho(r)$.

The experimental results, and the best fits to them according to the above shapes, are shown in Fig. 1. With 32 pieces of data, and three parameters to be fitted, the values of χ^2 in the two cases were 26 ($n=1$) and 30 ($n=2$).

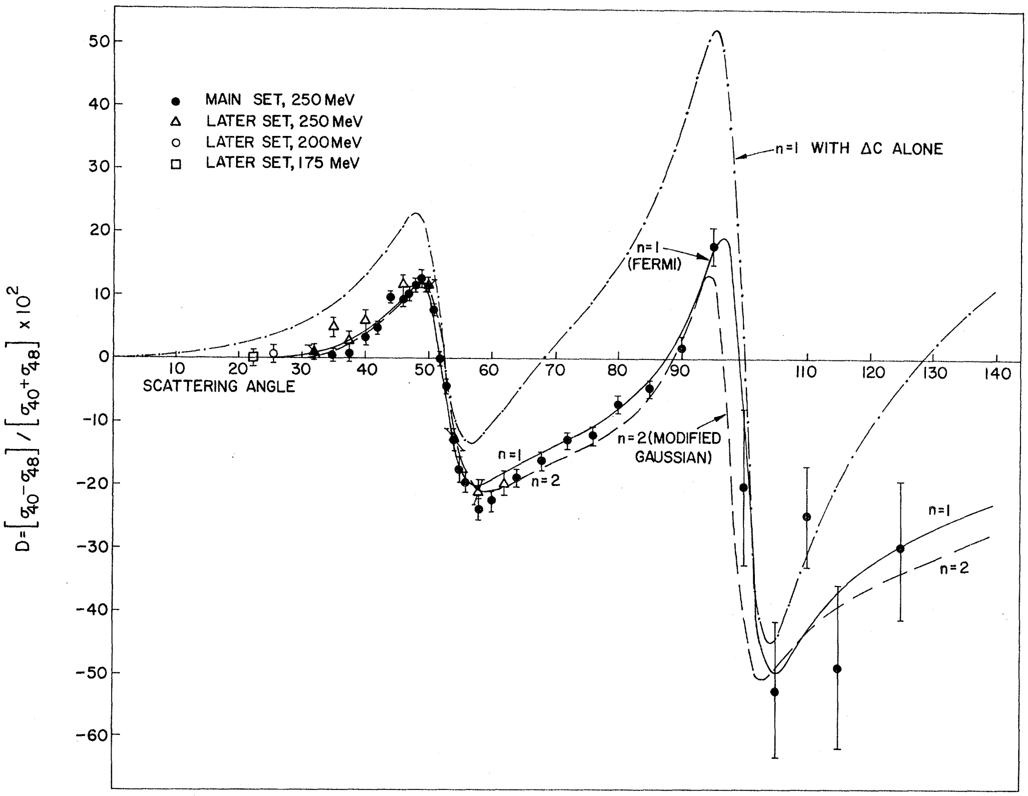

FIG. 1. Isotopic difference of 250-MeV electron-scattering cross sections from Ca^{40} and Ca^{48}. The curves are obtained by using charge distributions of the form (3) with parameter values given in Table I. They are least-squares best fits to the data indicated by closed circles. The points indicated by open triangles, an open circle, and an open square were obtained in a later run, and were at 250, 200, and 175 MeV, respectively. (See the text for comments on these points.) Another curve illustrates the effect of a change in radius alone.

Without the extra parameter w in (3), the value of χ^2 for $n=1$ is 61. Additional experimental points taken in a later run with Ti^{48} are included in the figure (not in the least-squares analysis) to verify the consistency of our results. Some points taken at lower energies to examine the behavior of $D(\theta)$ at small values of q, the recoil momentum, and thus to check on our measurements of target thickness, etc., are also shown. These points, actually measured at $\theta = 32°$, are plotted at the value of θ which at 250 MeV gives the same value of q. We have verified by our partial-wave analysis that this is a satisfactorily accurate procedure. The agreement with the 250-MeV results is very good.

The quantity $Q(r)$, expressing the difference in charge distributions between Ca^{40} and Ca^{48}, is shown in Fig. 2 for the two shapes $n=1$, $n=2$ in Eq. (3). The various parameters associated with the densities are given in Table I. The errors quoted there include, besides those indicated in Fig. 1, an over-all ±1% in the vertical scale to allow for uncertainty in our knowledge of the ratio of the target thicknesses. The third parameter w is small enough that $\rho(r)$ is flat near the center, and it is thus still possible to define the half-radius $r_{0.5}$ and the 90-10% skin thickness t. Their values are affected by w, however. We have included the Ca^{44} results for comparison. One sees from the table that the change in half-radius $\Delta r_{0.5}/r_{0.5}$ is ~4.5%, of comparable magnitude to the change in $A^{1/3}$ of 6.2%. The skin thickness changes negatively by a large amount: $\Delta t/t \sim -11\%$. But these numbers, while indicating a large effect, do not reveal the interesting feature displayed in Fig. 2, that the edge density of Ca^{48} is less than that of Ca^{40}. The over-all expansion of Ca^{48}, expressed by the $\Delta r_{0.5}/r_{0.5}$ result, is resisted, so far as the extreme edge is concerned, by some effect which push-

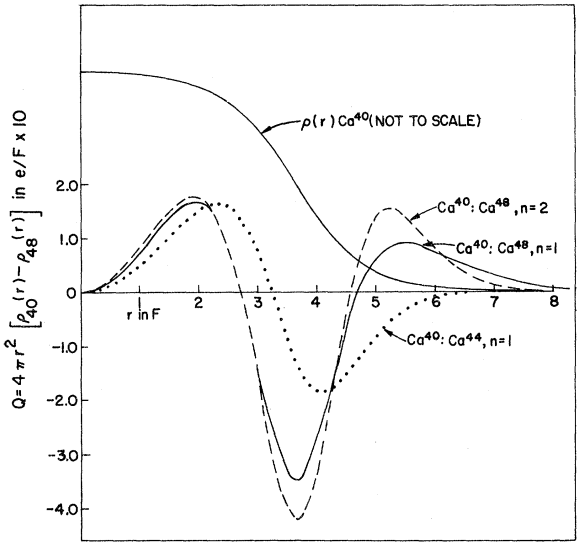

FIG. 2. The quantity $Q(r)$ [Eq. (2) of the text] for the best fits of Fig. 1. It includes the $4\pi r^2$ of the volume element, and is thus the actual amount of charge changed at a given r. The corresponding result for Ca^{40}:Ca^{44} is taken from Ref. 1. For comparison, the charge density $\rho(r)$ for Ca^{40} is plotted on the same horizontal scale. At $r=1$ F its value is $0.081e/F^3$.

es the charge distribution even closer in. The negative change of $\Delta t/t$ in the Ca^{40}-Ca^{44} comparison indicates the same tendency, but to a smaller extent. The net effect is that compared with Ca^{40}, Ca^{48} has a concentration of charge right in the region of the nuclear surface. As regards physical magnitude, we note that the total charge moved to this region, i.e., the negative part of $Q(r)$ in Fig. 2, is about $0.4e$. We emphasize that this is obtained as a phenomenological result, with reasonable assumed functional forms for the charge density, and has not involved the use of a nuclear model.

It is plausible, perhaps, to relate the sharper surface of the charge distribution to a more compact, less easily excited nucleus. Our results then imply that Ca^{48} is a more stable structure than Ca^{40}. This has been concluded previously from the single-particle nature of the Ca^{49} spectrum compared to the complex Ca^{41} spectrum. The presence in the Ca^{40} wave function of appreciable two-particle, two-hole configurations, as revealed by pick-up reactions, seems to be saying the same thing.[3]

In connection with associated experiments we note, from the table, that the muonic $2p \to 1s$ x-ray difference predicted by our results is negative. This as yet unmeasured quantity would by itself indicate that Ca^{48} is smaller than Ca^{40}, as regards mean-square radii. It will be interesting to have the experimental value, as an additional accurately known datum on these nuclei, and as a confirmation of our conclusions on the tail of the charge density.

For a possible comparison with nuclear scattering analyses, we note some preliminary results obtained with the single-particle shell model.[4] The hypothesis is made that the spatial wave function of this model gives directly (after inclusion of the proton size) the charge density. The Woods-Saxon potential in which the protons move has necessarily so many parameters that it could be adjusted to yield almost any desired nuclear charge density (proton density). We find that the shortening of the tail of the Ca^{48} density can be reproduced by, for example, having a considerably deeper well than in Ca^{40} (about 20% deeper) as well as a radius larger by $A^{1/3}$. We would expect

Table I. Isotopic changes in the parameters c, z, and w of the charge distributions [see Eq. (3)] of Ca^{48} and Ca^{44} compared with Ca^{40} and related quantities. The values for Ca^{40} are taken to be[1] $c=3.602$ F, $z=0.576$ F, $w=0$, for $n=1$ (Fermi); $c=3.373$ F, $z=2.20$ F, $w=0$ for $n=2$ (modified Gaussian). Errors in $\Delta c/c$, $\Delta z/z$, and Δw for the Ca^{40}-Ca^{48} comparison have maximum excursions of ±0.51%, ±1.33%, and ±0.20, respectively, but are correlated. Consequently, the errors on the derived quantities $\Delta r_{0.5}/r_{0.5}$ and $\Delta t/t$, where $r_{0.5}$ is the true half-radius and t the 90-10% skin thickness, are somewhat smaller, being about ±0.1 and ±1%, respectively. ΔE is the change predicted in the muonic $2p \to 1s$ x-ray energy. The Ca^{40}-Ca^{44} results are from Ref. 1, where only two parameter fits were made, so that Δw was not obtained.

Comparison	$\Delta c/c$ (%)	$\Delta z/z$ (%)	Δw	$\Delta r_{0.5}/r_{0.5}$ (%)	$\Delta t/t$ (%)	ΔE (keV)
Ca^{40}-Ca^{48}, $n=1$	1.94	−8.31	0.079	4.1	−12.4	−0.52
Ca^{40}-Ca^{48}, $n=2$	6.21	−5.53	0.023	4.9	−10.9	−0.48
Ca^{40}-Ca^{44}, $n=1$	2.18	−1.66	...	2.16	− 1.58	+0.70

that so large a change would be apparent in proton and alpha-particle scattering from these nuclei. Conversely, such scattering analyses, made with the object of examining the difference between the optical potentials of the two nuclei, could place restrictions on the variations allowable in our analysis of the electron scattering with this model. Other information, relating to the position of single-particle energy levels and of admixtures of higher configurations, is very relevant in this regard, of course.

A paper containing the details of our experiment and its analysis will be prepared soon.

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[1]R. Hofstadter, G. K. Nöldeke, K. J. van Oostrum, L. R. Suelzle, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 15, 758 (1965). This paper contains references to previous work of direct relevance.

[2]Because of the similarity in the excited states of these two nuclei, we hope that any contributions to the scattering from virtual nuclear excitation will cancel in our analysis, because we measure the difference between the nuclei. The only theoretical development in this direction since Ref. 1 that we are aware of is due to Rawitscher [G. Rawitscher, to be published], who allows for the effect of real excitation in depleting the elastic flux by including an imaginary part in the charge density. We are currently examining this suggestion.

[3]One of us (D.G.R.) acknowledges with thanks a stimulating conversation with E. Kashy and many helpful discussions with P. Axel on these points.

[4]J. M. McKinley, L. R. Mather, and D. G. Ravenhall, unpublished calculations.

INTENSITY FLUCTUATIONS IN LIGHT BEAMS WITH SEVERAL DEGREES OF FREEDOM

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The statistical properties of Gaussian partially polarized light have been investigated theoretically by several authors.[1-6] It has been shown that the intensity fluctuations of partially polarized light can be calculated under the assumption that the photons have $N=2/(1+P^2)$ degrees of freedom, where P is the degree of polarization. The distribution of the photoelectrons ejected by an unpolarized light beam ($P=0$) should show no differences compared to the distribution obtained if the beam is polarized and the area of the detector or its resolution time is so large that the average over two elementary beams is observed.

In this Letter we report measurements which establish these results experimentally. A convergent light beam is produced by a single-mode He-Ne laser and a lens. From this beam a Gaussian radiation source is generated by moving a random scatterer through it.[7-10] In order to get the probability density of the light intensity, the signal of a photomultiplier is chopped and analyzed by a multichannel analyzer.[11]

If the photomultiplier is illuminated coherently (detector aperture $\ll$ solid coherence angle, resolution time $\ll$ coherence time), we find an exponential curve for the probability density of linearly polarized chaotic light.[8-10,12,13] Figure 1 shows the probability densities of the photocurrent if the photomultiplier receives several elementary beams. The exponential curve with its maximum at $J=0$ measured with a point detector ($N=1$) becomes a Poisson curve with its maximum near the mean intensity $J=\langle J\rangle$ for large N. The probability density for detecting n photoelectrons in N elementary beams is given by[2,4,5]

$$p(n,N)=\frac{(n+N-1)!}{(N-1)!n!}\left(1+\frac{\bar{n}}{N}\right)^{-N}\left(1+\frac{N}{\bar{n}}\right)^{-n}, \quad (1)$$