EXISTENCE OF THE TRINEUTRON

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Recently, Ajdačić et al.[1] have reported on the possible existence of a bound trineutron state (abbreviated n^3) with about 1 MeV binding energy, through the observation of a peak in the proton distribution in the reaction $n(\mathrm{He}^3, p)3n$. The existence of such a bound state is of particular interest, since n^3, thanks to the Pauli principle and the absence of Coulomb effects, represents a rather clean system for theoretical analysis. While the possibility of three-body forces cannot be ruled out, it should be more interesting to relate the existence of n^3 to the role of n-n forces which are restricted by the Pauli principle to exist in ${}^1S_0$ and ${}^3P$ states at low and moderate energies. The case appears ideally suited for a three-body treatment with separable potentials in ${}^1S_0$ and ${}^3P$ states, of which the latter clearly plays the dominant role. Such an analysis, being free from the usual uncertainties of variational treatments,[2] should at least provide a reliable estimate of the minimum strength of the ${}^3P$ force needed to bind n^3. With the help of techniques described earlier,[3] we have carried out a three-body analysis of n^3 using an ordinary n-n force in the ${}^3P$ state. Our results suggest not only that the required strength of this force is well below the limit for a ${}^3P$ bound n^2 state, but that it is quite compatible with the magnitude of the ${}^3P$ phase shifts in nucleon-nucleon scattering obtained by some recent authors.[4,5] The most likely assignment of the quantum number (LSJ) to the state is $(1\,\frac{3}{2}\,\frac{1}{2})$, the other nearest candidates being $(1\,\frac{3}{2}\,\frac{3}{2})$, $(0\,\frac{1}{2}\,\frac{1}{2})$, and $(2\,\frac{1}{2}\,\frac{3}{2})$ in descending order of preference.

For the case $(0\,\frac{1}{2}\,\frac{1}{2})$ the equations are the same as those given in Ref. 3, except for the interchange of isospin and ordinary spin, and the consequent replacement of ${}^3S$ and ${}^1P$ forces by ${}^1S$ and ${}^3P$ forces, respectively. However, as the kernel for the mere s-wave part of the force is repulsive, the main force of interest is the p-wave term which is taken as

$$\langle \vec{p}\,|V|\,\vec{p}\,'\rangle = -3\lambda M^{-1}(\vec{p}\cdot\vec{p}\,')v(p)v(p'),$$
$$v(p) = (\beta^2 + p^2)^{-1}. \tag{1}$$

The ${}^1S_0$ force is the same as given by Yamaguchi.[6] The coupled equations were solved for the minimum strength parameter λ needed to provide zero binding energy in n^3. These figures, which are listed in Table I for several input values of the inverse range parameter β, with and without the coupling to the s-wave term, show that the inclusion of the s-wave part has a negligible effect on the eigenvalue λ.

For other (LSJ) assignments, the states which are found to be attractive are $(1\,\frac{3}{2}\,\frac{1}{2})$, $(1\,\frac{3}{2}\,\frac{3}{2})$, and $(2\,\frac{1}{2}\,\frac{3}{2})$, and their corresponding eigenvalue equations are single integral equations of the form

$$[\lambda^{-1} - h(P)]F(P) = \int d\vec{q}\,\xi K(\vec{P},\vec{q})F(q), \tag{2}$$

where the functions $h(P)$ and K depend only on the form factor $v(p)$ of the potential (1), but not on λ. The quantity ξ, which is an essentially geometrical factor involving angular correlations, has the average values 7/4:2:2:1 for

Table I. Minimum eigenvalue of λ needed to bind n^3 for different assignments of (LSJ), as functions of the inverse range parameter β. (α is the deuteron binding energy parameter.)

$\beta\alpha^{-1}$	$\alpha^{-1}\lambda_c = \pi^{-2}\beta\alpha^{-1}$	$\alpha^{-1}\lambda(0\,\frac{1}{2}\,\frac{1}{2})$ Coupled Eq.	$\alpha^{-1}\lambda(0\,\frac{1}{2}\,\frac{1}{2})$ Single Eq.	$\alpha^{-1}\lambda(1\,\frac{3}{2}\,\frac{1}{2}) = \alpha^{-1}\lambda(1\,\frac{3}{2}\,\frac{3}{2})$
5.0	0.507	0.291	0.292	0.272
6.0	0.608	0.306	0.308	0.290
7.0	0.709	0.322	0.323	0.307

the states $(0\,\frac{1}{2}\,\frac{1}{2})$, $(1\,\frac{3}{2}\,\frac{1}{2})$, $(1\,\frac{3}{2}\,\frac{3}{2})$, and $(2\,\frac{1}{2}\,\frac{3}{2})$, respectively.[7] This makes the states $(1\,\frac{3}{2}\,\frac{1}{2})$ and $(1\,\frac{3}{2}\,\frac{3}{2})$ the most attractive ones for n^3. The corresponding minimum eigenvalues of λ needed to bind n^3 are listed in Table I for different input values of β, and are seen to be only slightly different from those for the case $(0\,\frac{1}{2}\,\frac{1}{2})$. That these λ values are substantially below the critical strength $\lambda_C = \beta/\pi^2$ needed to bind the dineutron in the ^3P state is clear from the column for λ_C. A comparison calculation of λ with an input binding energy of 1 MeV, which was made for the case $(1\,\frac{3}{2}\,\frac{1}{2})$, showed negligible (~1%) variation from the zero-energy values listed in Table I.

The degeneracy between the $(1\,\frac{3}{2}\,\frac{1}{2})$ and $(1\,\frac{3}{2}\,\frac{3}{2})$ states can be removed by the introduction of spin-orbit and/or tensor forces in the ^3P state. It has been found[8] that if the sign of the spin-orbit force is so adjusted as to give a larger phase shift in $^3P_0$ than in $^3P_2$ (and this is indeed suggested by the analysis of Ref. 4), then the states $(1\,\frac{3}{2}\,\frac{1}{2})$ and $(1\,\frac{3}{2}\,\frac{3}{2})$ have their effective values of λ increased, respectively, to $\lambda + \frac{5}{6}\lambda_{LS}$ and $\lambda + \frac{1}{3}\lambda_{LS}$, where λ_{LS} is a (positive) coupling constant associated with the (p-wave) spin-orbit term. Such a spin-orbit force would then make the state $(1\,\frac{3}{2}\,\frac{1}{2})$ somewhat more attractive than $(1\,\frac{3}{2}\,\frac{3}{2})$, and hence, according to the present analysis, the most favored candidate for observation.

To discuss the physical implications of the results for λ given in Table I, we must compare the phase shifts predicted by our ^3P potential (adjusted to give a bound n^3 state) with the latest data given in Ref. 4 (which essentially incorporates the phenomenological analysis of Ref. 5). Table II, which gives a comparison of $k^3 \cot\delta$ obtained from the present analysis with the corresponding function for the $^3P_0$ phase shift given in Ref. 4, shows that there is a rough "matching" for different energies, if β is increased somewhat with energy. Since β is essentially an inverse range parameter, this means that increasing the energy has the effect of making the range shorter. To make this statement somewhat more quantitative, we have compared the behavior of the form factor $v^2(p) = p^2(p^2+\beta^2)^{-2}$ with that of an "analytical form factor" $\frac{3}{4}p^{-2}Q_1(1+\frac{1}{2}\beta_1{}^2p^{-2})$ in which the parameter β_1 has the precise interpretation of a boson mass exchanged between two nucleons via a Yukawa-type interaction.[9] These two functions broadly agree if β_1 is chosen somewhat smaller than β, a crude correspondence being $\beta_1 \approx 5\alpha \to \beta \approx 6\alpha$ and $\beta_1 \approx 6\alpha \to \beta \approx 7\alpha$, where α^2/M is the deuteron binding energy of 2.226 MeV. This implies an inverse range of a little more than two pion masses, which compares favorably with a central force of similar range needed in Ref. 4 to give the $^3P_0$ phase shifts.[10]

To summarize, our analysis suggests that a moderately attractive ^3P force, well below the requirement for a bound dineutron, is enough to produce a bound n^3 state. The range and strength of the force seem to be in good accord with the $^3P_0$ phase-shift data of Bryan and Scott. While a detailed numerical calculation of the effect of a p-wave spin-orbit force has not been carried out, a spin-orbit force of such a sign as to make the $^3P_0$ phase shifts larger than $^3P_2$ assigns to n^3 the quantum numbers $(LSJ) = (1\,\frac{3}{2}\,\frac{1}{2})$, with the second best candidate $(1\,\frac{3}{2}\,\frac{3}{2})$ closely following.

It looks unlikely that even if the existence of n^3 is eventually confirmed, it would be easy to make a direct measurement of its spin. However, a distinction between the various alternative assignments of (LSJ) could perhaps be made in terms of their effects on the H^3 and He^3 form factors. One of the problems here is how to understand the difference between the H^3 and He^3 charge form factors,[11] using a small (~1%) admixture of the S' state. Now it has been suggested[12] that an isobaric state of $T=\frac{3}{2}$ helps in this regard, so it should be interesting to test the detailed effects of the $J=\frac{1}{2}$ and $\frac{3}{2}$ assignments for its spin, on the form-factor problem.

As a final remark, the p-wave force mecha-

Table II. Comparison of $k^3 \cot\delta$ from the present analysis for different values of β with the corresponding quantity $k^3 \cot\delta$ of Bryan and Scott[a] for the $^3P_0$ phase shift for several values of the laboratory energy E in MeV.

E in lab (MeV)	$\alpha^{-3}k^3 \cot\delta$ (Calculated) $\beta\alpha^{-1}$ 5.0	6.0	7.0	$\alpha^{-3}k^3 \cot\delta(^3P_0)$ of Bryan and Scott[a]
10	49.68	112.6	216.1	61.5
20	54.99	121.5	229.4	86.29
40	70.83	144.4	260.5	156.0
60	93.75	173.8	298.1	261.9
80	123.8	210.0	342.1	419.8
100	160.6	252.7	392.2	664.9

[a]See Ref. 4.

nism suggested for the n^3 state has the advantage of disentangling the question of its existence from that of the more controversial n^4 state,[13] in which the $^1S_0$ force should play the bigger role, compared with its negligible effect on the n^3 problem. The nonexistence of n^4 need not therefore necessarily imply that of n^3. On the contrary, a clear detection of an n^3 state would throw valuable light on the nature of the p-wave N-N force.

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[3]A. N. Mitra and V. S. Bhasin, Phys. Rev. 131, 1265 (1963).

[4]Ronald A. Bryan and Bruce Scott, Phys. Rev. 135, B434 (1964).

[5]M. H. Hull et al., Phys. Rev. 128, 830 (1962).

[6]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[7]Ignoring the negligible effect of the $^1S_0$ force on the state $(0\ \frac{1}{2}\ \frac{1}{2})$.

[8]A. N. Mitra, to be published.

[9]A. N. Mitra, Phys. Rev. 123, 1892 (1961).

[10]Actually, Bryan and Scott have used two different masses ($2m_\pi$ and $4m_\pi$) for the scalar meson of zero isospin needed to fit the N-N phase shifts. In addition they have a spin-orbit potential of a shorter range (m_ω^{-1}). In this respect, the effective value of the range parameter needed in our analysis is somewhat larger than the ones required in Ref. 4. Now a shorter range parameter in our analysis would make things more favorable from the point of view of binding the trineutron, according to the well-known argument that of two potentials which fit scattering data, the one with the shorter range is more effective in providing three-body attraction. Our estimate of the minimum strength parameter needed to bind n^3 (insofar as it uses a rather "long range" force compared with Ref. 4) is therefore quite a conservative one.

[11]L. I. Schiff, Phys. Rev. 133, B802 (1964).

[12]T. A. Griffy, Phys. Letters 11, 155 (1964); K. Okamoto, Phys. Letters 11, 150 (1964).

[13]For a detailed set of references on this point, see Ref. 1.

SEARCH FOR STRUCTURE IN THE FAST-NEUTRON INTERACTION WITH U^{235}†

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It has recently been reported that the fission cross sections of U^{235} and U^{233} fluctuate in an energy-dependent manner throughout the incident energy interval 0.1 to 2.0 MeV.[1] The reported structure over the measured energy interval was characterized by a relative amplitude of 25% or more, a width of ~45 keV, and a spacing of 75-100 keV.[1] The effect was somewhat more pronounced for U^{233} than for U^{235} but quite clear in both cases. In particular, the reported U^{235} results obtained with experimental resolutions of 10 to 30 keV in the interval 0.4 to 0.6 MeV were characterized by several pronounced peaks, as illustrated in Fig. 1(a) taken directly from Ref. 1. This structure was attributed to intermediate resonances[1,2] or, less likely, to competition between fission and inelastic neutron scattering which varied with the availability of exit channels at the saddle point.[3] Experimentally, the reported phenomenon is somewhat surprising as, particularly, the U^{235} fission cross section has been widely employed as a flux monitor.[4] Many of the monitoring applications involved incident resolutions of $\lesssim$20 keV, and if fluctuations in the monitoring fission cross sections were present they should have appeared as energy-dependent variations in the primary phenomena under study—fast-neutron capture cross sections, for example. No such variations appear to have been reported. In view of the interesting physical interpretation of the reported phenomenon and the applied importance of the U^{235} fission cross section, it was decided to re-examine experimentally the elastic-scattering and fission cross sections of U^{235} from 0.4 to 0.6 MeV, a region where the fission cross-section variations were said to be particularly large.[1]

The first type of measurement employed fast time-of-flight techniques to determine the differential elastic-scattering cross sections of U^{235} at 10-keV intervals for 0.4 to 0.6 MeV.[5] Measurements were made at 8 angles between 25 and 150 deg with incident-neutron resolutions