tain $H_c(0)$.

[11]First-principle attempts at describing the effects of paramagnetism have been made by K. Maki [Physics 1, 127 (1964)], who has investigated the effect of paramagnetism on H_{c2} for type-II superconductors at all temperatures, for the case of a small gap parameter and weak paramagnetic limiting. For this case Maki gets

$$H_{c2}(0)^* = H_{c2}(0)H_p(0)[2H_{c2}(0)^2 + H_p(0)^2]^{-1/2}.$$

Because of the restrictions of small gap parameter and weak paramagnetic limiting, as mentioned above, it is not clear whether the experimental results can be described by this analysis.

[12]One must keep in mind that the Al films we have used were very disordered. In the 100-Å films the mean free path was estimated to be about 20 Å.

[13]R. A. Ferrell, Phys. Rev. Letters 3, 262 (1959).

[14]K. Maki has indicated that he and others have now considered the problem of the critical field in the presence of a strong Pauli effect as well as spin-orbit scattering.

FLUX PINNING AND FLUX-FLOW RESISTIVITY IN MAGNETICALLY COUPLED SUPERCONDUCTING FILMS

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It has recently been shown that when two superconducting films are placed sufficiently close together they become magnetically coupled[1]; i.e., if a dc current is passed along one film such that it enters a resistive yet superconducting state, a dc current and a dc voltage may be induced along the other film. This effect apparently takes place whether the films are in the mixed state[1] or in the intermediate state.[2] In this Letter, I wish to report some further observations on such a system; and, in particular, effects related to the concepts of flux pinning and flux-flow resistivity in type-II superconductors.

The appearance of a voltage in a type-II superconductor has been associated with the motion of quantized flux vortices (fluxons) perpendicular to the current direction.[3] A very simple criterion for the motion of fluxons has been established by Kim, Hempstead, and Strnad[4] who consider the average forces per unit length acting on one fluxon,

$$\eta v_L = (\varphi_0/c)J - F_p. \tag{1}$$

$(\varphi_0/c)J$ is the Lorentz force acting on the fluxon, where φ_0 is the flux quantum, c the velocity of light, and J the current density. F_p is the so-called pinning force. The pinning force is generally associated with lattice defects which, in effect, form energy barriers and tend to trap the fluxons. When the Lorentz force exceeds the pinning force, the fluxons start to move through the lattice with a velocity v_L. This motion is thought of as a viscous flow, and the fluxons will be subjected to a third force ηv_L where η is the viscosity coefficient. The observed electric field E along a type-II superconductor is taken to be proportional to v_L:

$$E = \frac{n\varphi_0 v_L}{c} = \frac{Bv_L}{c}, \tag{2}$$

where n is the fluxon density. By combining Eqs. (1) and (2), the flux-flow resistivity ρ_f is obtained:

$$\rho_f = dE/dJ = \varphi_0 B/c^2\eta. \tag{3}$$

Unfortunately, these simple formulas are not directly applicable to thin films and, in particular, not when the applied magnetic field is zero. (In most experiments dealing with type-II superconductors, the applied field is much larger than the self-field from the transport current.) In this Letter, I am mainly concerned with how the coupling between two films affects the flux pinning and the flux-flow resistivity. Thus, I shall rely upon analogous equations, even though they may not be valid in detail.

The samples were prepared by vacuum-depositing a film of tin onto a microscope glass slide, then insulating it with a thin layer of silicon oxide,[5] and finally depositing a film of tin on the top. The bottom Sn film is referred to as the primary; the top Sn film, which is narrower than the primary, is referred to as

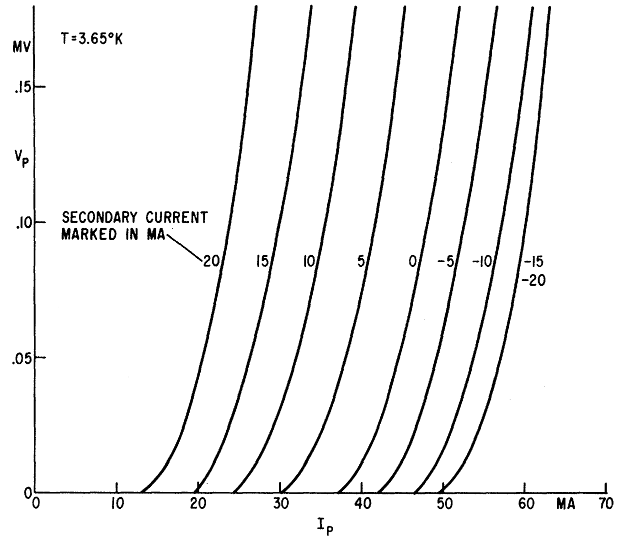

FIG. 1. The current-voltage characteristic of the primary film as a function of the secondary current. The secondary current is positive when it flows parallel to the primary.

the secondary. The effective lengths of the films are ~0.5 cm, while the widths are ~0.1 cm. Both Sn films are on the order of 1000 Å thick, while the silicon-oxide layer is approximately 150 Å thick.

Figure 1 shows several current-voltage characteristics of the primary film for different values of an applied current through the secondary film. As can be seen from the curves, passing a current through the secondary film changes the onset of a primary voltage and therefore the effective pinning force in the primary film.

Since the two films are spaced very close together, I shall assume that every fluxon threads both films. A schematic flux plot is shown in Fig. 2. The local forces acting on a fluxon may be written, in analogy with Eq. (1),

$$\eta_p v_{Lp} + \eta_s v_{Ls} = \frac{\varphi_0}{c} J_p + \frac{\varphi_0}{c} J_s - F_{pp} - F_{ps}, \tag{4}$$

where the extra subscripts refer to the primary (p) and secondary (s) films. From this equation, it immediately follows that the primary and secondary currents are of equal importance as driving forces for the flux motion. Thus, the condition for a voltage to appear in the primary can be written as

$$I_p > I_{0p} + I_{0s} - I_s. \tag{5}$$

I_p and I_s are the total currents through the primary and secondary films, respectively, and I_{0p} and I_{0s} are proportional to the sum over all the pinning forces in the two films. As seen from Fig. 1, this is in good agreement with the experimental results except when $I_s < -10$ mA. Implicit in Eq. (5), however, is the assumption that any force can be transmitted along one fluxon from the primary to the secondary film. This cannot be true as, for example, the whole effect depends upon the close spacing of the two films; if the films are too far apart, they will not be coupled at all. The maximum force which can be transmitted across the insulating layer in any sample depends only upon the magnetic field configuration in this region. For example, when the films are far apart, the magnetic field becomes uniform in the middle of the insulator and no force can be transmitted between the primary and secondary film. Since the maximum force manifests itself experimentally as a current I_c, the condition in Eq. (5) should be supplemented by the condition that

$$\text{if } |I_{0s} - I_s| > I_c,$$

$$\text{then } I_p > I_{0p} + I_c. \tag{6}$$

Note that in the case when the secondary current aids the primary current, only the difference between the Lorentz force and the pinning force in the secondary film needs to be transmitted across to the primary film.

Next I shall examine the flux-flow resistivity. In Fig. 3, both the primary voltage V_p and the secondary voltage V_s are plotted as a function of the primary current I_p for two different values of the load resistance R_s in the secondary circuit. As seen from the curves, the current-

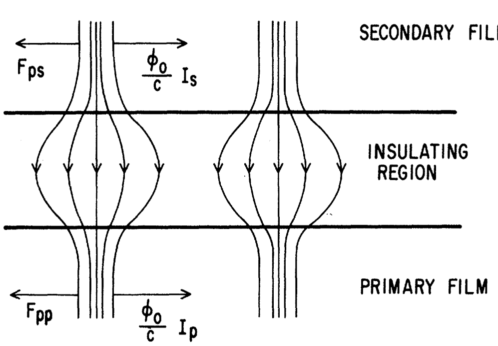

FIG. 2. A schematic flux plot of the region between the primary and secondary films.

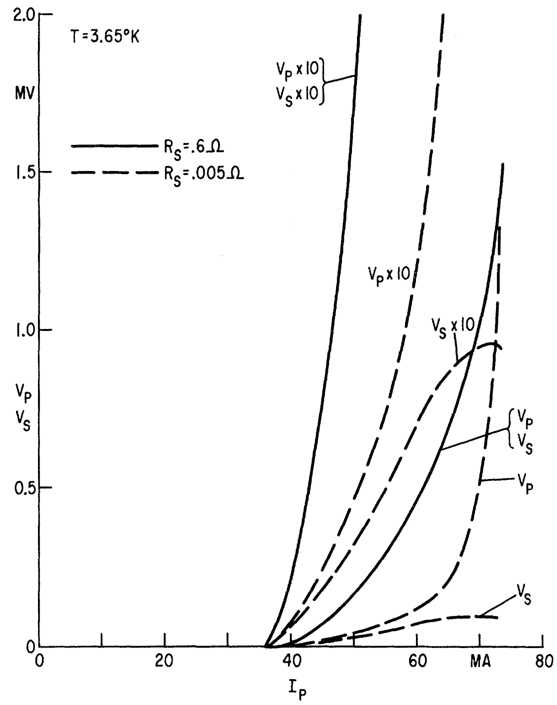

FIG. 3. The current-voltage characteristic of the primary film for two different values of the secondary load resistance R_S.

voltage characteristic of the primary film changes markedly with the secondary load resistance. Note that the onset of the primary voltage is not affected. Thus, as viewed from the primary, the secondary load resistance affects only the dynamics of the flux motion and not the pinning of the fluxons.

Unfortunately, the concept of flux flow is not quite as simple in a thin film as it is in a type-II superconductor where it is approximately a constant [Eq. (3)]. However, intuitively, the results described in the previous paragraph may be understood using Eq. (4). To a first approximation, the flux-flow velocity will be the same in the two films. Experimentally, this is true for a sufficiently large load resistance as the secondary and primary voltages are equal, as seen in Fig. 3. The net secondary current remains zero, irrespective of the secondary load resistance, until a voltage appears. It follows then from Eq. (4) that the load resistance can have no effect upon the onset of the voltage.

The load resistance R_S has a marked effect upon the flux-flow resistivity, however. Experimentally, I find that as long as the primary and secondary voltages are equal,

$$[R_f(R_S)]^{-1} \sim [R_f(\infty)]^{-1} + (R_S)^{-1}. \tag{7}$$

Thus, $R_f(\infty)$ may be regarded as a mutual resistance similar to the mutual inductance in a transformer. [More precisely, $1/R_f(\infty)$ is actually a sum of a self-conductance and a mutual conductance as the primary film is wider than the secondary film.] Note that even though $R_f(\infty)$ is not a constant, in analogy with Eq. (3), I expect $R_f(\infty)$ to be inversely proportional to $(\eta_S + \eta_p)$.

In the limit where the secondary voltage becomes constant, it is easy to understand the flux-flow resistivity. In this limit, the coupling force between the films has been exceeded; and the flux motion in the secondary film becomes independent of the flux motion in the primary. Thus, to a first approximation, R_f is simply inversely proportional to η_p as all the terms in Eq. (4) relating to the secondary circuit are constants. As seen from Fig. 3 there is indeed a break in the slope of the current-voltage characteristic at this point.

The effects reported here are more complex than these simple remarks indicate, and are not understood in detail. All I have hoped to do is to present a qualitative understanding of the experimental results.

I wish to thank Dr. C. P. Bean and Dr. M. D. Fiske for helpful suggestions.

[1]I. Giaever, Phys. Rev. Letters 15, 825 (1965).

[2]P. R. Solomon, Phys. Rev. Letters 16, 51 (1966).

[3]P. W. Anderson, Phys. Rev. Letters 9, 309 (1962).

[4]Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. 139, A1163 (1965).

[5]Silicon monoxide is evaporated onto the first tin film, not SiO_2 as unfortunately stated in Ref. 1.