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$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(Y_i - Y_{Pi})}{(Y_i)^{1/2}}\right]^2,$$

where Y_i = actual data in the ith channel, Y_{Pi} = predicted value (or theoretical value) in the ith channel, and N = total number of channels used in the fitting. In our case $N = 38$, and the number of parameters = 5, so that the number of degrees of freedom = 38−5 = 33. The χ^2 values are 1.547, 1.195, and 1.114 for the point nucleus, the single-particle model, and the configuration-mixing model, respectively.

FINE STRUCTURE AND ISOTOPE SHIFT IN MUONIC LEAD*

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The use of high-resolution lithium-drift germanium detectors[1] has made it possible to resolve clearly the fine structure in the K, L, and M x-ray lines of the heavy muonic atoms.[2-5] We report here our measurements and analyses of these lines in the case of lead. Comparison of two lead samples of different isotopic composition has made apparent an isotope shift in the $2p$-$1s$ transitions.

The experiment was carried out using the muon channel of the Chicago synchrocyclotron. The Ge detector[6] had an area of 5.2 cm^2 and an active thickness of 8 mm. It was at 90° to the beam and shielded from it. It viewed a 45° target 5 in.×5 in.×8 g/cm^2 thick in which normally 2×10^4 muons per second were stopped. A conventional array of coincidence and anticoincidence scintillation counters signalled the stopping of a muon in the target. A fast coincidence ($2\tau = 25$ nsec) was required between one of these telescope counter pulses and any Ge pulse in the energy range 400-8000 keV. The total rate of x-ray events in this range was 20 per second. The pulse-height analysis system, fed from three biased amplifiers, consisted of a pair of Nuclear Data 10-bit analog-to-digital converters (ADC's) whose output was accumulated in the Maniac III computer,[7] and a Victoreen 800-channel pulse-height analyzer. This latter had also been used in our previous run[2] and helped to intercalibrate the two.

Two pulsers were coupled in parallel to the Ge diode at the preamplifier input. The first, a precision mercury pulser monitored by a differential voltmeter, was used frequently for a detailed voltage calibration over the whole range of the analyzers. The second, a monitor pulser, fed tagged pulses of four different amplitudes to the computer once a second to provide a continuing check of the gain and bias changes in the system. The information thus provided made it possible to accumulate data corrected for drifts over long periods of time.

For energy calibration we used a number of known gamma-ray sources: the single and double escape peaks from the 6130-keV line of N^{16}, the 2614.5-keV line of ThC″, the 1273.6-keV line and the 511.0-keV annihilation line from Na^{22}, the 1172.8- and 1332.5-keV lines of Co^{60}, and a number of other known lines of lower energy. Measurements of these lines showed that the ratio of energy to pulse height expressed in volts was constant to within 1 part in 10^3 over the whole range of the spectrometer from 60 to 5600 keV. As a check on possible effects of line shape, counting rate, and gating which might be different for the x rays than for the gamma-ray sources we relied on the 511.0-keV annihilation line which was present in the x-ray spectrum for all heavy targets and the 2614.5-keV line of Pb^{208*} due to muon capture in Bi. These measurements verified our calibration of the Victoreen analyzer but showed that in the Nuclear Data ADC's the x-ray lines appeared shifted in energy by 8.9 ± 1.0 keV. This shift was independent of energy over the range of interest and was the result of improper timing on the gating of the ADC's.

We used targets with isotopic abundances measured[8] to be 52.31% Pb^{208}, 21.39% Pb^{207}, 24.93% Pb^{206}, and 1.37% Pb^{204} for the natural lead target and 2.73% Pb^{208}, 8.48% Pb^{207}, 88.72% Pb^{206}, and 0.07% Pb^{204} for the radio-lead target. For each target the data were combined into channels of standard energy intervals (usually 2 keV), taking due account of the variation in the pulser peak positions over the course of the runs.

The analysis of the data was complicated by the lack of symmetry in the line shapes obtained with our detectors. This was due presumably to inefficiencies in charge collection arising from imperfections in our relatively large crystals. We supposed that the line shape was basically Gaussian in form with a fraction f of the events reduced in amplitude by the inefficiency according to an exponential law. Thus, the number of events falling into chan-

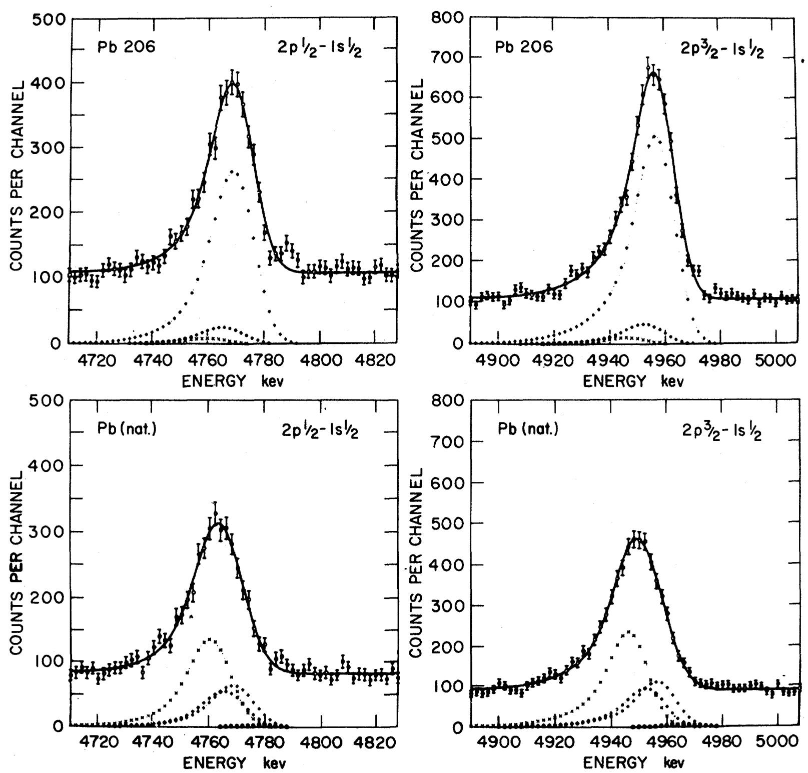

FIG. 1. $2p$-$1s$ transitions (double escape peaks) in natural lead and radio-lead. Solid curve is the fit obtained. Broken curves show the contributions, without background, of the 208, 207, 206, and 204 isotopes in order of increasing energy. Channel width was 2 keV. For $2p_{3/2}$-$1s_{1/2}$ the shape parameters $\lambda = 0.0515$, $\beta = 0.168$, $f = 0.484$ gave $\chi^2 = 96$ (120 channels fitted with 10 parameters). For $2p_{1/2}$-$1s_{1/2}$, $\lambda = 0.0424$, $\beta = 0.198$, $f = 0.460$ gave $\chi^2 = 104$.

nel c_k is given by

$$N(c_k)=Af\beta\sum_{k}^{l_{\max}}\exp[-\lambda(c_0-c_l)^2]\exp[-\beta(c_l-c_k)]$$
$$+A(1-f)\exp[-\lambda(c_0-c_k)^2]+a+b(c_0-c_k).$$

The last two terms take into account the background level and its slope under the line. The parameters were varied until a best fit was obtained according to the least-squares criterion.

In the case of the $2p$-$1s$ transitions, the lines obtained with the natural lead sample were obviously broader than those obtained with radio-lead. Our fitting was done by superposing four lines, one for each isotope, with amplitude proportional to its abundance and with spacing relative to Pb^{206} according to the rule $-\delta$ for Pb^{208}, -0.385δ for Pb^{207}, and $+\delta$ for Pb^{204}, with the object of determining δ, the (206-208) isotope shift. The factor 0.385 was chosen in accordance with the findings of Steudel[9] who obtained the relative isotope shift from the optical spectra. For each of the lines, e.g., $2p_{3/2}$-$1s_{1/2}$, a simultaneous fit was obtained for the two lead samples using common shape parameters λ, β, and f and position parameters c_0 (for Pb^{206}) and δ. The parameters a, b, and A were allowed to take their own best values separately for the two samples. The good fits, shown in Fig. 1, have χ^2's close to their expectation values.

Our results are given in Table I. For the $2p$-$1s$ and the $3d$-$2p$ transitions, we measured the double escape peaks and added 1022 keV to the observed energies. The errors quoted were compounded from uncertainties of 0.1% in the absolute calibration, 1.0 keV in the gating shift, and 1.0 keV in the peak fitting. In determining the separation of two near peaks only the peak fitting errors taken twice are important. The $3d$-$2p$ peaks were analyzed as single, as in the example shown in Fig. 2. The $3d$-$2p$ energy differences found for the two Pb samples were interpreted as isotope shifts, and a small correction was made for the isotopic composition in arriving at the values listed in Table I. In the case of the $4f$-$3d$ transitions, the full energy was measured directly and the results from both Pb samples averaged. Al-

Table I. Experimental and (calculated) muonic x-ray energies for Pb^{208} and Pb^{206} in keV. Intensity ratios given are $R_p=(2p_{3/2}\to 1s_{1/2})/(2p_{1/2}\to 1s_{1/2})$, $R_d=(3d_{5/2}+3d_{3/2}\to 2p_{3/2})/(3d_{3/2}\to 2p_{1/2})$ and $R_f=(4f_{7/2}+4f_{5/2}\to 3d_{5/2})/(4f_{5/2}\to 3d_{3/2})$.

Quantity	Pb^{208}	Pb^{206}	δ $E(206)-E(208)$	Natural Pb	Natural Pb[b]
$2p_{3/2}$-$1s_{1/2}$	5969.5 ± 5.2 (5970.8)	5979.7 ± 5.2 (5979.4)	10.2 ± 1.4 (8.6)	5973.7 ± 5.2	5972 ± 7
$2p_{1/2}$-$1s_{1/2}$	5783.7 ± 5.0 (5785.6)	5792.1 ± 5.0 (5793.8)	8.4 ± 1.4 (8.2)	5788.2 ± 5.0	5787 ± 7
Δp	185.8 ± 1.4 (185.2)	187.6 ± 1.4 (185.6)		185.5 ± 1.4	185 ± 2
R_p	1.8 ± 0.3	2.0 ± 0.3		1.8 ± 0.3	1.8 ± 0.4
$3d_{3/2}$-$2p_{1/2}$	2643.9 ± 2.0 (2644.1)	2645.1 ± 1.9 (2645.3)	1.2 ± 1.4 (1.2)	2644.3 ± 1.9	2643 ± 4
$3d_{5/2}$-$2p_{3/2}$	2500.2 ± 1.9 (2501.6)	2502.8 ± 1.8 (2502.5)	2.6 ± 1.4 (0.9)	2501.2 ± 1.8	2502 ± 3
Δp-Δd	143.7 ± 1.4 (142.5)	142.3 ± 1.4 (142.8)		143.1 ± 1.4	141 ± 5
R_d	1.9 ± 0.3	1.8 ± 0.3		1.9 ± 0.3	1.6 ± 0.5
$4f_{5/2}$-$3d_{3/2}$		970.7 ± 1.7 (971.7)			
$4f_{7/2}$-$3d_{5/2}$		937.2 ± 1.7 (938.0)		950.5 ± 1.7[a]	953 ± 3
Δd-Δf	33.5 ± 1.4 (33.7)			33.5 ± 1.4	•••
Δf		(9.1)			
R_f	1.5 ± 0.2	1.5 ± 0.2		1.5 ± 0.2	•••

[a]Average using statistical weights. [b]See Ref. 3.

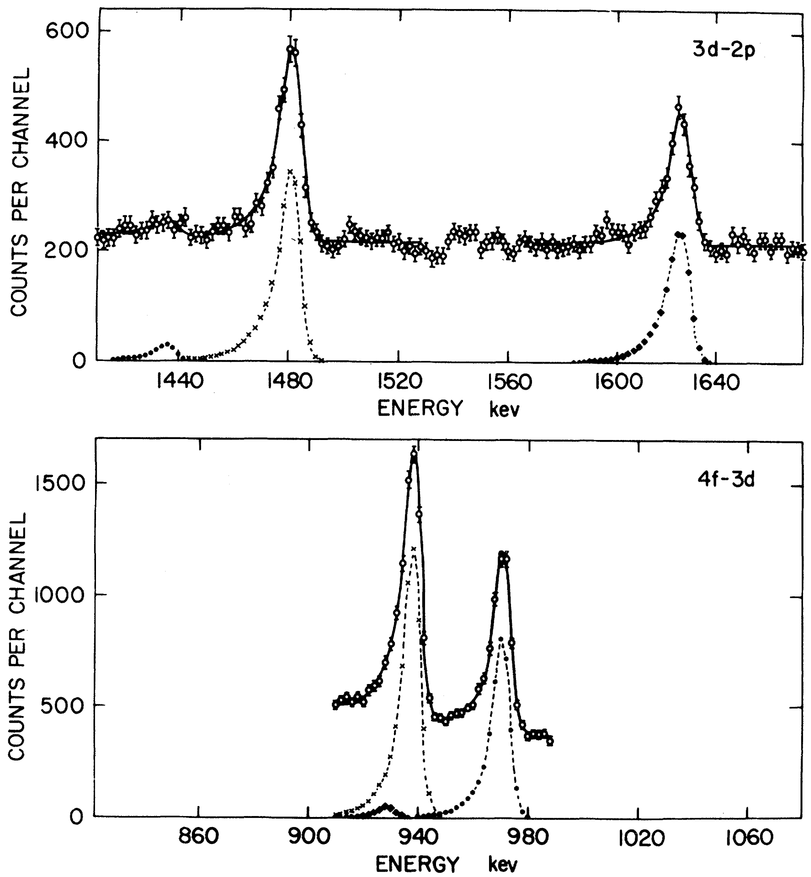

FIG. 2. $3d$-$2p$ (double escape peaks) and $4f$-$3d$ (full energy peaks) transitions in radio-lead sample. Solid curve is the fit obtained in each case. Broken curves show the contribution of the fine-structure components with background subtracted. Channel width was 2 keV. Parameters for $3d$-$2p$ were $\lambda = 0.204$; $\beta = 0.266$; $f = 0.558$; $\chi^2 = 65$ for 110 channels fitted with 12 parameters; for $4f$-$3d$, $\lambda = 0.280$, $\beta = 0.306$, $f = 0.455$; $\chi^2 = 21$ for 40 channels fitted with 10 parameters.

so listed are energies averaged to correspond to the composition of natural Pb. These are in quite good agreement with recent values from CERN,[3] and in general accord with earlier work.[10-13]

We have used our results to specify the size and shape of the nuclear charge distribution in the manner suggested by Hill and Ford.[14] We chose a distribution of the Fermi type,[15]

$$\rho(r) = \rho_0 \{1 + \exp[n(r/R - 1)]\}^{-1},$$
$$r_0 A^{1/3} = (5/3)^{1/2} \langle r^2 \rangle_{\mathrm{av}}^{1/2},$$

and found the exact eigenvalues of energy of the Dirac equation by numerical methods for several values of the parameters. The values of $1/n$ and r_0 which give a best fit to our data were determined by least squares from linearized expressions for the calculated transition energies giving their dependence on these quantities in the vicinity of $r_0 = 1.20$ F and $1/n = 1/15$, with coefficients as listed in Table II.

With these and the measured energies, together with their errors, as given in Table I we obtained[16]

$$r_0 = 1.195 \pm 0.003 \text{ F}; \quad 1/n = 0.072 \pm 0.010;$$
$$\text{correlation error } \langle \Delta r_0 \Delta 1/n \rangle_{\mathrm{av}} = 2.0 \times 10^{-5} \text{ F}.$$

These are in good accord with the Stanford electron-scattering results[15] as well as the x-ray work from CERN.[3] The calculated energies using these values of r_0 and $1/n$ are given in brackets in Table I. The accord with the measurements is quite good, but it should be noted that only the first three of the quantities listed in Table II have appreciable sensitivity to the shape factors.

Table II. Dirac energies in keV and first derivatives calculated for $Z=82$, $A=208$, $m_\mu=105.652$ MeV, $r_0=1.20$ F, and $1/n=1/15$, including the vacuum polarization correction.[14] For our solution, $R/A^{1/3}=1.130$ F, $n=13.9$, $r_0=1.195$ F, the uncorrected energy eigenvalues and the (vacuum polarization correction) were $1s_{1/2}$, 10535.3 (66.1); $2p_{1/2}$, 4784.6 (31.2); $2p_{3/2}$, 4602.0 (28.7); $3d_{3/2}$, 2162.3 (9.4); $3d_{5/2}$, 2120.3 (8.8); $4f_{5/2}$, 1197.3 (2.7); and $4f_{7/2}$, 1188.2 (2.7). The effect of the anomalous part of the magnetic moment of the muon has been neglected in this work.

Transition or splitting	$E(r_0)$	dE/dr_0	$dE/d(1/n)$
$2p_{1/2}$-$1s_{1/2}$	5759.2	−3929	+1140
Δp	184.1	−200	+37
$3d_{3/2}$-$2p_{1/2}$	2641.2	−646	−49
Δp-Δd	141.3	−194	+39
$4f_{5/2}$-$3d_{3/2}$	971.7	−12	−6
Δd-Δf	33.7	−6	−3
Δf	9.1	...	...

For Pb^{206} Table II continues to apply but refers to values of r_0 which are larger in the ratio $(208/206)^{1/3}$. Thus, if the Pb^{206} nucleus were also spherical in shape and had the same density of charge as Pb^{208} we would expect an isotope shift of 15.2 keV for the $2p_{1/2}$-$1s_{1/2}$ transition. Instead we find a shift which is 0.55 ± 0.08 of this. The Brix and Kopfermann[17] value obtained from optical spectra is 0.60 ±0.07. Application of the least-squares method for a best fit to our data gave the following values for Pb^{206}:

$$r_0=1.197\pm0.003\text{ F};\ 1/n=0.073\pm0.009;$$
$$\text{correlation error }\langle\Delta r_0\Delta 1/n\rangle_{\text{av}}=1.8\times10^{-5}\text{ F}.$$

Calculated energies corresponding to these values are given in brackets in Table I.

Such an analysis implies that the removal of two neutrons from Pb^{208} results in a decrease in the density of the nuclear charge. Alternatively, the observed isotope shifts can be accounted for if the Pb^{206} nucleus is distorted into an ellipsoidal form[18] without change in density. Thus, our calculation for a uniform distribution of charge ($n=\infty$) shows that for an ellipsoid with an intrinsic quadrupole moment $Q_0=3.5$ b the isotope shift is 8.4 keV in $2p_{1/2}$-$1s_{1/2}$ and 1.5 keV in $3d_{3/2}$-$2p_{1/2}$. However, such a quadrupole moment is 3 times larger than that deduced from the observed Coulomb-excitation transition probability.[19]

We can give only a rough measure of the line intensities in the fine structure. The efficiency of our detector varies appreciably over the fine-structure separation and is not known very well. The intensity ratios listed in Table I are those obtained from the analysis of the peaks after adding a correction for the efficiency of 7% for R_p, 18% for R_d, and −4% for R_f (includes a small correction for absorption in the target), these values having been estimated from the data of Ewan and Tavendale.[1] There is no significant difference from the statistical values in these ratios.[20]

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CROSS SECTION AND POLARIZATION OF BALMER-ALPHA RADIATION PRODUCED IN CHARGE-EXCHANGE COLLISIONS OF PROTONS WITH N_2†

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Measurements of the cross section for production of Balmer-alpha radiation in charge-exchange collisions of 2- to 35-keV protons with N_2 and the associated polarization of the Balmer-alpha radiation are reported here. The cross section has been measured twice previously.[1,2] However, because of the finite radiative lifetimes associated with the $n=3$ states of hydrogen and the relatively short collision chambers used, the previous measurements could, at best, rely on estimates of relative populations of $3s$, $3p$, and $3d$ states in determining the cross section. The measurements reported here were made at a point sufficiently far from the entrance of the proton beam into the collision chamber that a knowledge of the above relative populations was unnecessary. These measurements have revealed structure in the cross section.

Neglecting processes involving two or more collisions and contributions from cascade effects, the change in number density of excited hydrogen atoms can be described by the equation

$$dn_m/dz + A_m v^{-1} n_m = nn^+\sigma_m, \qquad (1)$$

where n_m is the number density of hydrogen atoms in the state m, A_m is the spontaneous transition probability from the state m to all lower energy states, v is the velocity of the protons, n is the number density of N_2 molecules, n^+ is the number density of protons in the beam, and σ_m is the cross section for excitation of the state m of hydrogen in charge-exchange collisions of protons with N_2. Under the conditions of our experiment, $A_m/v \gg \alpha$, where $\alpha = n\sigma_c$ and σ_c is the total charge-exchange cross section for protons in N_2. The solution of Eq. (1) subject to this condition is

$$n_m = n\sigma_m n^+ v A_m^{-1}[1-\exp(-A_m z/v)], \qquad (2)$$

where z is the distance from the entrance of the collision chamber. When $z \gg v/A_m$, as in this experiment, the only change in n_m with z is that due to the decrease in n^+ from charge-exchange collisions. Under such conditions the observed volume emission rate is given by

$$I = nn^+v(\sigma_{3s} + 0.118\sigma_{3p} + \sigma_{3d}) = nn^+v\sigma_\alpha, \qquad (3)$$

where σ_α is the cross section for the production of Balmer-alpha radiation and is the cross section reported here.

The Balmer-alpha detector was a photometer using a well-blocked interference filter having a bandpass of 12 Å centered at 6563 Å. The measurements were carried out at a point 1 m from the entrance of the proton beam into the collision chamber. Target gas pressures were about 10^{-4} Torr and the energy resolution of the 0.01- to 2.0-μA beam was 20 to 50 eV for proton energies below 15 keV.

The results are shown in Fig. 1 along with the uncorrected cross sections measured by Philpot and Hughes[1] and by Sheridan and Clark.[2] Each of our points represents an average of about 10 data points. The relative scatter in