at our disposal by Professor R. T. Siegel.

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MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE HYPERFINE EFFECTS IN Bi^{209} MUONIC X RAYS*

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The excellence of the negative muon as a test particle for probing the electric and magnetic fields of nuclei has been recognized[1] since 1949. The problem has been to achieve adequate precision and energy resolution in the study of the emitted x rays so information could be obtained not only on the form factor of the charge distribution, but also on the magnetic dipole ($M1$) and electric quadrupole ($E2$) moments as well. The recent development of a high-resolution solid-state gamma-ray detector of lithium-drifted germanium has brought these aims much nearer to their realization.

It is well known that, for atomic electrons, the magnetic dipole hyperfine-structure effects are of approximately equal importance to that of the electric quadrupole interaction:

$$\frac{\langle \mu_N \mu_a / r^3 \rangle}{\langle e^2 Q / r^2 \rangle} \sim 1,$$

where μ_N and μ_a are the magnetic moments of the nucleus and the atomic particle, respectively. However, since the magnetic moment of the muon is about 207 times smaller than that of the electron, the magnetic hfs effects in a muonic atom tend to be, therefore, two orders of magnitude smaller than the electric quadrupole interactions. Nevertheless, there are special exceptions wherein the magnetic and quadrupole hfs effects may be of the same order of magnitude; the Bi^{209} nucleus is one of those favorable cases.

${}_{83}Bi^{209}_{126}$ may be considered a spherical nucleus as it has only one proton outside of the doubly magic core. The ground-state spin is large ($I=\frac{9}{2}$), and its magnetic moment is measured to be 4.08 n.m. The quadrupole moment is $Q_{\text{expt}} \cong -0.4\times10^{-24}$ cm^2. Because of the high sensitivity of the muon orbit to the finite extent of the nucleus, the observed hfs should reflect the radial distribution of the magnetic dipole as well as the electric quadrupole moments inside the nucleus. The former effect was first theoretically treated by Bohr and Weisskopf[2] and has been known as the Bohr-Weisskopf effect. The latter effect is usually more pronounced in highly deformed nuclei, such as the ${}_{74}W$ isotopes and ${}_{92}U^{238}$, where a dynamic $E2$ hfs, in addition to the static $E2$ hfs, has been observed and reported.[3]

Muonic x rays of Bi^{209} have been studied recently by using Ge(Li) detectors in the muon beam of Nevis cyclotron and have been briefly reported.[4] The extraction and focusing of the muon beam to the target area is similar to that used in previous investigations.[5] The small size of the Ge(Li) detector placed a great premium on high geometrical efficiency. For this reason an arrangement was used in which the muon beam, the associated counters, the target material in which the muons stopped and the x rays were produced, and the Ge(Li) detectors were all in a straight line. To minimize the background contribution of electrons in the beam, as well as the bremsstrahlung accompanying them, two Cherenkov counters, one of water and the other of Lucite, were added to the existing counter telescope system. The Ge(Li) detectors[6] were prepared in our laboratories. Two detectors of sensitive volume 7 cm$^2\times$0.9 cm were mounted side by side on a common liquid-nitrogen-cooled support and in a common vacuum envelope (3.5 cm$\times$7.3 cm$\times$15.0 cm). The over-all energy resolution for γ rays is 3.8 keV at 1 MeV, 5.5 keV between 2 and 3 MeV, and 8 keV for 6 MeV. To fully utilize the high resolution of the detector, four multichannel analyzers of 400 to 1600 channels were employed to record the data simultaneously. To examine a specific region of the spectrum, a linear window amplifier was used.

The data processing and analyzing were carried out using a PDP4 computer. Figures 1(a) and 1(b) show the regions of K and L x rays of Bi^{209}. The 6.131-MeV γ ray[7] from N^{16} was shown in Fig. 1(a) for the purpose of comparing line shapes in that energy region. For this purpose, we recorded the N^{16} γ ray with the cyclotron beam on as well as with cyclotron beam off—also summed over several runs for an extended period. Owing to the high stability of the detector system, the apparent linewidth for an extended run was 8-9 keV (full width at half-maximum). The observed $K\alpha_2(2p_{1/2}\rightarrow1s_{1/2})$ line shows a linewidth (full width at half-maximum = 13.5 keV) which is a clear evidence of broadening over the natural linewidth. Since no electric quadrupole interaction is involved in the $2p_{1/2}\rightarrow1s_{1/2}$ transitions, the observed broadening must be attributed to the magnetic hfs effects. The lifetime of the $2p$ state in Bi^{209} has been estimated[8] to be $\sim10^{-18}$ sec; its contribution to the linewidth is, therefore, only ~1 keV. The $K\alpha_1(2p_{3/2}\rightarrow1s_{1/2})$ line is much broader than the $K\alpha_2$ line, and shows a large dip in the middle of the line (full width at half-maximum = 21 ± 1 keV). The two L x rays in Fig. 1(b) also show appreciable broadening. The occurrence of a nuclear γ ray (2.6145 MeV) from excited Pb^{208*} ($Bi^{209}+\mu^-\rightarrow Pb^{208*}+\nu+n$) between them provides a constant monitoring source for the natural line shape. The broadening of this nuclear γ line during the long run was less than 0.5 keV.

In calculating the hyperfine splitting in Bi^{209} due to both $M1$ and $E2$ effects, Le Bellac[9] assumed three different nuclear models: (1) point-nucleus model, which is the simplest, but probably most unrealistic (A_1'); (2) finite-nucleus model, but using single-particle shell-model wave functions (A_1^0); and (3) single-particle model with configuration mixing (A_1). This was used because the single-particle shell model failed to explain the observed magnetic moment. The displacement of the state $|\vec{I},\vec{J},\vec{F}\rangle$ ($\vec{F}=\vec{I}+\vec{J}$), is given by

$$(\Delta W_F)_{M1}=\frac{[F(F+1)-I(I+1)-J(J+1)]}{2IJ}A_1,$$

where A_1 is called the hfs constant and is model dependent. The values of A_1 calculated for the above three models are listed in Table I. It is interesting to see that the finite nuclear size effect reduces the value of A_1 by nearly 50%.

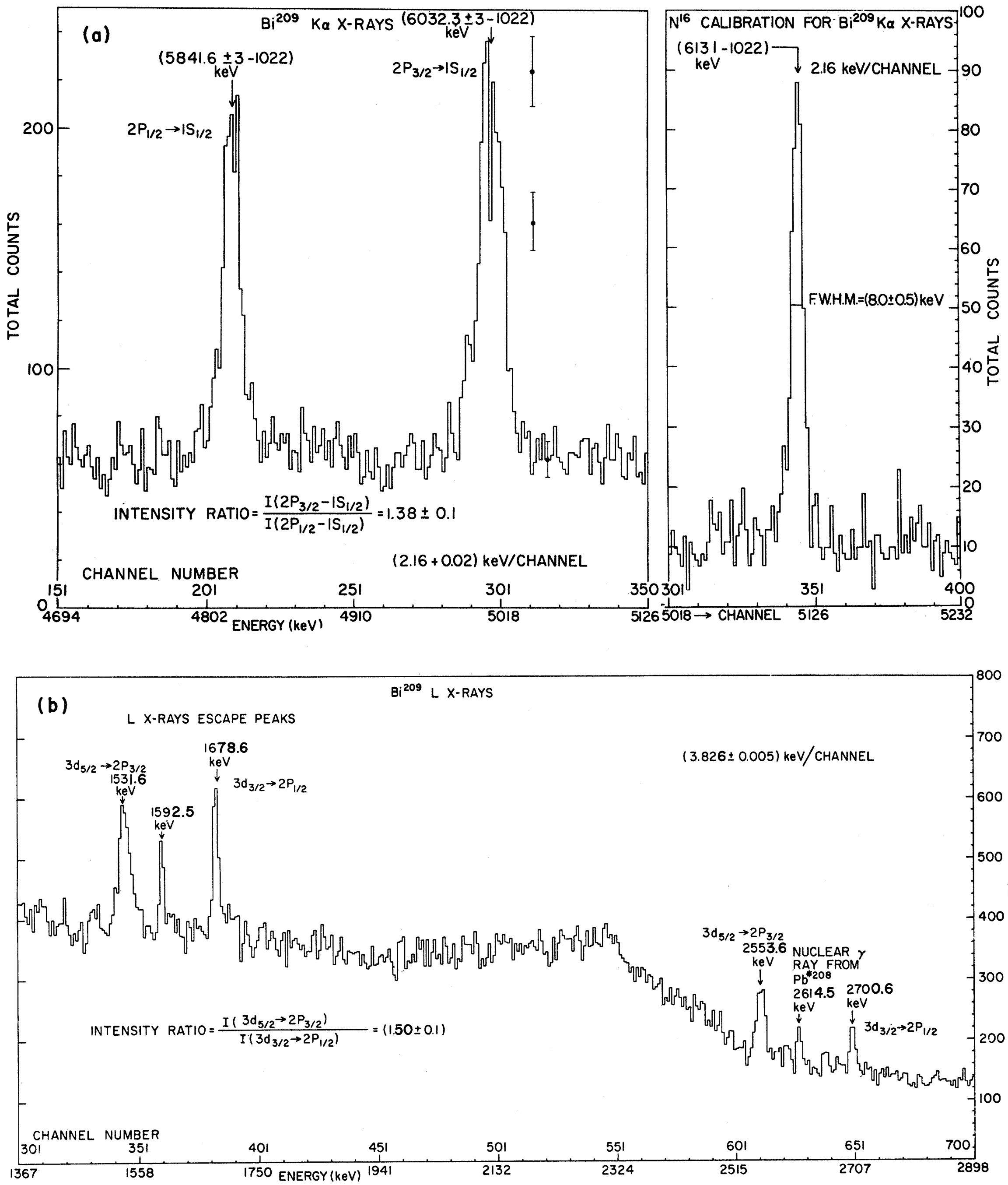

FIG. 1. (a) Spectrum of the K x rays of muonic Bi^{209}. To show the broadening and splitting observed in the K x-ray doublet, the linewidth of 6.130 MeV of O^{16} is inserted to the right for comparison. (b) Spectrum of the L x rays showing full-energy as well as double-escape peaks. The narrow line between the L x rays is a de-excitation γ ray from the reaction

$$Bi^{209} + \mu^- \rightarrow Pb^{208*} + n + \nu_\mu$$
$$\hookrightarrow Pb^{208} + \gamma(2.6145\ \mathrm{MeV}).$$

Table I. The magnetic hfs constants A_1 in Bi^{209} calculated by Le Bellac.[9]

Level	A_1', point nucleus	A_1^0, single particle	A_1, configuration mixing	$\frac{A_1'-A_1}{A'}$
$1s_{1/2}$	3.02	1.36	1.63	46%
$2p_{1/2}$	1.62	0.71	0.84	48%

In Fig. 2(a), we show the displacement $(\Delta W_F)_{M1}$ of the F levels in the $2p_{1/2}$ and the $1s_{1/2}$ states. The hyperfine splitting and the relative transition rate on the configuration-mixing model are shown in Fig. 2(b). In order to compare the theoretically expected composite line shape of the transition $2p_{1/2} \rightarrow 1s_{1/2}$ with the experimentally observed one, we composed the envelope of the four constituent lines by using a linewidth and line shape as determined by the calibration N^{16} γ-ray line for each component, using the theoretical line intensities. The comparison between the experimentally observed and the theoretically composed line for the configuration-mixing model is shown in Fig. 2(c). Of the three models, the configuration-mixing model yielded the best fit to the experimental values by χ^2 test.[10]

For the transition $2p_{3/2} \rightarrow 1s_{1/2}$, the analysis of the hfs splitting was carried out in a manner similar to that for the transition $2p_{1/2} \rightarrow 1s_{1/2}$, except that here both the magnetic and electric hfs must be taken into account. The results are shown in Fig. 3. The displacement of the state $|\vec{I}, \vec{J}, \vec{F}\rangle$ due to electric quadrupole moment is given by

$$(\Delta W_F)_{E2} = \frac{6[K(K+1)-\frac{4}{3}I(I+1)J(J+1)]}{2I(2I-1)2J(2J-1)}A_2,$$

where $K = F(F+1)-I(I+1)-J(J+1)$; A_2 is also model dependent. Le Bellac adopted the value of $A_2 = -2.74$ keV, which is calculated by using

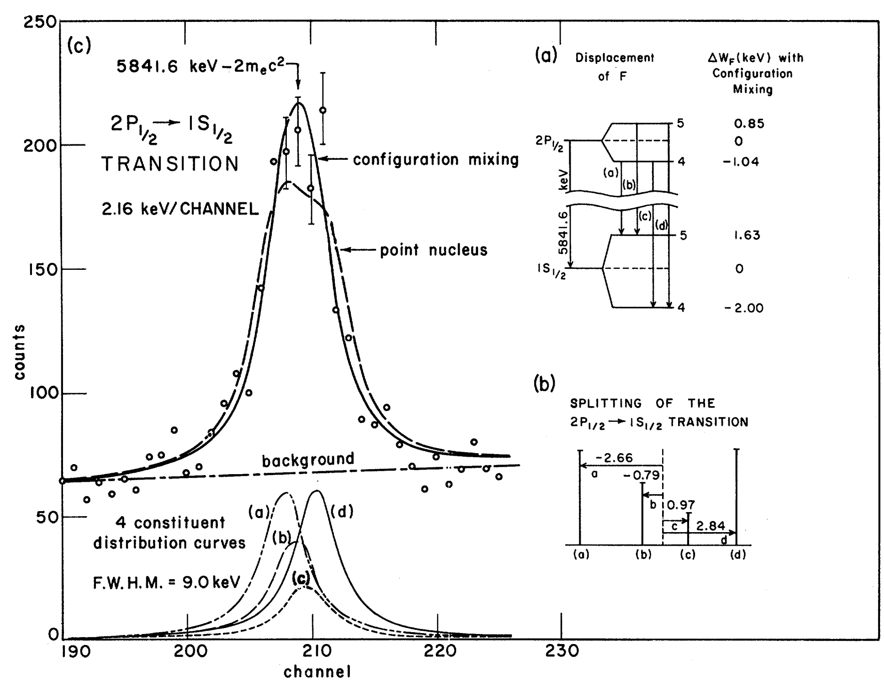

FIG. 2. (a) $M1$ hfs splitting of the $2p_{1/2}$ and $1s_{1/2}$ levels, calculated using the value of A_1 from Ref. 9. (b) Hyperfine structure of the transition $2p_{1/2} \rightarrow 1s_{1/2}$. The relative intensities are assumed to be statistical. (c) Fit of the experimental points with point-nucleus model and configuration-mixing model. The four constituent lines shown are calculated using the configuration-mixing model.

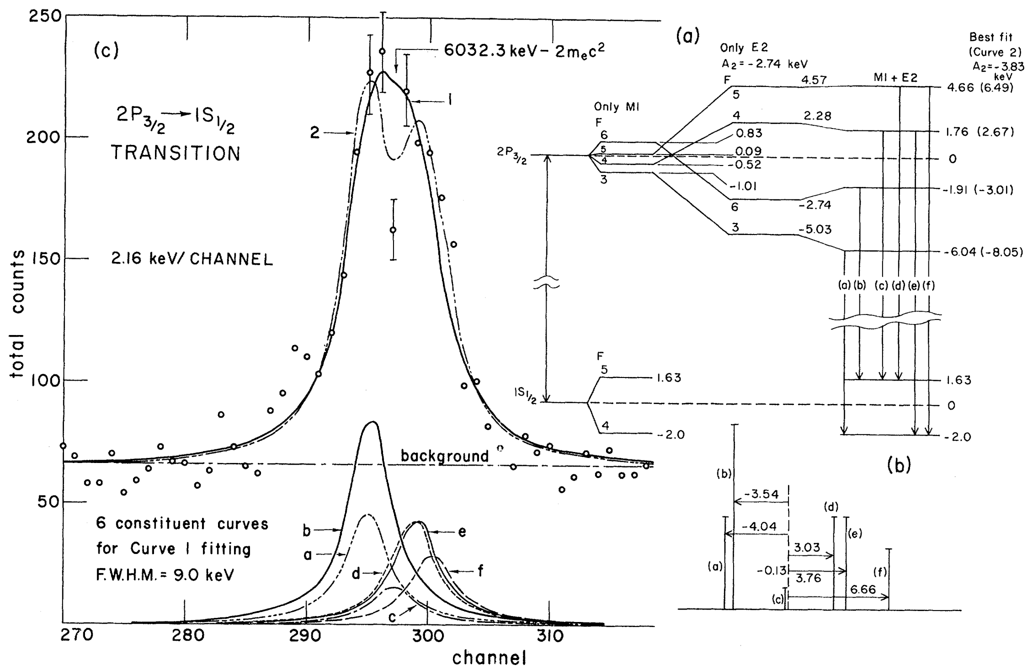

FIG. 3. (a) Combined $M1$ and $E2$ hfs splitting of the $2p_{3/2}$ and $1s_{1/2}$ levels calculated with values of A_1 and A_2 given in Ref. 9. (b) Hyperfine structure of the transition $2p_{3/2} \rightarrow 1s_{1/2}$. The relative intensities are assumed to be statistical. (c) Fit of the experimental points. Curve 1 (solid line) is the sum of 6 lines calculated from values given in Ref. 9. Curve 2 (dashed line) is obtained by increasing the $E2$ hfs constant to $A_2 = -3.83$ which results in minimum χ^2.

single-particle wave functions. Although the single-particle wave functions yield a calculated Q value of -0.44×10^{-24} cm^2 which is comparable to the measured $Q_{\text{expt}} = -0.40 \times 10^{-24}$ cm^2, the Q_{expt} is actually derived from measurements with atomic electrons so it is rather insensitive to the finite size effect. In order to take into account the volume effect of A_2, more sophisticated calculations are needed. However, it is interesting to see that our experimental results fit much better if the hfs constant for the $E2$ effect (A_2) is equal to -3.8 keV. Furthermore, in the calculation of the relative intensities, the level densities are assumed to be proportional to the statistical factors. In view of the intensity anomalies[4,5] observed in the K_α doublets,

$$\frac{I(2p_{3/2} \rightarrow 1s_{1/2})}{I(2p_{1/2} \rightarrow 1s_{1/2})} = 1.38 \pm 0.1,$$

and L multiplets,

$$\frac{I(3d_{5/2} \rightarrow 2p_{3/2})}{I(3d_{3/2} \rightarrow 2p_{1/2})} = 1.50 \pm 0.1,$$

the validity of statistical assumptions may be questioned.

The energies of the higher order x rays and their relative intensities ($I_K : I_L : I_M : I_N$) have also been measured. The interpretation of the charge distribution of the Bi^{209} nucleus in terms of the measured x-ray energies has been investigated. A detailed discussion of our results on Bi^{209} muonic atoms will be submitted elsewhere.

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$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(Y_i - Y_{Pi})}{(Y_i)^{1/2}}\right]^2,$$

where Y_i = actual data in the ith channel, Y_{Pi} = predicted value (or theoretical value) in the ith channel, and N = total number of channels used in the fitting. In our case $N = 38$, and the number of parameters = 5, so that the number of degrees of freedom = 38−5 = 33. The χ^2 values are 1.547, 1.195, and 1.114 for the point nucleus, the single-particle model, and the configuration-mixing model, respectively.

FINE STRUCTURE AND ISOTOPE SHIFT IN MUONIC LEAD*

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The use of high-resolution lithium-drift germanium detectors[1] has made it possible to resolve clearly the fine structure in the K, L, and M x-ray lines of the heavy muonic atoms.[2-5] We report here our measurements and analyses of these lines in the case of lead. Comparison of two lead samples of different isotopic composition has made apparent an isotope shift in the $2p$-$1s$ transitions.

The experiment was carried out using the muon channel of the Chicago synchrocyclotron. The Ge detector[6] had an area of 5.2 cm^2 and an active thickness of 8 mm. It was at 90° to the beam and shielded from it. It viewed a 45° target 5 in. × 5 in. × 8 g/cm^2 thick in which normally 2×10^4 muons per second were stopped. A conventional array of coincidence and anticoincidence scintillation counters signalled the stopping of a muon in the target. A fast coincidence ($2\tau = 25$ nsec) was required between one of these telescope counter pulses and any Ge pulse in the energy range 400-8000 keV. The total rate of x-ray events in this range was 20 per second. The pulse-height analysis system, fed from three biased amplifiers, consisted of a pair of Nuclear Data 10-bit analog-to-digital converters (ADC's) whose output was accumulated in the Maniac III computer,[7] and a Victoreen 800-channel pulse-height analyzer. This latter had also been used in our previous run[2] and helped to intercalibrate the two.

Two pulsers were coupled in parallel to the Ge diode at the preamplifier input. The first, a precision mercury pulser monitored by a differential voltmeter, was used frequently for a detailed voltage calibration over the whole range of the analyzers. The second, a monitor pulser, fed tagged pulses of four different amplitudes to the computer once a second to provide a continuing check of the gain and bias changes in the system. The information thus provided made it possible to accumulate data corrected for drifts over long periods of time.

For energy calibration we used a number of known gamma-ray sources: the single and