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EXPERIMENTAL OBSERVATION OF $M1$ AND $E2$ HYPERFINE STRUCTURE IN THE MUONIC K AND L X RAYS OF BISMUTH*†

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In this Letter we summarize an experimental investigation of the hyperfine structure of muonic Bi, conducted with particular emphasis on the magnetic effect.

The hfs of muonic atoms differs qualitatively, in certain points, from that of electronic (ordinary) atoms. Some of these points are as follows: (1) The magnetic ($M1$) interaction is in general much smaller than the electric quadrupole ($E2$) interaction. (2) For deformable nuclei, the hfs ($E2$) and fine-structure splittings can become comparable to each other, and to nuclear excitation energies; the usual static description no longer applies then.[1] (3) For both hf interactions, the effects of finite source size are larger (reduction of about 50% for high Z), and rather sensitive to the details of the source distributions ($M1$ and $E2$ densities). (4) All muonic atoms are hydrogenlike, and the muon wave functions can be computed reliably (once the scalar nuclear charge distribution is known).

The uncertainties in electronic wave functions are such that ordinary hf spectroscopy has been superseded as a source of nuclear $M1$ moments μ_I, and yields only moderately accurate values of Q, the $E2$ moment. Effects of finite source size[2] can be studied reliably only through the comparison of isotopes,[3] and for $M1$ moments only.

In view of (3) and (4), muonic spectroscopy offers, at least theoretically, the possibility of (a) investigating the effects of finite $M1$ distributions for individual isotopes, and (b) of determining $E2$ moments with fair accuracy (limited primarily by the uncertainties in $E2$ distribution!) by an independent technique. Experimentally, the Ge(Li) detector, whose use in muonic x-ray spectroscopy was pioneered by Anderson et al.,[4] brings (b) into the realm of feasibility. Investigations of the type (a) however require exceptional precautions even with the highest resolution Ge(Li) detectors — because of point (1) above.

Simple estimates show that Bi^{209} ($Z=83$, $I=\frac{9}{2}$, $\mu_N=4.04$ n.m., $Q\approx-0.4$) is probably the most suitable nucleus for a first study of the $M1$ hf effects in a muonic atom.[5,6] Although more accurate predictions of the relevant hf splittings are given in Table I, we first present these estimates for clarity's sake.

The $2p_{1/2}\to 1s_{1/2}$ transition ($K\alpha_2$) is split by the $M1$ interaction only. The splitting is largest in the ground state, and is given (for a point dipole) by the Fermi formula

$$\Delta W_1(1s_{1/2}) \equiv A_1^{\text{point}}[(2I+1)/I]$$
$$=(8\pi/3)\psi^2(0)\mu_0^{\,2}\mu_I(m/M)[(2I+1)/I]. \quad (1a)$$

Table I. Parameters used to calculate the muonic x-ray spectra from Bi^{209}.

State	Constants[a] (keV) A_1	A_2
$1s_{1/2}$	1.6^b (3.2)	...
$2p_{1/2}$	0.8^b (1.6)	...
$2p_{3/2}$	0.8^b (1.6)	-4.0^d [-4.5]
$3d_{3/2}$	0.2^c	-0.6^d
$3d_{5/2}$	0.3^c	-0.7^d

[a]See LeBellac (Ref. 9) for a definition of A_1 and A_2. Values in parentheses are for a point dipole.

[b]As given in Ref. 9. T. T. Bardin et al., Ref. 5, also found these values to give satisfactory agreement with experiment.

[c]Scaled from the p-state values in Ref. 9.

[d]Computed, with $Q=-0.46$ b, using a $\delta(r-R)$ distribution for ρ_2 [S. Raboy, C. C. Trail, J. A. Bjorkland, R. D. Ehrlich, R. J. Powers, and V. L. Telegdi, Nucl. Phys. 73, 353 (1965)]. The value in brackets fits the K data better. Ref. 9 gave $A_2(2p_{3/2})=-2.7$ keV.

Here, $\psi^2(0)$ comes from the "contact" interaction between the moments; to allow for the finite extension of the nuclear $M1$ moment, one has to average $\psi^2(\vec{r})\delta(\vec{r}-\vec{R})$ over its distribution $\vec{M}(\vec{R})$. Now the corresponding average $\langle\psi^2(r)\rangle$ over the nuclear charge distribution $\rho(R)$ is familiar from muon capture (which is also a contact interaction!), viz. $\langle\psi^2(r)\rangle \equiv (Z_{\mathrm{eff}}^4/Z)(1/\pi a_0^3)$; Z_{eff}^4 is tabulated.[7] Thus one has,[8] assuming that $\vec{M}(R)$ has the same isotropic distribution as $\rho(R)$, instead of A_1^{point} above,

$$A_1^{\mathrm{finite}} = (\tfrac{2}{3})(Z_{\mathrm{eff}}^4/Z)\mu_I(m/M)\alpha^4 mc^2. \qquad (1b)$$

This assumption is, however, only approximately valid. The hf interaction depends, as Bohr and Weisskopf (B-W)[2] first pointed out, on the detailed structure of $\vec{M}(\vec{R})$; later Winston[8] and LeBellac[9] discussed muonic hfs specifically. We shall reserve the term "B-W effect" for corrections δA to the trivial volume effect ($M \sim \rho$).

Numerically, (1b) predicts for Bi $\Delta W(1s_{1/2}) \approx 3.5$ keV. Although one also expects some splitting in the $2p_{1/2}$ state, it is already clear that experimentally (with 8- to 12-keV resolution) the hfs will lead merely to a broadening of the Bi $K\alpha_2$ "line." The same conclusion holds for the $L\beta_1$ ($3d_{3/2} \to 2p_{1/2}$) transition.[10]

The $K\alpha_1$ transition ($2p_{3/2} \to 1s_{1/2}$) involves both $M1$ and $E2$ hfs. The ratio of the hf constants is roughly given by

$$A_1/A_2 \approx 4(\mu_I/Q)(\hbar/mc)^2(m/M) = -0.17 \text{ for Bi.} \qquad (2)$$

Thus, although Q is small, the $K\alpha_1$ line will be strongly affected by the $E2$ hfs.

The quantitative study of phenomena on the basis of a change in linewidth or line shape poses delicate experimental problems. Past experience in this laboratory convinced us that it is almost impossible to record "static" comparison lines (say, γ rays from radioactive sources) under exactly the same conditions as beam-induced transitions (muonic x rays or capture γ's). Only the comparison of simultaneously recorded events of this latter type appears reliable. We therefore used for reference (a) in the K series, the muonic x rays from a Pb^{206} target,[11] and (b) in the L series, a 2.615-MeV transition in Pb^{208} that follows μ capture in Bi^{209}.[12] Adopting a method due to Cohen et al.,[13] we used a detector that viewed several targets (here Pb^{206} and Bi) at once; stops in a given target were labeled electronically. For each photon event satisfying the logic criteria,[14] both its pulse height and its timing (with respect to the stopped muon) were measured in digital form. A 100-Mc/sec "digitron"[15] was used for the timing. The two-parameter spectra, together with the target "labels," were stored on magnetic tape. This procedure enabled us to record the pulse-height distributions of prompt events, accidentals, and capture γ rays simultaneously. An estimate of the "prompt" background (presumably due to a residual e^- contamination of the "muon" beam) was obtained using comparison targets (Ta for the K series, Ti for the L series) which could not give appreciable amounts of muonic x rays in the energy intervals of interest. To ensure against drifts, the linear amplification system was kept at constant temperature (±0.2°C). Although our experiments did not aim primarily at accurate absolute energy determinations, the runs were interspersed with pulser and source (ThC$''$, O^{16*}) calibrations.

Figure 1 shows the experimental points in the $K\alpha$ doublet regions of Bi (top) and Pb^{206} (bottom); flat contributions from accidentals, capture γ rays, and "prompt" background were subtracted before plotting. The curves drawn through the Pb^{206} data represent the detector response under real running conditions; the

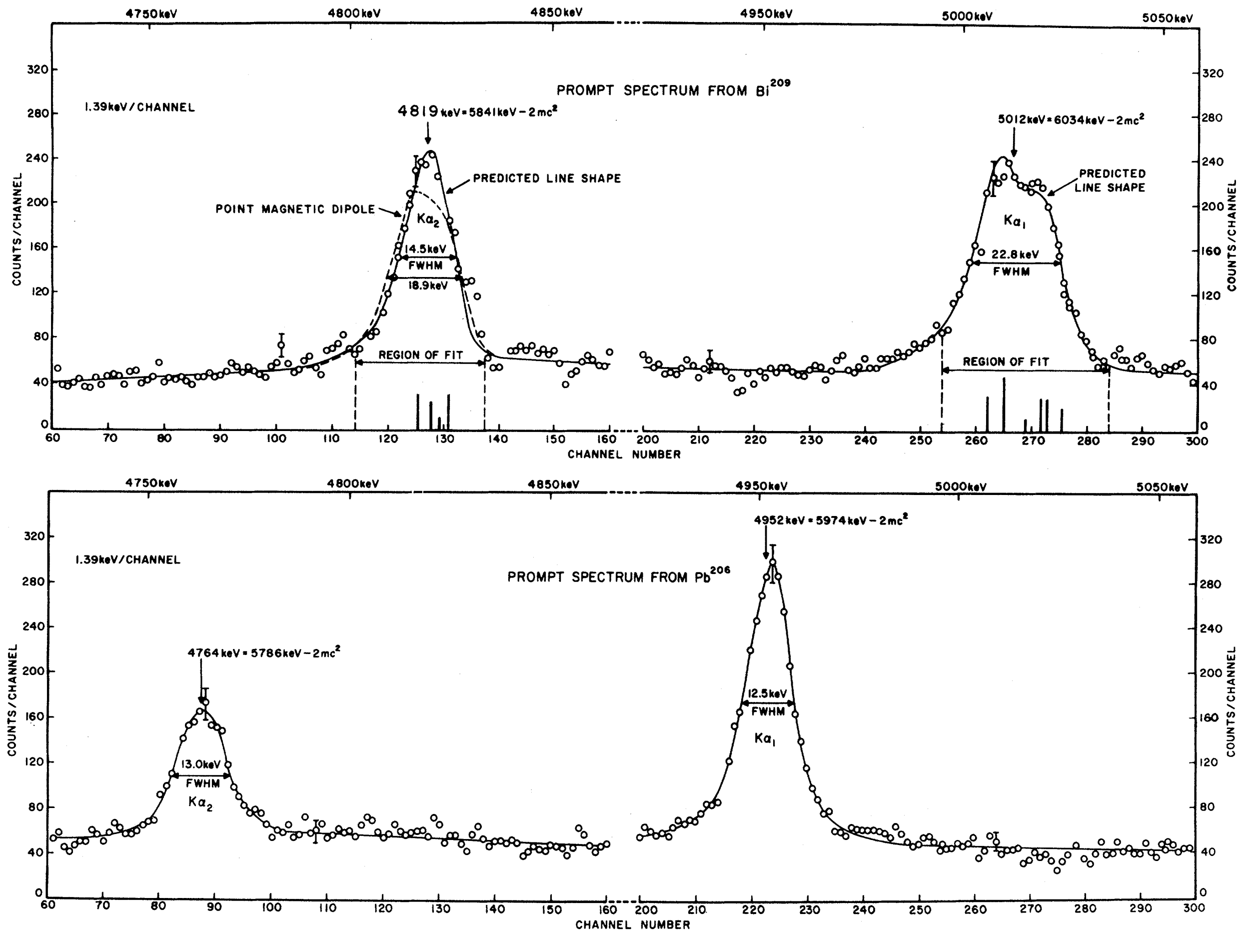

FIG. 1. Comparison of muonic K x rays in Bi^{209} and Pb^{206}. The line spectra shown at the base line are the hfs patterns calculated with the parameters for an extended dipole listed in Table I. The solid line represents the theoretical predictions normalized in the regions of fit and obtained by folding the observed Pb^{206} $K\alpha_1$ line shape into these spectra. The dashed curve in the Bi $K\alpha_2$ region is the expected line shape for a point magnetic dipole. In the $K\alpha_2$ x-ray region the dashed curve gives a χ^2 of 45.6 for 25 degrees of freedom while the solid line (extended dipole) gives a χ^2 of 30.2. The uncertainties in the indicated γ energies are less than 2 keV. The absolute energies quoted are referenced to the 6.131-MeV O^{16*} line.

Pb $K\alpha_1$ line shape shown was used to compute the "theoretical" line shapes for the two Bi transitions, adopting the hf constants given in Table I. The width of the Bi $K\alpha_2$ line (pure $M1$ hfs) rules out the point-dipole model[16]; the experiment is not sufficiently sensitive to verify the B-W effect proper. The Bi $K\alpha_2$ line shape agrees with the anticipated composite effect of $M1$ and $E2$ hfs, allowing for a volume effect in the latter as well. The energies of the four K transitions shown in Fig. 1 agree well with recent measurements. Our data also confirm the anomalous $K\alpha_1/K\alpha_2$ intensity ratio (1.42 ± 0.10) for Bi. For Pb the ratio is 2.02 ± 0.15.

Figure 2 (top) shows the experimental points near the L lines of Bi, observed as "prompt" radiations; the corrections mentioned in connection with Fig. 1 were applied here also. Fig. 2 (bottom) shows the prompt (a) and the delayed (b) events in an energy range between these two lines. The 2.615-keV capture γ ray from Pb^{208} emerges clearly in the delayed spectrum. The "bump" at 1616 keV in the spectrum (2) is tentatively attributed to a $6p \rightarrow 3s$ transition, but we remark that Pb $L\alpha$ spectra observed

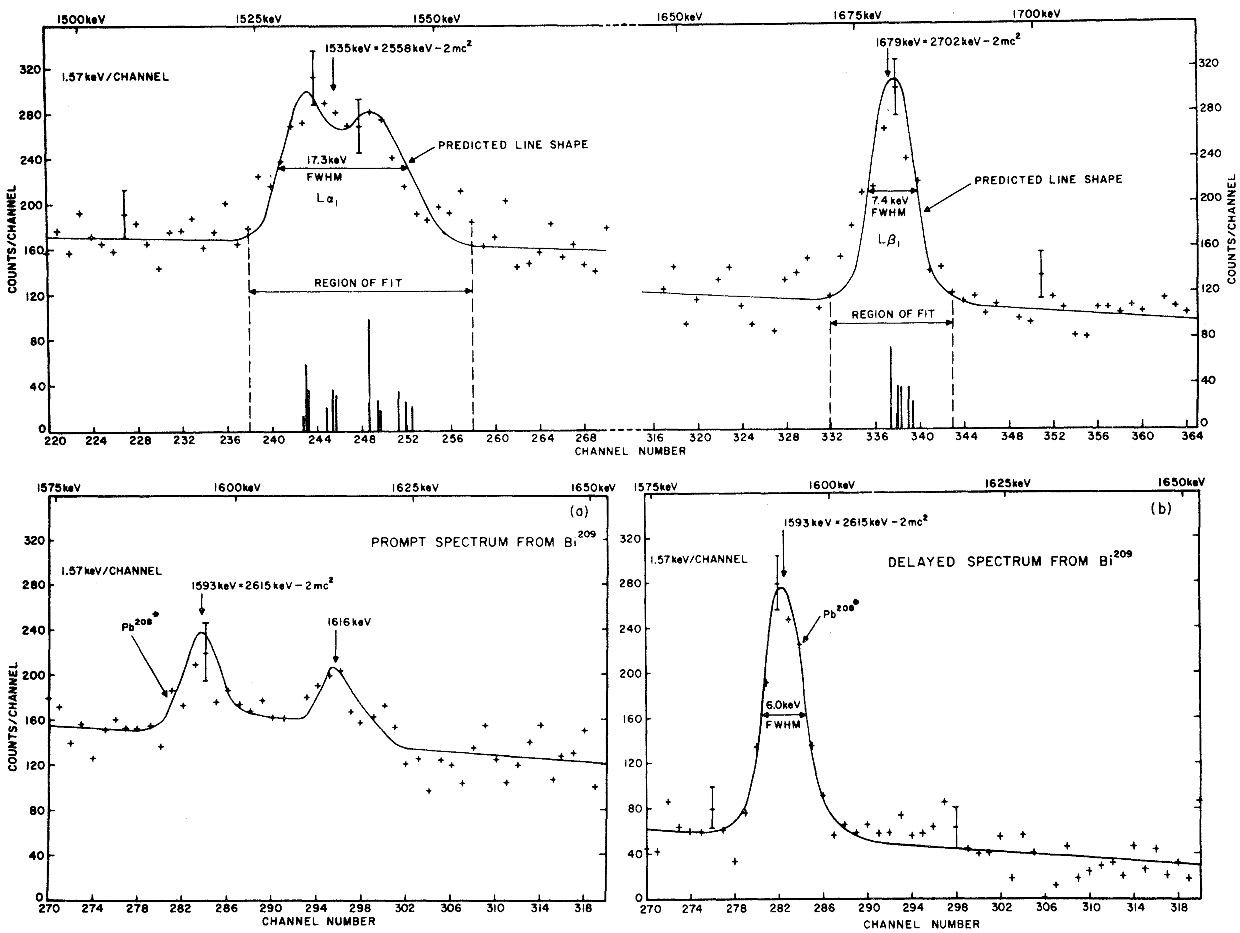

FIG. 2. (Top) Muonic L x-ray spectrum for Bi^{209}. The line spectra drawn at the base line are the hfs patterns calculated with the parameters for an extended dipole listed in Table I. The solid line represents the theoretical predictions normalized in the region of fit and obtained by folding in the observed line shape of the 2.615-MeV γ ray from Pb^{208}. (Bottom) Delayed (a) and prompt (b) spectra in the energy region between the two Bi L lines. The time window (with respect to the muon stop) is −15 to +25 nsec for the "prompt" events and +35 to +145 nsec for the delayed events. Here again the uncertainties in the indicated γ energies are less than 2 keV.

here (not given) show no corresponding "bump." The "theoretical" Bi L lines, computed using the line shape in Fig. 2(b) and the constants in Table I, are seen to agree well with experiment, although the best fit requires a slightly (10%) different A_2 from that used for matching the $K\alpha_1$ line shape. Again the absolute energies and the $L\alpha_1/L\beta_1$ intensity ratio (1.74 ± 0.15) are in agreement with recent determinations.[5] Our value for the $L\alpha_1/L\beta_1$ intensity ratio has been corrected for the energy dependence of (a) the pair-production cross section in germanium (+17%), (b) the self-absorption in the target (+1%), and (c) the probability for escape of secondary electrons (−1%). It is not clear whether previously reported values for this ratio include this correction. (The correction for the $K\alpha$ x rays is only 1%.)

In view of the anomalous intensity ratio of the $K\alpha$ doublet members (and the theoretical arguments[17] invoked to explain it), one should perhaps be warned against using straightforward static theory to analyze their hfs. Conversely, the reasonable agreement between such an analysis and experiment might be used to rule out certain explanations.

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[2]A. Bohr and V. F. Weisskopf, Phys. Rev. 77, 94 (1950).

[3]For a full discussion, see H. Kopfermann, Nuclear Moments (Academic Press, Inc., New York, 1958).

[4]H. L. Anderson, C. K. Hargrove, E. P. Hincks, and A. J. Tavendale, Proceedings of the International Conference on High-Energy Physics, Dubna, 1964 (to be published).

[5]The muonic x rays from Bi have already been investigated with Ge(Li) detectors by groups at Chicago (Anderson et al.), CERN, and Columbia. Spectra obtained by the last two groups were presented at the International Conference on Elementary Particles and Nuclear Structure in Brussels, 13-16 September 1965; at the 1966 New York American Physical Society Meeting two bismuth papers were presented: S. Raboy, R. E. Cote, R. Guso, R. A. Carrigan, Jr., A. Gaigalas, R. B. Sutton, and C. C. Trail, Bull. Am. Phys. Soc. 11, 129 (1966) and T. T. Bardin, R. Barrett, R. C. Cohen, S. Devons, D. Hitlin, E. Macagno, C. Nissim-Sabat, J. Rainwater, K. Runge, and C. S. Wu, Bull. Am. Phys. Soc. 11, 130 (1966).

[6]It is perhaps interesting to note that the first quantitative study (Back and Goudsmit, 1927) of atomic ($M1$) hfs was that of the Bi I spectrum.

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[9]M. LeBellac, Nucl. Phys. 40, 645 (1963).

[10]Allowance has, of course to be made for the $E2$ hfs of the $3d$ state.

[11]Radiogenic lead containing 88% Pb^{206}.

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[14]A full discussion of the experimental procedure will be submitted elsewhere by one of us (R. J. P.).

[15]R. A. Lundy, Rev. Sci. Instr. 34, 146 (1963). The instrument used here was a newer model developed by T. A. Nunamaker.

[16]The volume effect in the $E2$ hfs was already discussed (nonrelativistically) in J. A. Wheeler's fundamental paper on muonic x rays, Phys. Rev. 92, 812 (1953). The relevant relativistic formulas are given by LeBellac, Ref. 9.

[17]W. B. Rolnick, Phys. Rev. 132, 1110 (1963); J. Hüfner, thesis, University of Heidelberg, 1965 (unpublished); W. Frati and J. Rainwater, Ref. 12.

MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE HYPERFINE EFFECTS IN Bi^{209} MUONIC X RAYS*

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The excellence of the negative muon as a test particle for probing the electric and magnetic fields of nuclei has been recognized[1] since 1949. The problem has been to achieve adequate precision and energy resolution in the study of the emitted x rays so information could be obtained not only on the form factor of the charge distribution, but also on the magnetic dipole ($M1$) and electric quadrupole ($E2$) moments as well. The recent development of a high-resolution solid-state gamma-ray detector of lithium-drifted germanium has brought these aims much nearer to their realization.

It is well known that, for atomic electrons, the magnetic dipole hyperfine-structure effects are of approximately equal importance to that of the electric quadrupole interaction:

$$\frac{\langle \mu_N \mu_a / r^3 \rangle}{\langle e^2 Q / r^2 \rangle} \sim 1,$$