INEQUALITY FOR ELECTRON AND MUON SCATTERING FROM NUCLEONS*

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Adler[1] has derived a fixed-momentum-transfer sum rule for inelastic neutrino scattering from a nucleon. By isotopic rotation this may be turned into a useful inequality for inelastic electron-nucleon (or μ-nucleon) scattering:

$$\lim_{E\to\infty} \frac{d(\sigma_p+\sigma_n)}{d(q^2)} > \frac{2\pi\alpha^2}{q^4} = \frac{1}{2} \lim_{E\to\infty} \frac{d\sigma_p^{NS}}{d(q^2)}. \quad (1)$$

E and E' are incident and final laboratory energies of the electron, θ the scattering angle, and $q^2 = 4EE'\sin^2(\theta/2)$. σ_p is the total (elastic + inelastic) electron-proton cross section and σ_p^{NS} the cross-section from a point, spinless proton. More generally, writing

$$\frac{d(\sigma_p+\sigma_n)}{d(q^2)dE'} = \frac{E'}{E}\left\{\cos^2\frac{\theta}{2}F_1(q^2, E-E') + \sin^2\frac{\theta}{2}F_2(q^2, E-E')\right\}, \quad (2)$$

the inequality, Eq. (1), is

$$\int_0^\infty d\nu\, F_1(q^2,\nu) > \frac{2\pi\alpha^2}{q^4}. \quad (3)$$

Equation (1) has the classic structure of a sum rule except for the factor of 2 mismatch. It occurs because half of the point cross section comes from the isoscalar current, about which isotopic-spin commutation rules (the basic input to the sum rule) shed no light.

The key to arriving at Eq. (1) lies in the inequality

$$\langle p|Q^2|p\rangle + \langle n|Q^2|n\rangle > \tfrac{1}{2}\sum_{n;\,T=\frac{1}{2}} |\langle p|T^+|n\rangle|^2 - \tfrac{1}{3}\sum_{n;\,T=\frac{3}{2}} |\langle p|T^-|n\rangle|^2 = \tfrac{1}{2}\langle p|[T^+,T^-]|p\rangle = \tfrac{1}{2}. \quad (4)$$

Using the idea expressed in Eq. (4) (on current densities instead of charges) and Adler's calculation,[2] it is straightforward to obtain Eq. (3) and Eq. (1).

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[1]S. L. Adler, to be published; and Phys. Rev., to be published.

[2]The main assumptions in that derivation are (a) local commutation relations for the isovector charge densities, and (b) unsubtracted dispersion relations for certain components of the odd part of the forward-scattering amplitude of the isovector current from a nucleon. These points will be discussed in a forthcoming paper.

QUANTUM FIELD THEORY AND APPROXIMATE SYMMETRIES

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We want to point out in this note some difficulties facing the idea of approximate symmetries in particle physics. Using the currently accepted language,[1] the idea can be expressed as follows: There is a set of self-adjoint operators, the "charges" (space integral of local currents), obeying the commutation rules of some Lie algebra, and therefore representing the generators of some Lie group (the "symmetry" group). It is not assumed that these charges are constants of the motion; this would imply an exact symmetry, which is not realized in nature. Only the weaker requirement is made, that physical particles can still be represented by irreducible representations of the group, without requiring that the masses be degenerate, as would follow from an exact symmetry. The lack of symmetry also implies