NONLEPTONIC DECAYS OF HYPERONS*

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Recently Sugawara[1] and Suzuki[2] have shown that the current commutation relations based on the quark model,[3] and the partial conservation of axial-vector currents (PCAC),[4] lead to predictions about the nonleptonic hyperon decays which are compatible with experiment as far as the s-wave amplitudes are concerned. In this paper we show that the above two assumptions plus octet dominance suffice to determine both s- and p-wave amplitudes. At the end we shall also comment on the $K_{2\pi}$ decays.

We shall assume that the nonleptonic weak interaction Hamiltonian H_w is represented either by a product of quark currents in the Cabibbo form,

$$H_w \sim J_\mu{}^\dagger J_\mu + J_\mu J_\mu{}^\dagger, \qquad J_\mu = i\bar{q}\gamma_\mu(1+\gamma_5)(\lambda_1+i\lambda_2)q\cos\theta + i\bar{q}\gamma_\mu(1+\gamma_5)(\lambda_4+i\lambda_5)q\sin\theta, \tag{1}$$

as was done by the above authors, or by a sum of products of charged as well as neutral currents so that the $|\Delta S|=1$ part of H_w behaves as

$$H_w(|\Delta S|=1) \sim d_{bij} J_\mu{}^{(i)} J_\mu{}^{(j)}, \qquad J_\mu{}^{(i)} = i\bar{q}\gamma_\mu(1+\gamma_5)\lambda_i q = V_\mu{}^{(i)} + A_\mu{}^{(i)}. \tag{2}$$

In the former case dynamical enhancement of the octet part will have to be assumed in addition.[5]

The nonleptonic decay amplitude can be written as

$$(2k_0)^{1/2}\langle B^{(a)}\pi^{(b)}(k)|H_w(0)|B^{(c)}\rangle = i\int d^4x\, e^{-ikx}(\mu^2-\Box^2)\langle B^{(a)}|[\varphi^{(b)}(x),H_w(0)]|B^{(c)}\rangle\theta(-x_0), \tag{3}$$

where $\varphi^{(b)}(x)$ is the pion field. From the PCAC relation

$$\partial_\mu A_\mu{}^{(b)}(x) = c\varphi^{(b)}(x) \quad [c=(2m_N\mu^2/g_{\pi NN})(-G_A/G_V)], \tag{4}$$

we obtain

$$\begin{aligned} &c\int d^4x\langle B^{(a)}|[\varphi^{(b)}(x),H_w(0)]|B^{(c)}\rangle\theta(-x_0) \\ &= \int d^4x\langle B^{(a)}|[\partial_\mu A_\mu{}^{(b)}(x),H_w(0)]|B^{(c)}\rangle\theta(-x_0) \\ &= \lim_{k\to 0}\{\int d^4x\langle B^{(a)}|[\partial_\mu(A_\mu{}^{(b)}(x)e^{ikx}),H_w(0)]|B^{(c)}\rangle\theta(-x_0) \\ &\quad -ik_\mu\int d^4x\langle B^{(a)}|[A_\mu{}^{(b)}(x)e^{ikx},H_w(0)]|B^{(c)}\rangle\theta(-x_0)\},\quad (\mathrm{Im}k_0<0) \\ &= \langle B^{(a)}|[\int d^3x A_0{}^{(b)}(\vec{x}),H_w(0)]|B^{(c)}\rangle - \sum_{E_n=m(B_8)}\int d^3x[\langle B^{(a)}|A_0{}^{(b)}(\vec{x})|n\rangle\langle n|H_w(0)|B^{(c)}\rangle \\ &\qquad -\langle B^{(a)}|H_w(0)|n\rangle\langle n|A_0{}^{(b)}(\vec{x})|B^{(c)}\rangle]. \end{aligned} \tag{5}$$

From Eqs. (3) and (5) follows

$$(c/\mu^2)\lim_{k\to 0}(2k_0)^{1/2}\langle B^{(a)}\pi^{(b)}(k)|H_w(0)|B^{(c)}\rangle$$
$$=\langle B^{(a)}|[\int d^3x A_0{}^{(b)}(\vec{x}),H_w(0)]|B^{(c)}\rangle-\sum_{E_n=m(B_8)}\int d^3x[\langle B^{(a)}|A_0{}^{(b)}(\vec{x})|n\rangle\langle n|H_w(0)|B^{(c)}\rangle$$
$$-\langle B^{(a)}|H_w(0)|n\rangle\langle n|A_0{}^{(b)}(\vec{x})|B^{(c)}\rangle], \tag{6}$$

if we assume that both sides of Eq. (6) satisfy unsubtracted dispersion relations. The sum must be taken only over the states degenerate in energy and spin with the initial and final states, i.e., the octet baryons B_8.

Equation (6) is identical with the formula for soft pion emission (as induced by H_w) developed by Nambu and Schrauner[6] on the basis of explicit chiral invariance of strong interactions. According to this work, the second term corresponds to "pole diagrams" where the meson is emitted by the initial or final baryon. These diagrams are important in the sense that in the limit of degeneracy $E_n\to m(B_8)$ and zero meson mass $\mu\to 0$, the p-wave meson-emission amplitude remains finite because of a vanishing energy denominator.[7] This justifies the retention of the pole terms even though in the above derivation they would seem to survive only under strict degeneracy.

Working in the above limit, Eq. (6) can now be expressed in terms of the spurion $\langle B^{(a)}|H_w(0)\times|B^{(c)}\rangle=S^{(ac)}$ and the pion-baryon coupling constants alone. The CP invariance implies that $S^{(ac)}$ is a scalar spurion.[2] The first term (Sugawara and Suzuki) gives only S waves since $[\int d^3x A_0{}^{(b)}(\vec{x}),H_w(0)]$ is effectively a scalar spurion due to CP invariance.[2] The second term, interpreted as pole diagrams, gives only p waves (induced by the scalar spurion).[8] The final result can be cast in the form

$$(c/\mu^2)(2k_0)^{1/2}\langle B^{(a)}\pi^{(b)}|H_w(0)|B^{(c)}\rangle$$
$$=A_{abc}\langle 1\rangle-B_{abc}\langle\vec{\sigma}\rangle\cdot\Delta\vec{p}/\Delta m. \tag{7}$$

Here

$$A_{abc}/\sqrt{2}=(D+F)\operatorname{tr}(\bar{B}_a[M_b,\lambda_6]B_c)-(D-F)\operatorname{tr}(\bar{B}_aB_c[\lambda_6,M_b]),$$
$$B_{abc}/\sqrt{2}=(D+F)(d+f)\operatorname{tr}(\bar{B}_a[M_b,\lambda_6]B_c)+(D-F)(d-f)\operatorname{tr}(\bar{B}_aB_c[\lambda_6,M_b])$$
$$-\tfrac{4}{3}Dd[\operatorname{tr}(\bar{B}_aM_b)\operatorname{tr}(B_c\lambda_6)-\operatorname{tr}(\bar{B}_a\lambda_6)\operatorname{tr}(B_cM_b)]. \tag{8}$$

$\bar{B}$, B, and M are normalized 3×3 baryon and meson matrices, and

$$\langle B'^{(a)}|H_w|B^{(c)}\rangle=(2DD_{6ac}+2FF_{6ac})\bar{u}'u,$$
$$\langle B'^{(a)}|A_\mu{}^{(b)}|B^{(c)}\rangle$$
$$=(2dD_{bac}+2fF_{bac})i\bar{u}'\gamma_\mu\gamma_5u. \tag{9}$$

In deriving Eq. (7), we have made the simplifying assumption $m_\Lambda=m_\Sigma$, namely that the intermediate states in Eq. (6) have either $m=m^{(a)}$ or $m=m^{(c)}$. There is an ambiguity as to whether (a) we should stick to the degeneracy limit $|\Delta p/\Delta m|=1$; or (b) take the actual values $|\Delta p/\Delta m|\approx q_\pi/190$ MeV $[\Delta m\approx(m_\Xi-m_N)/2]$. The A's and B's are listed in Table I. They satisfy the three $|\Delta I|=\frac{1}{2}$ sum rules as well as the Lee-Sugawara relation[9]

$$\Lambda_-{}^-+2\Xi_-{}^-=\sqrt{3}\Sigma_0{}^+, \tag{10}$$

together with $A(\Sigma_+{}^+)=0$.

Equation (8) contains only four parameters: D, F, d, and f. We have tried two different fits to 10 (s- and p-wave) amplitudes under the prescriptions (a) and (b) above.[10] $\Lambda_0{}^0$ and $\Xi_0{}^0$ are ignored as the $|\Delta I|=\frac{1}{2}$ rule is well satisfied for Λ and Ξ. The solutions are (fit b in

Table I. Coefficients A and B of Eq. (8).

	A	B
Λ_-^0	$-(D+3F)/\sqrt{3}$	$-[2(D+F)(d+f)+(D-F)(d-f)]/\sqrt{3}$
Λ_0^0	$(D+3F)/\sqrt{6}$	$[2(D+F)(d+f)+(D-F)(d-f)]/\sqrt{6}$
Ξ_-^-	$(-D+3F)/\sqrt{3}$	$[(D+F)(d+f)+2(D-F)(d-f)]/\sqrt{3}$
Ξ_0^0	$(D-3F)/\sqrt{6}$	$-[(D+F)(d+f)+2(D-F)(d-f)]/\sqrt{6}$
Σ_+^+	0	$\sqrt{2}(\frac{4}{3})Dd$
Σ_0^+	$-(D-F)$	$(D-F)(d-f)$
Σ_-^-	$\sqrt{2}(D-F)$	$\sqrt{2}[(\frac{4}{3})Dd-(D-F)(d-f)]$

brackets)

$$D=0.95(1.1)\times r\times 10^{-7}\mu,$$
$$F=-2.2(-2.2)\times r\times 10^{-7}\mu,$$
$$d=1.02(1.95),\quad f=0.48(1.10), \tag{11}$$

where $r=(-G_A/G_V)$ appears through the coefficient c in Eq. (4). Table II displays the comparison with experiment. The agreement is reasonable except for $B(\Sigma_+^+)$ in fit a. We must bear in mind that the $|\Delta I|=\frac{1}{2}$ rule for the Σ's and the Lee-Sugawara relation are not quite satisfied by experiment in the case of p waves.

Eq. (11) gives $d+f(=-G_A/G_V)=1.5(3.0)$ and $d/f=2.1(1.8)$, which are to be compared with their expected values 1.18 and 1.7 (leptonic decay data[11]), or $\frac{5}{3}$ and $\frac{3}{2}$ [SU(6) theory]. The ratio $D/F=-0.42(-0.5)$ for the spurion is,[12] interestingly enough, comparable to those for the strong and electromagnetic splittings, viz. $D/F\sim -0.33$, suggesting a universal coupling of spurions.

We may thus conclude that the nonleptonic decays can be described reasonably well by means of the four parameters. Their values are theoretically satisfactory except that $d+f$ is uncertain by a factor of 2 and tends to be too large. The deviation from the sum rules in p waves must be attributed to various corrections to our formulas.

Finally we remark that the same procedure may be applied to K decays, relating any two processes which differ by an extra π emission. In this case, however, there are no pole diagrams. Now let us assume the universal spurion coupling, and relate $K\to 2\pi$ decays to the transition $K\to\pi$ due to a spurion D'. D' may be determined by requiring that D'/D (or D'/F) is equal to the corresponding one for the strong (Okubo-Gell-Mann) splitting. The $K_1^0\to 2\pi$ decay rate predicted from this[13] is $\sim 2\times 10^{10}$/sec as compared to the experimental 1.1×10^{10}/sec. The $K_{2\pi}^{\pm}$ decays are forbidden since we have assumed a strict $|\Delta I|=\frac{1}{2}$ rule.

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Table II. Comparison with experiment.[10] The alternative theoretical values correspond to the two solutions in Eq. (11).

		Λ_-^0	Ξ_-^-	Σ_+^+	Σ_0^+	Σ_-^-
$(\mu^2/c)A\times 10^7$	Expt.	3.3 ± 0.1	-4.4 ± 0.1	-0.1 ± 0.2	-1.9 ± 0.3 -3.8 ± 0.3	4.2 ± 0.1
	Theory	3.3(3.2)	−4.4(−4.5)	0(0)	−3.2(−3.3)	4.5(4.7)
$(\mu^2/c)\lvert\Delta p/\Delta m\rvert\times B\times 10^7$	Expt.	1.2 ± 0.1	0.9 ± 0.1	4.2 ± 0.2	3.7 ± 0.3 1.5 ± 0.2	-0.5 ± 0.6
	Theory	1.2(1.2)	0.9(0.9)	1.8(4.0)	1.7(2.8)	−0.6(−0.1)
Decay rate,	Expt.	2.6	5.7	6.2	6.4	6.3
$\Gamma\times 10^9$ sec	Theory	2.6	5.7(5.9)	1.0(5.6)	5.0(7.0)	7.3(7.8)
Asymmetry α	Expt.	0.66 ± 0.05	-0.41 ± 0.05	-0.05 ± 0.08	-0.79 ± 0.1	-0.16 ± 0.21
	Theory	0.66	−0.40(−0.39)	0	−0.83(−0.99)	−0.26(−0.04)

respectively.

[7]We can see this, also, by observing that the invariant amplitude (6) is to be evaluated in a reference frame in which the emitted meson is at rest. As the meson mass decreases, the baryon velocities increase, and hence the matrix elements of A_0 remain finite.

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[13]There is a peculiar paradox in that the answer depends on to which of the three mesons we apply the PCAC formula. This is due to the fact that the PCAC formula becomes exact only in the limit where the meson represented by the axial current becomes massless and the corresponding chiral SU(2) symmetry is exact. In the SU(3)-symmetry limit (all masses equal), the $K_1^0 \rightarrow 2\pi$ amplitude vanishes in agreement with Gell-Mann's general argument (Ref. 9). The finite values quoted here are obtained because we are off the symmetry point. More details about this problem will be reported elsewhere.