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RELATION BETWEEN D/F AND G_A/G_V FROM CURRENT ALGEBRA

R. Gatto

Istituto di Fisica dell'Università, Firenze, Italy

L. Maiani

Istituto Superiore di Sanità, Roma, Italy

and

G. Preparata

Sezione di Firenze dell'Istituto Nazionale di Fisica Nucleare, Firenze, Italy

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We examine in this note the inclusion of higher baryonic resonances in the saturation of the chiral U(3)⊗ U(3) commutation rules of Gell-Mann's current algebra.[1] We find a consistent solution if the relation

$$-\frac{G_A}{G_V}=\frac{1}{3}\,\frac{D+F}{D-F} \tag{1}$$

is verified. Eq. (1) is well satisfied experimentally.

Recent work has led to satisfactory calculations of G_A/G_V and D/F from sum rules over experimental meson-nucleon cross sections, obtained from the current algebra.[2] We follow here the approach suggested by Lee[3] and Dashen and Gell-Mann,[4] by saturating the commutation relations among stable and resonant baryon states and looking for a consistent solution.

In selecting the important resonant multiplets, we have been guided by a recent discussion[5] of higher baryonic resonances, suggesting the relevance of a classification according to the representation $\underline{20}$ of SU(6) with orbital angular momentum $L=1$. We thus include among the initial, final, and intermediate states, besides the $\underline{56}$ baryon states [multiplets $(\underline{8},\frac{1}{2}^+)$ and $(\underline{10},\frac{3}{2}^+)$ in the notation $(\mathrm{SU}(3),J^P)$], the states of $\underline{20}$ with $L=1$ [multiplets $(\underline{1},\frac{1}{2}^-)$, $(\underline{1},\frac{3}{2}^-)$, $(\underline{1},\frac{5}{2}^-)$, $(\underline{8},\frac{1}{2}^-)$, and $(\underline{8},\frac{3}{2}^-)$]. The commutation relation

$$[F_\lambda{}^5,F_\mu{}^5]=-\sqrt{3}\begin{pmatrix}8&8&8a\\ \lambda&\mu&\nu\end{pmatrix}F_\nu, \tag{2}$$

where $F_\lambda{}^5$ is an axial-vector generator and

$$\begin{pmatrix}8&8&8a\\ \lambda&\mu&\nu\end{pmatrix}$$

is a SU(3) Clebsch-Gordan coefficient, is taken between states with momentum $\vec{p}_i$ and $\vec{p}_f$, in the limit $\vec{p}_i = \vec{p}_f \to \infty$.[6] From invariance arguments it follows that there is only one independent matrix element in this limit for each transition. A particle of half-integer spin J is described by a Rarita-Schwinger spinor $\psi_{\mu\nu\cdots\lambda}$ with $J-\frac{1}{2}$ Lorentz indices $\mu, \nu, \cdots, \lambda$ and subject to the conditions (i) complete symmetry and zero traces; (ii) $\gamma_\mu \psi_{\mu\nu\cdots\lambda} = 0$; (iii) $P_\mu \psi_{\mu\nu\cdots\lambda} = 0$; (iv) $(\not{p}-m)\psi_{\mu\nu\cdots\lambda} = 0$. The matrix element of F^5 between states J^P and $J'^{P'}$ (with $|J-J'| \leq 1$) is uniquely determined in the limit $\vec{p}_i = \vec{p}_f \to \infty$ from the above conditions, according to the following rules[7]: For $J=J'$ and $PP'=+1$ the only possible coupling between the spinors ψ and φ is $i\bar{\varphi}_{\mu\cdots\lambda}\gamma_4\gamma_5\psi_{\mu\cdots\lambda}$; for $J=J'$ and $PP'=-1$ the coupling is $\bar{\varphi}_{\mu\cdots\lambda} \times \gamma_4 \psi_{\mu\cdots\lambda}$; for $J=J'+1$, $PP'=+1$ the coupling is $\bar{\varphi}_{4\nu\cdots\lambda}\psi_{\nu\cdots\lambda}$; and for $J=J+1$, $PP=-1$ one has $i\bar{\varphi}_{4\nu\cdots\lambda}\gamma_5\psi_{\nu\cdots\lambda}$. We note also that the operator F^5 has the selection rule $\Delta h = 0$, where h is the helicity, and that the matrix elements between states of helicity h are related to those between states of helicity $-h$ by the equation

$$\langle J^P h | F^5 | J'^{P'} h \rangle = -PP' \langle J^P -h | F^5 | J'^{P'} -h \rangle. \quad (3)$$

Eq. (3) is obtained by applying a parity operation followed by a 180° rotation around an axis orthogonal to the momentum. All the above conclusions hold in general. The particular matrix elements of interest here depend upon the helicities as follows:

$$\langle \tfrac{1}{2}^\pm | F^5 | \tfrac{1}{2}^\pm \rangle \propto \chi^\dagger \sigma_3 \chi', \quad \langle \tfrac{3}{2}^\pm | F^5 | \tfrac{3}{2}^\pm \rangle \propto \chi_K^\dagger \sigma_3 \chi_K',$$

$$\langle \tfrac{1}{2}^\mp | F^5 | \tfrac{1}{2}^\pm \rangle \propto \chi^\dagger \chi, \quad \langle \tfrac{3}{2}^\pm | F^5 | \tfrac{3}{2}^\mp \rangle \propto \chi_K^\dagger \chi_K',$$

$$\langle \tfrac{3}{2}^\pm | F^5 | \tfrac{1}{2}^\mp \rangle \propto \chi_3^\dagger \sigma_3 \chi', \quad \langle \tfrac{3}{2}^- | F^5 | \tfrac{5}{2}^- \rangle \propto \chi_i^\dagger \chi_{i3}',$$

$$\langle \tfrac{3}{2}^\pm | F^5 | \tfrac{1}{2}^\pm \rangle \propto \chi_3^\dagger \chi', \quad \langle \tfrac{3}{2}^\dagger | F^5 | \tfrac{5}{2}^- \rangle \propto \chi_i^\dagger \sigma_3 \chi_{i3}',$$

$$\langle \tfrac{5}{2}^- | F^5 | \tfrac{5}{2}^- \rangle \propto \chi_{iK}^\dagger \sigma_3 \chi_{iK}',$$

where $\chi_{i\cdots K}$ are Pauli spinors for each index $i, \cdots, k = 1, 2, 3$, are symmetric and traceless in the indices $i, \cdots, k$, and satisfy $\sigma_i \chi_{i\cdots K} = 0$.

We write the states as $|R\rho, J^P h\rangle$, where R denotes the dimensionality of the SU(3) representation; ρ, the set of quantum numbers I, I_3, Y; J^P, the spin and the parity; and h, the helicity. The matrix elements of the axial charges can then be written as

$$\langle R\rho, J^P h | F_\lambda^5 | R'\rho', J'^{P'} h' \rangle = \delta_{hh'} C(J^P J'^{P'}, h) \times \sum G_\xi (RR' | J^P J'^{P'}) \begin{pmatrix} R' & 8 & R_\xi \\ \rho' & \lambda & \rho \end{pmatrix}, \quad (4)$$

where $C(J^P J'^{P'}, h)$ are the values assumed for the different cases of $J^P, J'^{P'}$ by the couplings of the preceding discussion, $G_\xi(RR'|J^P J'^{P'})$ are the unknown coupling coefficients, and the sum includes all the representations R_ξ connected through the Clebsch-Gordan coefficient in (4). For $J^P = J'^{P'} = \frac{1}{2}^+$ our normalization coincides with that of Lee.[3] From Eq. (3), $C(J^P, J'^{P'}, h) = (-1)^{PP'+1} C(J^P J'^{P'}, -h)$. Also one has $C(J^P J'^{P'}, h) = C(J'^{P'} J^P, h)$. The matrix elements of the vector charges can similarly be written as

$$\langle R\rho, J^P h | F_\nu | R'\rho', J'^{P'} h' \rangle = \delta_{RR'} \delta_{JJ'} \delta_{\rho\rho'} \delta_{hh'} f(R) \begin{pmatrix} R' & 8 & R_a \\ \rho' & \nu & \rho \end{pmatrix}, \quad (5)$$

where $f(\underline{8}) = \sqrt{3}$ and $f(\underline{10}) = \sqrt{6}$. Substituting into Eq. (2), taken between states R, J^P and R', and $J'^{P'}$, we obtain the equation

$$(R/Q)^{1/2} \xi_1(R'RQ_\beta) \xi_2(R'QR_\beta) \xi_3(R'R^*Q_\beta{}^*) [1 - \xi_1(88Q_\alpha)] \sum_{j\pi T\xi\xi'} T G_\xi (TR | j^\pi J^P) G_{\xi'}(TR' | j^\pi J'^{P'}) \times (T\xi'\xi | \beta_{\mathrm{II}}(R8R'8) | Q\alpha\beta) C(J_j^{P\pi}, h) C(J'^{P'} j^\pi, h) = -\sqrt{3}\, \delta_{Q8} \delta_{\alpha a} \delta_{\beta a} \delta_{RR'} \delta_{JJ'} \delta_{PP'} f(R) R. \quad (6)$$

Equation (6) is written in de Swart's notations[8]; in particular, $(T\xi'\xi | \beta_{\mathrm{II}}(R8R'8) | Q\alpha\beta)$ is a crossing matrix, the symbols $\xi_1(R'RQ_\beta)$, etc., are phases, $Q\alpha\beta$ takes on the values 1, 8_{aa}, 8_{as}, 8_{sa}, 8_{ss}, 10, 10*, and 27. The values of (R, J^P), $(R'J'^{P'})$, and (T, j^π) vary over the selected set of states. The compact form (6) specialized to our case gives a set of 48 equations for the unknown transition strengths $G_\xi(RR'|J^P J'^{P'})$. Solution of a set of 48 nonlinear equations is a lengthy and not usually

Table I. Solution of the current-algebra equations, Eq. (6). The strength $G_\xi(RR'|J^PJ'^{P'})$ is that of the transition between the states J^P of the SU(3) representation R and $J'^{P'}$ of the representation R' [for $R=R'=\underline{8}$ the index ξ distinguishes between symmetric (s) and antisymmetric (a) coupling]. The parameters a and d satisfy $0 \leq a \leq 1$ and $|d| \leq 4/\sqrt{3}$. The symbols $\alpha, \beta, \gamma, \delta, \epsilon, \eta$ denote arbitrary signs ± 1. For $a=1$ the solution reproduces for the $(\underline{8}, \frac{1}{2}^+)$ and $(\underline{10}, \frac{3}{2}^+)$ multiplets the well-known SU(6) results.

$$
\begin{aligned}
G_a(8\ 8|\tfrac{1}{2}^+\ \tfrac{1}{2}^+) &= 2/\sqrt{3}a \\
G_s(8\ 8|\tfrac{1}{2}^+\ \tfrac{1}{2}^+) &= (5/3)^{1/2}\tfrac{1}{3}(1+2a) \\
G(8\ 10|\tfrac{1}{2}^+\ \tfrac{3}{2}^+) &= \epsilon 2(15a)^{1/2} \\
G_a(8\ 8|\tfrac{1}{2}^+\ \tfrac{3}{2}^-) &= 0 \\
G_s(8\ 8|\tfrac{1}{2}^+\ \tfrac{3}{2}^-) &= \tfrac{2}{3}\eta[10(1-a)]^{1/2} \\
G_a(8\ 8|\tfrac{1}{2}^+\ \tfrac{1}{2}^-) &= \epsilon\delta \times 2[a(1-a)/3]^{1/2} \\
G_s(8\ 8|\tfrac{1}{2}^+\ \tfrac{1}{2}^-) &= \epsilon\delta \times \tfrac{2}{3}[5a(1-a)/3]^{1/2} \\
G(1\ 8|\tfrac{3}{2}^-\ \tfrac{1}{2}^+) &= \alpha\beta\epsilon\delta \times 4[(1-a)(80+d^2)/3(80+3d^2)]^{1/2} \\
G(1\ 8|\tfrac{1}{2}^-\ \tfrac{1}{2}^+) &= -\beta\epsilon\delta \times 4d[2(1-a)/3(80+3d^2)]^{1/2} \\
G(10\ 10|\tfrac{3}{2}^+\ \tfrac{3}{2}^+) &= \sqrt{6} \\
G(8\ 10|\tfrac{3}{2}^-\ \tfrac{3}{2}^+) &= 0 \\
G(8\ 10|\tfrac{1}{2}^-\ \tfrac{3}{2}^+) &= \delta[10(1-a)]^{1/2} \\
G_a(8\ 8|\tfrac{3}{2}^-\ \tfrac{3}{2}^-) &= 0 \\
G_s(8\ 8|\tfrac{3}{2}^-\ \tfrac{3}{2}^-) &= (5/3)^{1/2} \\
G_a(8\ 8|\tfrac{3}{2}^-\ \tfrac{1}{2}^-) &= 0 \\
G_s(8\ 8|\tfrac{3}{2}^-\ \tfrac{1}{2}^-) &= -\epsilon\delta\eta \times 4(5a/3)^{1/2} \\
G(1\ 8|\tfrac{1}{2}^-\ \tfrac{3}{2}^-) &= \alpha(80+d^2)^{1/2} \\
G(1\ 8|\tfrac{3}{2}^-\ \tfrac{3}{2}^-) &= d \\
G(1\ 8|\tfrac{5}{2}^-\ \tfrac{3}{2}^-) &= \gamma\sqrt{2}(16/3-d^2)^{1/2} \\
G_a(8\ 8|\tfrac{1}{2}^-\ \tfrac{1}{2}^-) &= (2/\sqrt{3})(1-a) \\
G_s(8\ 8|\tfrac{1}{2}^-\ \tfrac{1}{2}^-) &= (5/3)^{1/2}(1-\tfrac{2}{3}a) \\
G(1\ 8|\tfrac{1}{2}^-\ \tfrac{1}{2}^-) &= \beta \times 4d[2a/3(80+3d^2)]^{1/2} \\
G(1\ 8|\tfrac{3}{2}^-\ \tfrac{1}{2}^-) &= -\alpha\beta \times 4[2a(80+d^2)/(80+3d^2)]^{1/2}
\end{aligned}
$$

successful task. Among the various solutions, the one reported in Table I (including some arbitrary choices of signs) appears as physically significant. The strengths reported in Table I are all in principle susceptible of physical interpretations (through weak transitions by neutrinos; and by relating them to strong couplings by the Goldberger-Treiman argument). A very simple and important conclusion can be derived by noting that $G_a(8\ 8|\frac{1}{2}^+\ \frac{1}{2}^+)$ and $G_s(8\ 8|\frac{1}{2}^+\ \frac{1}{2}^+)$ are given in terms of the single parameter a. This implies the relation (1).

The above extension of the current-algebra approach can be applied also to meson states, including besides the states of $\underline{35}$ of SU(6) those of $\underline{35}$ with $L=1$,[9] and leading to predictions about the widths and branching ratios of the different decay modes. An interesting application concerns also the commutation relations of the magnetic and electric moments. It would be useful if mathematical methods could be devised to solve the cumbersome nonlinear sets of equations, compactly expressed by Eq. (6). A more ambitious program would contemplate the saturation of the commutator algebra with representations of noncompact groups, except for the apparently prohibitive mathematical difficulties.

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