and the only intermediate state it connects to the $K^+$ state is the $\pi^0$. Thus in the SU(3)-symmetry limit ($M_k = M_\pi$), we find

$$F_+(0) = (M_\pi^2/C_\pi)g. \qquad (21)$$

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‡On leave of absence from the Tata Institute of Fundamental Research, Colaba, Bombay 5, India.

[1]S. Fubini and G. Furlan, Physics 1, 229 (1965); G. Furlan, F. Lannoy, C. Rosetti, and G. Segrè, Nuovo Cimento 38, 1747 (1965), and to be published; S. Fubini, G. Furlan, and C. Rosetti, to be published; S. Okubo, University of Rochester Report No. UR-875-97, 1965 (unpublished); V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, University of California Radiation Laboratory Report No. UCRL 16367, 1965 (unpublished).

[2]S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); Phys. Rev. 140, B736 (1965); W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965); L. K. Pandit and J. Schechter, Phys. Letters 19, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, Phys. Letters 19, 59 (1965); A. Sato and S. Sasaki, to be published; C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); V. S. Mathur and L. K. Pandit, University of Rochester Report No. UR-875-103, 1965 (unpublished); I. J. Muzinich and S. Nussinov, Phys. Letters 19, 248 (1965); V. S. Mathur and L. K. Pandit, to be published; S. Okubo, to be published; H. Sugawara, Phys. Rev. Letters 15, 870 (1965); R. Oehme, University of Chicago Report No. COO-264-287, 1965 (unpublished); S. L. Adler, to be published.

[3]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[4]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[5]J. J. Sakurai, Ann. Phys. 11, 1 (1960).

[6]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963). A result similar to Eq. (17) has been derived on the basis of a broken SU(3) model by R. Oehme [Ann. Phys. 33, 108 (1965)] and by R. Oehme and G. Segrè [Phys. Letters 11, 94 (1964)]. However, both the motivation and the technique involved are quite different from ours. In particular our main results contained in Eqs. (15) and (16) cannot be obtained on the basis of this model.

[7]See, for example, J. D. Jackson, Lectures in Theoretical Physics, Brandeis University, 1962 (W. A. Benjamin, Inc., New York, 1963).

[8]The appearance of $\rho$ and $\kappa$ terms in the single particle contributions to $F_+(0)$ and, presumably to $F_+(-[q-q']^2)$ is not surprising as these states appear in the crossed channels of the amplitudes for $K+\pi \rightarrow K+\pi$.

[9]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1965).

[10]S. Okubo, see Ref. (1).

[11]M. Gell-Mann, Physics 1, 63 (1964); R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965).

[12]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).

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# EXPERIMENTAL STUDY OF $\pi$-$\pi$ INTERACTION IN THE REACTION $\pi^- + p \rightarrow \pi^0 + \pi^0 + n$ AT INCIDENT $\pi^-$ ENERGY OF 378 MeV*

Tin Maung, Kenneth M. Crowe, and Ned T. Dairiki

Lawrence Radiation Laboratory, University of California, Berkeley, California
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In the study of the reactions

$$p + d \rightarrow \mathrm{He}^3 + 2\pi \qquad (1a)$$

and

$$p + d \rightarrow \mathrm{H}^3 + 2\pi, \qquad (1b)$$

Abashian, Booth, and Crowe (ABC)[1] observed a peak in the momentum distribution of $\mathrm{He}^3$, which was subsequently interpreted as a strong $S$-wave $I=0$, $\pi$-$\pi$ scattering-length effect. We have completed an experiment in which we measured the differential neutron time-of-flight distribution from the reaction

$$\pi^- + p \rightarrow n + \pi^0 + \pi^0 \qquad (2)$$

at a $\pi^-$ laboratory kinetic energy of 378 MeV, to look for this effect.

We measured the neutron spectrum at 45 deg (lab) in coincidence with the $\gamma$ rays produced in Reaction (2). The experimental setup is as shown in Fig. 1. Negative pions, produced at an internal target of the 184-inch cyclotron were momentum analyzed and focused on a 3-in.-diameter by 10-in.-long liquid-hydrogen target, which was completely surrounded by scintillation counters. The pion beam was designed to have less than ±1.5 deg of angular divergence. 14 liquid-scintillation neutron counters and six lead-scintillator sandwich counters were used to detect the neutrons and $\pi^0$-decay $\gamma$ rays,

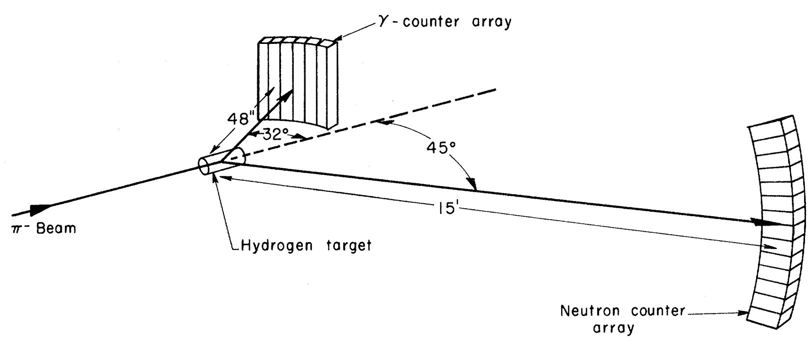


FIG. 1. Experimental arrangement showing the location of the six gamma counters and the 14 neutron counters.

respectively. When the direction of the neutron is fixed, the charge-exchange $\pi^0$ has a unique energy and direction in space. Therefore, only one of the two $\gamma$ rays from the charge-exchange $\pi^0$ is detected on any one side of this direction. We utilized this fact and placed the $\gamma$-ray detectors on one side of this space direction. Two or more $\gamma$ rays were then required in coincidence with an interacted pion signal before the gate for the neutron time-of-flight measurement was opened. The charge-exchange neutron contamination was thus reduced by a factor of about 100.

Resolution of the neutron time-of-flight system was measured by turning off the $\gamma$ requirement, and was found to be 4.5 nsec (full width at half-maximum). Calibration of the time-of-flight system was checked by measuring the prompt $\gamma$ ray and the charge-exchange neutron times of flight. The calibration agrees within ±0.25 nsec with the calculated values. The measured total cross section for Reaction (2) is 1.40± 0.21 mb at 500 MeV/$c$, in agreement with the published data of Barish et al.[2]

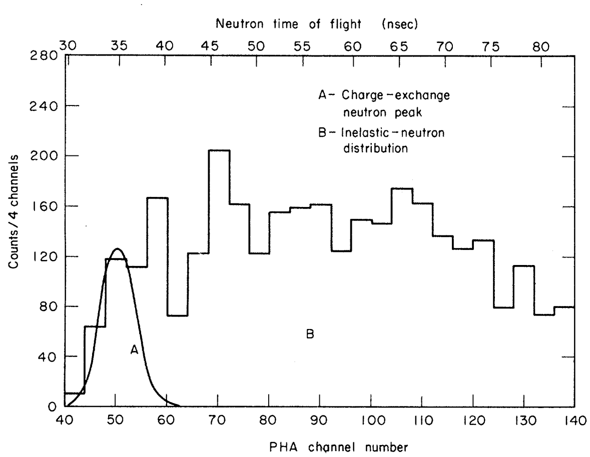


FIG. 2. Figure showing the subtraction of the charge-exchange neutron peak from the raw data.

Superimposed on the raw time-of-flight data are the following: (a) the charge-exchange neutron peak, which although reduced in magnitude by the gamma requirement, appears around $\beta = 0.458$; (b) the inelastic neutron distribution starting at $\beta = 0.41$. Assuming a Gaussian resolution function and using the measured neutron resolution width, we have subtracted the charge-exchange peak from the raw data to separate out the inelastic distribution. The subtraction is presented in Fig. 2. Because of the relatively long flight path for the neutrons, there is very little overlap between the times of flight of the charge-exchange and the inelastic neutrons.

A strong $I = 0$ two-pion interaction around a di-pion mass of 400 MeV was observed in several pion-production experiments.[2-6] To explain the experimental results, Brown and Singer proposed a two-pion resonance with an effective mass of 400 MeV and a width of 100 MeV.[7,8] The data of the $K_{e4}^+$ decay experiment of Birge et al.[9] do not show any evidence of the sigma. The appearance of the sigma-type enhancement in the inelastic pion-nucleon interactions has been explained most recently by Dalitz and Moorhouse as being caused by a strong $P_{11}$ $\pi$-$N$ absorption.[10] The presence of this enhancement can mask the $\pi$-$\pi$ scattering-length effect present in the data. We therefore chose to limit the available $\pi$-$\pi$ effective mass to a maximum of 316 MeV to minimize its contribution.

The inelastic neutron time-of-flight data are presented in Fig. 3. Also shown in the figure are the following calculated distributions:

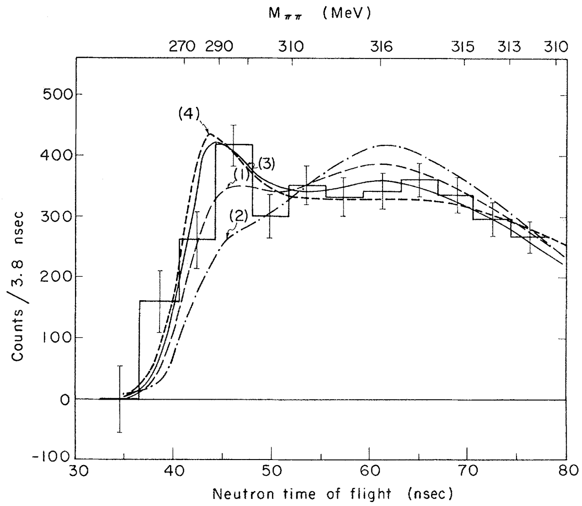


FIG. 3. Measured inelastic neutron time-of-flight spectrum, presented together with the following calculated distributions: (1) (P. S.), (2) (P. S.) $\times\sigma$, (3) (P. S.) $\times$ [20 $(\pi\pi)+\sigma$] for $a_{S0}=2$ pion Compton wavelengths, (4) (P. S.) $\times$ $(\pi\pi)$ for $a_{S0}=2$ pion Compton wavelengths. The $\chi^2$ values for the best fit of the four curves are 10.6, 35.6, 5.7, and 6.5, respectively, for seven degrees of freedom.

(1) relativistically invariant phase space (P.S.); (2) phase space multiplied by a Brown-Singer-type resonance (called sigma) between the two pions at a di-pion mass of 400 MeV with a full width of 100 MeV, (P.S.)$\times\sigma$; (3) phase space multiplied by (a) the effect due to an $S$-wave $\pi$-$\pi$ scattering length of 2.0 pion Compton wavelengths, as given by the $S$-dominant solution of the $\pi$-$\pi$ effective-range equation of Chew and Mandelstam,[11] plus (b) a sigma resonance of 400 MeV and full width 100 MeV, (P.S.)$\times[\alpha(\pi\pi)+\beta(\sigma)]$; and (4) phase space multiplied by the effect due to an $S$-wave $\pi$-$\pi$ scattering length of 2.0 pion Compton wavelengths, as in (3) above, (P.S.)$\times(\pi\pi)$.

In the calculated distributions we have folded in (i) the efficiencies of the neutron counters, (ii) the enhancement due to the Bose symmetrization effect between the two pions,[12] with a Bose radius of 1.0 pion Compton wavelength, and (iii) the neutron center-of-mass angular distribution of the form $(1.0+A\cos\theta^*+B\cos^2\theta^*)$. For the best fit we obtain the values of the coefficients $A$ and $B$ as 3.6 and 3.7, respectively.

Since we do not know the relative contributions of sigma and the $\pi$-$\pi$ scattering length, we interpret the data as being a combination of the two, and vary their weights. The weights for the minimum $\chi^2$ were $\alpha=20$ and $\beta=1$. The remaining two parameters for the fit were then $R$ (the radius) and $a_{S0}$ (the $S$-wave $\pi$-$\pi$ scattering length). The $\chi^2$ values obtained for different values of $R$ and $a_{S0}$ are presented in Table I.

The value of $a_{S0}$ that we can deduce from the present analysis is dependent on $R$. Assuming the value of $R$ to be 1.0 pion Compton wavelength, we are able to obtain with 90% confidence a lower limit of 0.65 pion Compton wavelength for the magnitude of the $I=0$, $S$-wave $\pi$-$\pi$ scattering length. Since the momentum transfer is reasonably low—$\Delta^2$ being between 2 and 7 in units of $m_\pi^2$—we also used the Chew-Low formula[13] in the physical region to calculate the $\pi$-$\pi$ cross section. At low momentum transfers ($\Delta^2<3$) the data do not agree with a pure one-pion-exchange model. Also, the variation of $m_{\pi\pi}$ over the range of our data makes this method of analysis not very reliable. However, using only the data with $\Delta^2$ between 3 and 7, we can conclude that the $\pi$-$\pi$ cross section is 60 mb or larger.

Within the present statistics our results agree with the analysis of Booth and Abashian,[1] whose method we followed in the interpretation of the data. The uncertainty in the Bose radius makes it difficult to obtain better quantitative results at present. More information at different laboratory angles and incident pion energies is necessary, and further work is now in progress.

We are grateful to Professor G. F. Chew for his interest and helpful suggestions. Thanks are also due to Donald Myer, Allen Peters, M. Piccioti, J. Shively, and Lloyd Themes for their help during the data-taking phase of the experiment, and to the 184-inch cyclotron crew headed by Mr. J. Vale for providing many hours of steady beam.

Table I. Minimum values of $\chi^2$ as a function of $R$ and $a_{S0}$ for seven degrees of freedom.

| | $R$ | | | | |
|---|---|---|---|---|---|
| $a_{S0}$ | 0 | 1.0 | 2.0 | 3.0 | 4.0 |
| 0 | 26.6 | 20.2 | 10.3 | 7.76 | 11.1 |
| 0.5 | 15.6 | 11.2 | 7.64 | 9.71 | 15.0 |
| 1.0 | 10.3 | 7.56 | 6.25 | 7.24 | 11.0 |
| 1.5 | 6.63 | 6.18 | 6.01 | 7.55 | 11.9 |
| 2.0 | 5.80 | 5.68 | 5.90 | 7.92 | 12.8 |
| 3.0 | 5.10 | 5.22 | 5.98 | 8.80 | 14.6 |
| 4.0 | 4.92 | 5.19 | 6.31 | 9.67 | 16.1 |

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960); N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961); N. E. Booth, A. Abashian, and K. M. Crowe, Rev. Mod. Phys. 35, 393 (1961); A. Abashian, N. E. Booth, K. M. Crowe, R. E. Hill, and E. H. Rogers, Phys. Rev. 132, 2296 (1963); N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. 132, 2309 (1963); N. E. Booth and A. Abashian, Phys. Rev. 132, 2314 (1963).

[2]B. C. Barish, R. J. Kurz, V. Perez-Mendez, and J. Solomon, Phys. Rev. 135, B416 (1964).

[3]Yu. A. Batusov, C. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 115.

[4]T. D. Blokhinseva, V. G. Grebinnik, V. A. Zhukov, G. Libman, L. L. Nemenov, G. I. Selivanov, and Y. Yung-Fang, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 117.

[5]J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. 130, 2481 (1963).

[6]I. M. Blair, G. Conforto, C. Rubbia, G. Torelli, and E. Zavattini, Phys. Letters 11, 79 (1964).

[7]L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962); Phys. Rev. 133, B812 (1964).

[8]H. Faier and L. M. Brown, Bull. Am. Phys. Soc. 10, 467 (1965).

[9]R. W. Birge, R. P. Ely, Jr., G. Gidal, G. E. Kalmus, A. Kerman, W. M. Powell, U. Camerini, D. Cline, W. F. Fry, J. G. Gaidos, D. Murphree, and C. T. Murphy, Phys. Rev. 139, B1600 (1965).

[10]R. H. Dalitz and R. G. Moorhouse, Phys. Letters 14, 159 (1965).

[11]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[12]G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).

[13]G.F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

# RELATION BETWEEN $D/F$ AND $G_A/G_V$ FROM CURRENT ALGEBRA

R. Gatto

Istituto di Fisica dell'Università, Firenze, Italy

L. Maiani

Istituto Superiore di Sanità, Roma, Italy

and

G. Preparata

Sezione di Firenze dell'Istituto Nazionale di Fisica Nucleare, Firenze, Italy

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We examine in this note the inclusion of higher baryonic resonances in the saturation of the chiral U(3)⊗ U(3) commutation rules of Gell-Mann's current algebra.[1] We find a consistent solution if the relation

$$-\frac{G_A}{G_V}=\frac{1}{3}\frac{D+F}{D-F} \tag{1}$$

is verified. Eq. (1) is well satisfied experimentally.

Recent work has led to satisfactory calculations of $G_A/G_V$ and $D/F$ from sum rules over experimental meson-nucleon cross sections, obtained from the current algebra.[2] We follow here the approach suggested by Lee[3] and Dashen and Gell-Mann,[4] by saturating the commutation relations among stable and resonant baryon states and looking for a consistent solution. In selecting the important resonant multiplets, we have been guided by a recent discussion[5] of higher baryonic resonances, suggesting the relevance of a classification according to the representation 20 of SU(6) with orbital angular momentum $L=1$. We thus include among the initial, final, and intermediate states, besides the 56 baryon states [multiplets $(\underline{8},\frac{1}{2}^+)$ and $(\underline{10},\frac{3}{2}^+)$ in the notation $(\mathrm{SU}(3),J^P)$], the states of 20 with $L=1$ [multiplets $(\underline{1},\frac{1}{2}^-)$, $(\underline{1},\frac{3}{2}^-)$, $(\underline{1},\frac{5}{2}^-)$, $(\underline{8},\frac{1}{2}^-)$, and $(\underline{8},\frac{3}{2}^-)$]. The commutation relation

$$[F_\lambda{}^5,F_\mu{}^5]=-\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \lambda & \mu & \nu\end{pmatrix}F_\nu, \tag{2}$$

where $F_\lambda{}^5$ is an axial-vector generator and

$$\begin{pmatrix}8 & 8 & 8a\\ \lambda & \mu & \nu\end{pmatrix}$$

is a SU(3) Clebsch-Gordan coefficient, is tak-