ance of massless particles may similarly be obtained if the two fields $\varphi_1(x)$ and $\varphi_2(x)$ are both coupled to a third field $A(x)$ which is invariant under the transformation induced by Q $\{[Q, A(x)]=0\}$. In this case we may define fields

$$\Psi_{ayz}(x) \equiv \psi_{ay}(x) A(x+z) \tag{11}$$

and repeat the preceding arguments with $\Psi_{ayz}(x)$ replacing $\psi_{ay}(x)$. The conclusion is that massless particles arise if the three-point functions $\langle 0|\varphi_1(0)\varphi_1(y)A(z)|0\rangle$ and $\langle 0|\varphi_2(0)\varphi_2(y)A(z)|0\rangle$ are not identical. The three-point functions may in turn be expressed in terms of two-point functions and vertices. If the two points happen to coincide, any difference in the vertices still leads to massless particles. Similar use of the freedom to choose composite fields for the ψ's of Eqs. (1) and (2) may be made in other cases to derive various conditions for the appearance of massless particles.

The results obtained above are important in connection with, among others, the attempts to explain the μ-e splitting by spontaneous symmetry breaking.[9-11] Since the SU(2) group which rotates between the μ and e fields is broken, massless particles with quantum numbers of μ^+e^- and μ^-e^+ should appear, provided our conditions are satisfied.

The essential point to check is whether the generator Q of the symmetry transformation commutes with the Hamiltonian. The usual way of finding commutators of products of field operators at the same point is dubious as pointed out by Schwinger[12]; more careful procedures must be employed. It should be noted that the time independence of Q is not guaranteed by its being the space integral of the time component of a conserved current.[13]

[1] J. Goldstone, Nuovo Cimento 19, 154 (1961).
[2] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).
[3] S. A. Bludman and A. Klein, Phys. Rev. 131, 2364 (1963).
[4] R. F. Streater, Phys. Rev. Letters 15, 475 (1965).
[5] Weaker assumptions than Streater's are made by N. Fuchs [Phys. Rev. Letters 15, 911 (1965)] whose result is that there exist "states" $|0'\rangle$ of energy and momentum zero which are not Lorentz invariant. His treatment of these $|0'\rangle$ appears incorrect because of the points raised in Ref. 2, Sec. III.
[6] A. Katz and Y. Frishman, to be published.
[7] The arguments of Klein and Lee [Phys. Rev. Letters 12, 266 (1964)] for the appearance of spurious states fail to hold under our conditions. This is because they work in a finite volume, while in Ref. 6 we work in an infinite volume from the outset, making use of spectral expansions and distributions, thus avoiding suspect procedures of extrapolating from a finite volume. Because of similar reasons, Gilbert's arguments [Phys. Rev. Letters 12, 713 (1964)] for the appearance of spurions in nonrelativistic theories do not apply under our conditions.
[8] This need not in general be true. We shall discuss the scope of this assumption elsewhere, where a method for defining $q(\vec{x})$ will also be given.
[9] M. Baker and S. L. Glashow, Phys. Rev. 128, 2462 (1962).
[10] Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Nuovo Cimento 34, 946 (1964); and to be published.
[11] R. Arnowitt and S. Deser, Phys. Letters 13, 256 (1964); Phys. Rev. 138, B712 (1965).
[12] J. Schwinger, Phys. Rev. Letters 3, 296 (1959).
[13] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

ALGEBRA OF CURRENTS AND K_{e3} DECAY*

V. S. Mathur,† S. Okubo, and L. K. Pandit‡

Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 8 December 1965)

By exploiting the algebra of currents according to the method[1] suggested by Fubini and Furlan, many interesting results[2] have been obtained recently. The present note will be devoted to a study of the K_{e3} decay.

The matrix element of the strangeness-changing vector current between K and π states is expressible in terms of form factors defined as

$$\langle \pi^0(q')|[V_\mu(0)]_1{}^3|K^+(q)\rangle$$
$$= (2q_c{}' \times 2q_0 V^2)^{-1/2}\{F_+([q-q']^2)[q_\mu + q_\mu{}'] + F_-([q-q']^2)[q_\mu - q_\mu{}']\}, \tag{1}$$

where we have used standard SU(3) tensor notation. For K_{e3} decay, only the form factor F_+ is important. Although the momentum-transfer dependence of the form factor $F_+([q-q']^2)$ may be studied by dispersion theory, one cannot, however, fix the value of $F_+(0)$, which appears either as a subtraction constant or as a scale factor. The algebra of currents, through its nonlinear structure, provides[3] us with a means of fixing the scale and hence allows $F_+(0)$ to be calculated.

The algebra of currents we employ is that obtained from the quark model. Denoting the vector and axial (pseudo-) vector current densities by $[V_\mu(x)]_b{}^a$ and $[P_\mu(x)]_b{}^a$, respectively, the corresponding "charges" are defined by:

$$A_b{}^a(t) = i\int_{x_0=t} d^3x [V_4(x)]_b{}^a,$$

$$B_b{}^a(t) = i\int_{x_0=t} d^3x [P_4(x)]_b{}^a; \quad a, b = 1, 2, 3. \tag{2}$$

We shall first use the commutation relation

$$[B_1{}^2(t), B_2{}^3(t)] = A_1{}^3(t). \tag{3}$$

Taking the matrix element of (3) between K^+ and π^0 states we have

$$\sum_n \{\langle\pi^0(q')|B_1{}^2(t)|n(k)\rangle\langle n(k)|B_2{}^3(t)|K^+(q)\rangle - \langle\pi^0(q')|B_2{}^3(t)|n(k)\rangle\langle n(k)|B_1{}^2(t)|K^+(q)\rangle\} = \langle\pi^0(q')|A_1{}^3(t)|K^+(q)\rangle. \tag{4}$$

The sum over intermediate states on the left-hand side will now be approximated by a ρ^+ state for the first term, and by $K^*(888)$ and $\kappa(725)$ for the second term. Using partial conservation of axial-vector currents,[4] whereby

$$i\partial_\mu [P_\mu(x)]_1{}^2 = (C_\pi/\sqrt{2})\pi^+(x),$$

$$i\partial_\mu [P_\mu(x)]_1{}^3 = C_K K^+(x), \tag{5}$$

we obtain in the standard manner the following sum rule:

$$F_+(0) = \frac{C_\pi C_K}{M_\pi{}^2 M_K{}^2}\left[G_{\rho\pi\pi}G_{\rho KK}\frac{1}{M_\rho{}^2} + G_{K^*K\pi}{}^2\frac{1}{M_{K^*}{}^2} + \frac{G_{\kappa K\pi}{}^2}{(M_\kappa{}^2 - M_\pi{}^2)(M_\kappa{}^2 - M_K{}^2)}\right], \tag{6}$$

where the various G's denote the indicated coupling constants. Making the assumption that ρ is coupled to a conserved isospin current,[5] we relate $G_{\rho KK}$ to $G_{\rho\pi\pi}$. The coupling constants $G_{\rho\pi\pi}$, $G_{K^*K\pi}$ and $G_{\kappa K\pi}$ are fixed, respectively, from the widths of ρ, K^*, and κ. The proportionality constants in Eq. (5) are given by

$$C_\pi/M_\pi{}^2 = g_A{}^N 2M_N/G_{NN\pi},$$

$$C_K/M_K{}^2 = g_A{}^\Lambda (M_N + M_\Lambda)/G_{KN\Lambda}, \tag{7}$$

$g_A{}^N$ and $g_A{}^\Lambda$ being the N and Λ β-decay axial-vector renormalization constants. Using the values $g_A{}^N = 1.18$, $g_A{}^\Lambda = -0.79$, and (in the absence of any definite determination) the SU(3) value of $G_{KN\Lambda}{}^2/4\pi \approx 13$, we obtain the effective decay constant involving Cabibbo's factor[6] $\sin\theta_V$ (0.26):

$$f_+(0) \equiv F_+(0)\sin\theta_V \simeq 0.20. \tag{8}$$

The experimental value[7] for $f_+(0)$ is $\simeq 0.16$. The contribution of κ to the sum rule is not crucial, being only about 5% for a κ width of about 10 MeV. We consider our result in satisfactory agreement with experiment in view of the approximations involved.[8]

On the basis of SU(3) symmetry and conservation of vector currents (CVC) for the pionic form factor involved in the π_{e3} decay, one obtains[6] $f_+(0) = 0.13$. Our result (8) is somewhat larger than this value, but should not be regarded as implying departures from the Ademollo-Gatto theorem,[9] in view of the approximations made in the analysis, especially since not all the parameters involved are known accurately from experiments. On the contrary, the fact that our result is reasonably close to the experimental value is indicative of the efficacy of the current algebra approach and shows that the approximations made are fairly reasonable.

Instead of calculating the value of $F_+(0)$, as done above, one can directly relate K_{l3} and K_{l2} amplitudes using the algebra of currents. This relation is very interesting since it is exact, excepting for the usual continuation of the squared four-momentum of the π^0 to the value zero (the so-called "zero-mass" pion continuation).

For this purpose we employ the standard reduction technique so that the matrix element

$$\langle\pi^0(q')\text{ out}|A_1{}^3(0)|K^+(q)\text{ in}\rangle$$
$$= i\int d^4x \frac{e^{-iq'x}}{(2q_0'V)^{1/2}}(M_\pi{}^2-\Box)$$
$$\times\langle 0|\theta(x_0)[\pi^0(x),A_1{}^3(0)]|K^+(q)\rangle. \quad (9)$$

Because of the appearance of the causal commutator, the right-hand side of Eq. (9) may be continued to the unphysical value $q'=0$. Integrating by parts and continuing to $q'=0$, we obtain

$$\lim_{q'\to 0}(2q_0'V)^{1/2}\langle\pi^0(q')|A_1{}^3(0)|K^+(q)\rangle$$
$$=(M_\pi{}^2/C_\pi)\langle 0|[B_1{}^1(0)-B_2{}^2(0),A_1{}^3(0)]|K^+(q)\rangle, \quad (10)$$

where we have used the following relations incorporating partial conservation of axial-vector currents:

$$B_j{}^i(0)=\int d^4x\,\theta(x_0)\partial_\mu[P_\mu(x)]_j{}^i$$
$$=\frac{C_\pi}{i\sqrt{2}}\int d^4x\,\theta(x_0)\pi_j{}^i(x). \quad (11)$$

The term $B_j{}^i(t=+\infty)$ will not contribute here because of the unequal masses involved in the matrix elements.[10] Using the commutation relation of the charges,

$$[B_j{}^i(t),A_l{}^k(t)]=\delta_j{}^kB_l{}^i-\delta_l{}^iB_j{}^k, \quad (12)$$

we obtain the result

$$\lim_{q'\to 0}(2q_0'V)^{1/2}\langle\pi^0(q')|A_1{}^3(0)|K^+(q)\rangle$$
$$=-(M_\pi{}^2/C_\pi)\langle 0|B_1{}^3(0)|K^+(q)\rangle. \quad (13)$$

This relation is essentially an analog of the Kroll-Ruderman theorem.[10] The matrix element on the left-hand side is related to K_{l3} decay according to Eqs. (1) and (2) and that on the right-hand side is related to the matrix element relevant for K_{l2} decay, namely

$$\langle 0|[P_\mu(0)]_1{}^3|K^+(q)\rangle=(2q_0V)^{-1/2}gq_\mu. \quad (14)$$

The decay K_{l2} is characterized by the constant g. Hence, Eq. (13) leads to the relation

$$F_-(-M_K{}^2)+F_+(-M_K{}^2)=(M_\pi{}^2/C_\pi)g. \quad (15)$$

Similarly, one may, of course, also relate the π_{l3} and π_{l2} form factors:

$$F_+{}^{(\pi)}(-M_\pi{}^2)=2(M_\pi{}^2/C_\pi)g^{(\pi)}. \quad (16)$$

Equations (15) and (16) imply that

$$[F_++F_-]/F_+{}^{(\pi)}=\tfrac{1}{2}g/g^{(\pi)}. \quad (17)$$

Neglecting F_-, Eq. (17) is identical to the result of Cabibbo with $\theta_V=\theta_A$ derived using SU(3) symmetry.[6] Our derivation of (17), of course, makes no use of SU(3) invariance at all. It is remarkable that our result agrees with the SU(3) result excepting for small terms arising due to SU(3) breaking. One knows that the vector-current renormalization is very small; our result then implies that also the axial-vector-current renormalization is small. This situation is perhaps understandable in view of the assumption of a "zero-mass" pion, whereby the strangeness-preserving axial-vector current is strictly speaking "divergenceless." This may imply a generalization of the Ademollo-Gatto theorem,[9] so that the strangeness-changing axial-vector current may acquire renormalization only as a second-order $\overline{W}_3$ [chiral SU(3) $\otimes$ SU(3)] symmetry[11] breaking effect.

From Eq. (15) now, neglecting F_-, we may calculate the ratio of the decay rates K_{e3} and $K_{\mu 2}$. Using the value $g_A{}^N=-1.18$ with Eq. (7) for C_π, we obtain

$$\Gamma(K_{e3}{}^+)/\Gamma(K_{\mu 2}{}^+)\simeq 0.13, \quad (18)$$

which is in reasonable agreement with experiments.[12] From Eqs. (8) and (15) we also obtain the value of $\xi\equiv F_-/F_+$ to be ≈ -0.35. In view of our approximations this is at best a crude estimate.

The relationship of Eq. (15) to an SU(3) calculation can be seen more explicitly by use of the following commutator:

$$[2^{-1/2}\{B_1{}^1(t)-B_2{}^2(t)\},A_1{}^3(t)]=2^{-1/2}B_1{}^3(t). \quad (19)$$

Taking the matrix element of (19) between K^+ and vacuum states, and using only a single-pion intermediate-state contribution, we obtain in the standard manner, in the limit of conserved strangeness-changing vector current,

$$F_+(-[M_K-M_\pi]^2)=2\frac{M_\pi{}^2}{C_\pi}\left(\frac{M_K}{M_K+M_\pi}\right)g. \quad (20)$$

In the SU(3)-symmetry limit $A_1{}^3$ (being an SU(3) generator) becomes time independent and the only intermediate state it connects to the K^+ state is the π^0. Thus in the SU(3)-symmetry limit ($M_k = M_\pi$), we find

$$F_+(0) = (M_\pi^2/C_\pi)g. \qquad (21)$$

We wish to thank Prof. R. E. Marshak for his interest in this work.

---

*Work supported in part by the U. S. Atomic Energy Commission.

†On leave of absence from the Centre of Advanced Studies in Theoretical Physics and Astrophysics, University of Delhi, Delhi, India.

‡On leave of absence from the Tata Institute of Fundamental Research, Colaba, Bombay 5, India.

[1]S. Fubini and G. Furlan, Physics 1, 229 (1965); G. Furlan, F. Lannoy, C. Rosetti, and G. Segrè, Nuovo Cimento 38, 1747 (1965), and to be published; S. Fubini, G. Furlan, and C. Rosetti, to be published; S. Okubo, University of Rochester Report No. UR-875-97, 1965 (unpublished); V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, University of California Radiation Laboratory Report No. UCRL 16367, 1965 (unpublished).

[2]S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); Phys. Rev. 140, B736 (1965); W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965); L. K. Pandit and J. Schechter, Phys. Letters 19, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, Phys. Letters 19, 59 (1965); A. Sato and S. Sasaki, to be published; C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); V. S. Mathur and L. K. Pandit, University of Rochester Report No. UR-875-103, 1965 (unpublished); I. J. Muzinich and S. Nussinov, Phys. Letters 19, 248 (1965); V. S. Mathur and L. K. Pandit, to be published; S. Okubo, to be published; H. Sugawara, Phys. Rev. Letters 15, 870 (1965); R. Oehme, University of Chicago Report No. COO-264-287, 1965 (unpublished); S. L. Adler, to be published.

[3]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[4]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[5]J. J. Sakurai, Ann. Phys. 11, 1 (1960).

[6]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963). A result similar to Eq. (17) has been derived on the basis of a broken SU(3) model by R. Oehme [Ann. Phys. 33, 108 (1965)] and by R. Oehme and G. Segrè [Phys. Letters 11, 94 (1964)]. However, both the motivation and the technique involved are quite different from ours. In particular our main results contained in Eqs. (15) and (16) cannot be obtained on the basis of this model.

[7]See, for example, J. D. Jackson, Lectures in Theoretical Physics, Brandeis University, 1962 (W. A. Benjamin, Inc., New York, 1963).

[8]The appearance of ρ and κ terms in the single particle contributions to $F_+(0)$ and, presumably to $F_+(-[q-q']^2)$ is not surprising as these states appear in the crossed channels of the amplitudes for $K+\pi \to K+\pi$.

[9]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1965).

[10]S. Okubo, see Ref. (1).

[11]M. Gell-Mann, Physics 1, 63 (1964); R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965).

[12]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).

EXPERIMENTAL STUDY OF π-π INTERACTION IN THE REACTION $\pi^- + p \to \pi^0 + \pi^0 + n$ AT INCIDENT π^- ENERGY OF 378 MeV*

Tin Maung, Kenneth M. Crowe, and Ned T. Dairiki

Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 18 November 1965)

In the study of the reactions

$$p + d \to He^3 + 2\pi \qquad (1a)$$

and

$$p + d \to H^3 + 2\pi, \qquad (1b)$$

Abashian, Booth, and Crowe (ABC)[1] observed a peak in the momentum distribution of He^3, which was subsequently interpreted as a strong S-wave $I=0$, π-π scattering-length effect. We have completed an experiment in which we measured the differential neutron time-of-flight distribution from the reaction

$$\pi^- + p \to n + \pi^0 + \pi^0 \qquad (2)$$

at a π^- laboratory kinetic energy of 378 MeV, to look for this effect.

We measured the neutron spectrum at 45 deg (lab) in coincidence with the γ rays produced in Reaction (2). The experimental setup is as shown in Fig. 1. Negative pions, produced at an internal target of the 184-inch cyclotron were momentum analyzed and focused on a 3-in.-diameter by 10-in.-long liquid-hydrogen target, which was completely surrounded by scintillation counters. The pion beam was designed to have less than ±1.5 deg of angular divergence. 14 liquid-scintillation neutron counters and six lead-scintillator sandwich counters were used to detect the neutrons and π^0-decay γ rays,