COROLLARIES OF THE GOLDSTONE THEOREM

Yitzhak Frishman and Amnon Katz

Physics Department, Weizmann Institute of Science, Rehovoth, Israel
(Received 26 January 1966)

It is the purpose of this note to present corollaries of the Goldstone theorem on broken symmetries and massless particles.[1-3] Streater[4] has recently shown that if a symmetry transformation mixes fields $\varphi_1(x)$ and $\varphi_2(x)$ corresponding to different masses, massless particles follow.[5] This result, which Streater considers "a significant generalization" of the Goldstone theorem, as well as stronger results, are corollaries of Goldstone's theorem. The stronger results include the following: Any difference between the two-point functions of the two fields, $\langle\varphi_1(x)\varphi_1(y)\rangle$ and $\langle\varphi_2(x)\varphi_2(y)\rangle$, or between the vertices coupling the two fields to a third field which is invariant under the symmetry transformation, entails massless particles.

Goldstone's symmetry-breaking condition requires that a set of fields $\psi_a(\vec{x}, t)$ which transform under the symmetry operation according to

$$i[Q, \psi_a(\vec{x}, t)] = \sum_b u_{ab}{}'\psi_b(\vec{x}, t) \tag{1}$$

satisfy

$$\sum_b u_{ab}{}'\langle 0|\psi_b(\vec{x}, t)|0\rangle \neq 0; \tag{2}$$

$|0\rangle$ is the invariant vacuum state which satisfies $H|0\rangle = \vec{P}|0\rangle = 0$ (H is the Hamiltonian, $\vec{P}$ is the total momentum); Q is the generator of the symmetry transformation.

We have been able to show[6] that massless particles (as distinct from spurious states) do indeed follow from the above condition under the following circumstances[7]: (1) The generator Q satisfies

$$[H, Q] = 0, \quad [\vec{P}, Q] = 0. \tag{3}$$

(2) Q may be expressed as an integral over a density,[8]

$$Q = \int q(\vec{x}) d^3x, \tag{4}$$

where $q(\vec{x})$ satisfies

$$i[\vec{P}, q(\vec{x})] = (\partial/\partial\vec{x}) q(\vec{x}). \tag{5}$$

The only requirements on the fields $\psi_a(\vec{x}, t)$ is that they satisfy

$$i[\vec{P}, \psi_a(\vec{x}, t)] = (\partial/\partial\vec{x})\psi_a(\vec{x}, t),$$
$$-i[H, \psi_a(\vec{x}, t)] = (\partial/\partial t)\psi_a(\vec{x}, t). \tag{6}$$

(No requirements of relativistic invariance are necessary. In their absence "massless particles" are modes of excitation with energy tending to zero along with the momentum.)

Suppose, now, that we have two fields $\varphi_1(x)$, $\varphi_2(x)$ and that a generator Q satisfying conditions (1) and (2) above induces between them the transformation

$$i[Q, \varphi_1(x)] = \varphi_2(x), \quad i[Q, \varphi_2(x)] = -\varphi_1(x). \tag{7}$$

Consider the new fields

$$\psi_{1y}(x) \equiv \varphi_1(x)\varphi_1(x+y), \quad \psi_{2y}(x) \equiv \varphi_2(x)\varphi_2(x+y),$$
$$\psi_{3y}(x) \equiv \varphi_1(x)\varphi_2(x+y), \quad \psi_{4y}(x) \equiv \varphi_2(x)\varphi_1(x+y). \tag{8}$$

They transform among themselves under the transformation generated by Q. They also satisfy Eq. (6). We may therefore use them as the fields ψ in Goldstone's symmetry-breaking condition [Eqs. (1) and (2)]. We have

$$i[Q, \psi_{3y}(x)] = \psi_{2y}(x) - \psi_{1y}(x). \tag{9}$$

The vacuum expectation value of the right-hand side of Eq. (9) is nothing but the difference between the two-point functions of the fields $\varphi_1(x)$ and $\varphi_2(x)$ at space-time separation y:

$$\langle 0|\psi_{2y}(x) - \psi_{1y}(x)|0\rangle$$
$$= \langle 0|\varphi_2(0)\varphi_2(y)|0\rangle - \langle 0|\varphi_1(0)\varphi_1(y)|0\rangle. \tag{10}$$

If the two-point functions of the two fields are not identical, the last expression differs from zero for some choice of y. Goldstone's symmetry-breaking condition is then satisfied for the fields $\psi_{ay}(x)$ for that choice of y, and massless particles follow.

Our conditions above are weaker than Streater's, who demands a difference in mass between the two fields. Also his restrictions on the Lehmann spectral functions are not necessary.

A different sufficient condition for the appearance of massless particles may similarly be obtained if the two fields $\varphi_1(x)$ and $\varphi_2(x)$ are both coupled to a third field $A(x)$ which is invariant under the transformation induced by Q $\{[Q, A(x)]=0\}$. In this case we may define fields

$$\Psi_{ayz}(x) \equiv \psi_{ay}(x) A(x+z) \tag{11}$$

and repeat the preceding arguments with $\Psi_{ayz}(x)$ replacing $\psi_{ay}(x)$. The conclusion is that massless particles arise if the three-point functions $\langle 0 | \varphi_1(0)\varphi_1(y)A(z) | 0\rangle$ and $\langle 0 | \varphi_2(0)\varphi_2(y)A(z) | 0\rangle$ are not identical. The three-point functions may in turn be expressed in terms of two-point functions and vertices. If the two points happen to coincide, any difference in the vertices still leads to massless particles. Similar use of the freedom to choose composite fields for the ψ's of Eqs. (1) and (2) may be made in other cases to derive various conditions for the appearance of massless particles.

The results obtained above are important in connection with, among others, the attempts to explain the μ-e splitting by spontaneous symmetry breaking.[9-11] Since the SU(2) group which rotates between the μ and e fields is broken, massless particles with quantum numbers of μ^+e^- and μ^-e^+ should appear, provided our conditions are satisfied.

The essential point to check is whether the generator Q of the symmetry transformation commutes with the Hamiltonian. The usual way of finding commutators of products of field operators at the same point is dubious as pointed out by Schwinger[12]; more careful procedures must be employed. It should be noted that the time independence of Q is not guaranteed by its being the space integral of the time component of a conserved current.[13]

[1]J. Goldstone, Nuovo Cimento 19, 154 (1961).

[2]J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[3]S. A. Bludman and A. Klein, Phys. Rev. 131, 2364 (1963).

[4]R. F. Streater, Phys. Rev. Letters 15, 475 (1965).

[5]Weaker assumptions than Streater's are made by N. Fuchs [Phys. Rev. Letters 15, 911 (1965)] whose result is that there exist "states" $|0'\rangle$ of energy and momentum zero which are not Lorentz invariant. His treatment of these $|0'\rangle$ appears incorrect because of the points raised in Ref. 2, Sec. III.

[6]A. Katz and Y. Frishman, to be published.

[7]The arguments of Klein and Lee [Phys. Rev. Letters 12, 266 (1964)] for the appearance of spurious states fail to hold under our conditions. This is because they work in a finite volume, while in Ref. 6 we work in an infinite volume from the outset, making use of spectral expansions and distributions, thus avoiding suspect procedures of extrapolating from a finite volume. Because of similar reasons, Gilbert's arguments [Phys. Rev. Letters 12, 713 (1964)] for the appearance of spurions in nonrelativistic theories do not apply under our conditions.

[8]This need not in general be true. We shall discuss the scope of this assumption elsewhere, where a method for defining $q(\vec{x})$ will also be given.

[9]M. Baker and S. L. Glashow, Phys. Rev. 128, 2462 (1962).

[10]Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Nuovo Cimento 34, 946 (1964); and to be published.

[11]R. Arnowitt and S. Deser, Phys. Letters 13, 256 (1964); Phys. Rev. 138, B712 (1965).

[12]J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

[13]G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

ALGEBRA OF CURRENTS AND K_{e3} DECAY*

V. S. Mathur,† S. Okubo, and L. K. Pandit‡

Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 8 December 1965)

By exploiting the algebra of currents according to the method[1] suggested by Fubini and Furlan, many interesting results[2] have been obtained recently. The present note will be devoted to a study of the K_{e3} decay.

The matrix element of the strangeness-changing vector current between K and π states is expressible in terms of form factors defined as

$$\langle \pi^0(q') | [V_\mu(0)]_1{}^3 | K^+(q)\rangle = (2q_c{}' \times 2q_0 V^2)^{-1/2}\{F_+([q-q']^2)[q_\mu + q_\mu{}'] + F_-([q-q']^2)[q_\mu - q_\mu{}']\}, \tag{1}$$