region. The existence of $E2$ absorption in the region of 20-30 MeV has been shown by the earlier experiments of Sorokin et al.[11] We therefore conclude that the theory is fully compatible with the available experimental data, and we also believe that the observed cross section exceeding the value ascribable to the high-energy tail of the giant dipole resonance is indeed the giant quadrupole resonance.

We acknowledge fruitful discussions with H. Arenhövel and stimulating criticism from E. G. Fuller and E. Hayward.

[1]M. Danos, W. Greiner, and B. C. Kohr, Report, University of Freiburg in Braunschweig, 1964 (to be published).

[2]M. Danos, W. Greiner, and B. C. Kohr, Phys. Letters 12, 344 (1964).

[3]M. Danos and W. Greiner, Phys. Rev. 134, B287 (1964); Phys. Letters 8, 113 (1964); Phys. Rev. 138, B876 (1965).

[4]H. Arenhövel, W. Greiner, and M. Danos, to be published; see also H. Arenhövel, Dissertation, Universität Frankfurt am Main, 1965, (unpublished).

[5]R. L. Bramblett, J. T. Caldwell, G. F. Auchanpaugh, and S. C. Fultz, Phys. Rev. 129, 2723 (1963).

[6]R. L. Bramblett, J. T. Caldwell, R. R. Harvey, and S. C. Fultz, Phys. Rev. 133, B868 (1964).

[7]M. Danos and W. Greiner, Phys. Rev. 138, B876 (1965).

[8]P. Axel, J. Miller, C. Schuhl, G. Tanan, and C. Tzara, to be published.

[9]E. Ambler, E. G. Fuller, and H. Marshak, Phys. Rev. 138, B117 (1965).

[10]G. S. Mutchler, W. Bertozzi, S. Kowalski, L. P. Sargent, and W. Turchinetz, Bull. Am. Phys. Soc. 10, 541 (1965), and private communication.

[11]Yu. I Sorokin, V. G. Shevchenko, and B. A. Yur'ev, Zh. Eksperim. i Teor. Fiz. 43, 1600 (1962) [translation: Soviet Phys.—JETP 16, 1127 (1963)].

METHOD TO TEST FOR TIME-REVERSAL INVARIANCE VIOLATION IN K DECAYS*

David Cline

Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 17 January 1966)

Recent evidence for CP-invariance violation in K_2^0 decay[1] has spurred experimental tests of CP and time-reversal invariance in several other physical systems.[2] In this note we suggest another physical system for which it may be fruitful to test for time-reversal invariance and we report a crude experimental limit on time-reversal invariance violation in this system.

The most general matrix element for $\pi^+\pi^0\gamma$ decay of the K^+ meson can be written as a sum of three terms[3]:

$$M(\pi^+\pi^0\gamma) = \alpha f_1 + \epsilon f_2 + m f_3, \tag{1}$$

where the terms f_i $(i=1,2,3)$ represent Lorentz-invariant factors consisting of products of the four-vectors for the system and the terms α, ϵ, and m represent form factors corresponding to inner bremsstrahlung, direct electric-dipole transitions, and direct magnetic-dipole transitions, respectively. α is also the off-mass-shell amplitude for $K^+ \to \pi^+ + \pi^0$ decay.

Time-reversal invariance requires that the form factors be relatively real except for final-state interactions.[4] Relaxing the requirement of time-reversal invariance and including final-state interactions leads to the substitutions[4]

$$\alpha \to \tilde{\alpha} \exp[i(\delta_2 + \varphi_2)],$$
$$\epsilon \to \tilde{\epsilon} \exp[i(\delta_1 + \varphi_1)],$$
$$m \to \tilde{m} \exp[i(\delta_1 + \varphi_1')],$$

where δ_2 and δ_1 are the $T=2$ and $T=1$ $\pi\pi$ scattering phase shifts, respectively, and $(\varphi_2, \varphi_1, \varphi_1')$ are time-reversal noninvariance phases.

The positive-pion spectrum for $\pi^+\pi^0\gamma$ decay is

$$dB(\pi^+)/dT(\pi^+) = \Gamma_{\pi 2}[B + \gamma \cos\Delta I + (\gamma^2 + \beta^2)E], \tag{2}$$

where $\gamma = \tilde{\epsilon}/\tilde{\alpha}$, $\beta = \tilde{m}/\tilde{\alpha}$, and $\Delta = (\delta_1 - \delta_2) + (\varphi_1 - \varphi_2)$, and where B, I, and E are known functions of the pion kinetic energy $T(\pi^+)$. The functions B, I, and E are tabulated in Ref. 3. $\Gamma_{\pi 2}$ is the $K_{\pi 2}$ decay rate.

In deriving Eq. (2) a summation over photon polarization removes all cross terms involving the magnetic-dipole transition ($\tilde{m}$). In Eq. (2) the rescattering of pions from inner bremsstrahlung decay has been neglected since the effects are expected to be quite small.[5] To test $\pi^+\pi^0\gamma$ decay for time-reversal invariance the phase difference $(\varphi_1-\varphi_2)$ must be determined from a fit to the experimental π^+ spectrum. In addition γ must obviously be nonzero.

The basis for extracting time-reversal invariance information from $\pi^+\pi^0\gamma$ decay rests on two plausible hypotheses: (1) Since only $T=2$ and $T=1$ $\pi\pi$ final states are involved, the effects of final-state interactions are expected to be negligible $(\delta_1-\delta_2<10°)$.[6] (2) The direct-emission terms (γ,β) are of the order of magnitude of unity since the inner bremsstrahlung process comes about through a $\Delta T\neq\frac{1}{2}$ transition and the direct processes can come about through intermediate states that satisfy the $\Delta T=\frac{1}{2}$ rule.[7]

The second hypothesis can be directly checked in $\pi^+\pi^0\gamma$ experiments. In addition, to relate the results of a test of time-reversal invariance in $\pi^+\pi^0\gamma$ decay to other $K\rightarrow\pi+\pi$ decay modes two further assumptions must be made: (3) The phase φ_2 (off mass shell) is the same as the phase for $K_{\pi 2}{}^+$ decay. (4) The phase φ_1 comes about mainly by $\Delta T=\frac{1}{2}$ processes and is thus a measure of time-reversal noninvariance in such processes. Thus $\varphi_1-\varphi_2$ measures the difference in phase between $\Delta T=\frac{1}{2}$ and $\Delta T\neq\frac{1}{2}$ amplitudes.

We know of no way to justify assumption (3); however, assumption (4) can be justified on the basis of a simple model for the electric-dipole direct-emission process.[7] It should be noted that independent of assumptions (3) and (4), the observation of a nonzero phase difference $(\varphi_1-\varphi_2)$ is evidence for time-reversal noninvariance in $\pi^+\pi^0\gamma$ decay. Note that even if there are no time-reversal invariance violating effects in the radiative matrix elements $(\varphi_1=\varphi_1{}'=0)$, the experiment is still sensitive to time-reversal noninvariance in the nonradiative amplitude $(\varphi_2\neq 0)$.

Experimentally we have collected a sample of 63 $\pi^+\pi^0\gamma$ decays from a recent stopping K^+ run in the Lawrence Radiation Laboratory heavy-liquid chamber. In a previous report on the first 18 events,[8] it was shown that the branching ratio and spectrum were not consistent with large values of γ and β. The method used to collect our present sample of events is the same as reported in Ref. 8. A detailed report of the experimental method will be presented elsewhere.

Only $\pi^+\pi^0\gamma$ events with π^+ kinetic energy in the range 55-90 MeV were detected in this experiment. A detailed background study shows that the sample has a background contamination of less than 15%. In addition, the energy distribution of the observed background events is uniform and therefore is not expected to change the shape of the observed π^+ spectrum appreciably. The resulting spectrum with the corrections folded in is shown in Fig. 1. A three-parameter likelihood-function fit to the data was made using Eq. (2). The form factors γ and β were assumed to be independent of $T(\pi^+)$.[3] For all values of γ and cosΔ for which the likelihood function is reasonably large, the parameter β is close to zero. Thus there is no evidence for a direct magnetic-dipole contribution to $\pi^+\pi^0\gamma$, in agreement with some theoretical calculations[9] and in disagreement with others.[7] For simplicity β was set equal to zero for a final likelihood fit to the data. A contour plot of this likelihood function is shown in Fig. 2.

Table I gives the best values of γ, β, and cosΔ for the cases of $\beta=0.0$ and $\cos\Delta=1.0$. The observed value of γ is consistent with the theoretical calculations of Pepper and Ueda[7] but our statistics are clearly inadequate to definitely prove that some direct emission process is present in $\pi^+\pi^0\gamma$ decay.[10] Within these limited statistics there is also no evidence for time-reversal invariance violation in $\pi^+\pi^0\gamma$ decay.

The present sample of $\pi^+\pi^0\gamma$ events is clear-

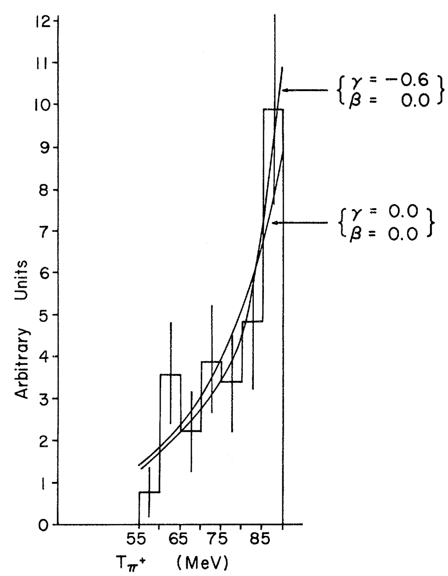

FIG. 1. Corrected π^+ spectrum for 63 $\pi^+\pi^0\gamma$ events for π^+ kinetic energies between 55 and 90 MeV. For reference the theoretical curves for $\gamma=\mp 0.6$, $\beta=0.0$, $\cos\Delta=1.0$, and $\gamma=0.0$, $\beta=0.0$, and $\cos\Delta=1.0$ are presented.

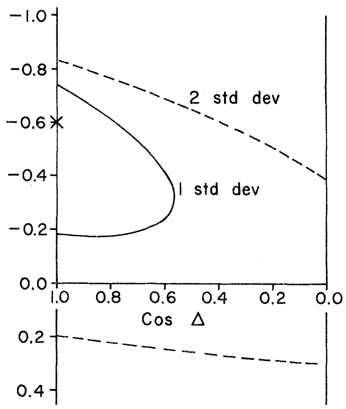

FIG. 2. Contour plot of likelihood function as a function of γ and $\cos\Delta$. The cross denotes the maximum of the likelihood function. The solid and dashed lines indicate the contour for a likelihood function reduced by the factors of $e^{-1/2}$ and e^{-2} times the maximum value, respectively.

ly too small to allow a detailed test of time-reversal invariance; however, an increase in the size of the sample by an order of magnitude should make it possible to test the hypothesis of time-reversal noninvariance in the nonleptonic decay using the method suggested here. A study of the photon-pion angular correlations should also be sensitive to the presence of time-reversal noninvariance.

Table I. Best values of γ and β for $\cos\Delta = 1.0$ and best values of γ and $\cos\Delta$ for $\beta = 0.0$. The value of $\varphi_1 - \varphi_2$ in column 4 is obtained by assuming $\cos\Delta = 0.55$ and $\delta_1 - \delta_2 = 10°$. The choice of $\delta_1 - \delta_2 = 10°$ is explained in Ref. 6.

γ	β	$\cos\Delta$	$\varphi_1-\varphi_2$
$-0.6^{+0.4}_{-0.15}$	0.0 ± 0.27	Assumed to be 1.0	...
$-0.6^{+0.4}_{-0.15}$	Assumed to be 0.0	>0.55	Less than 62°

I wish to thank Professor Wilson Powell and his group for helping us obtain the K^+ exposure and for the loan of the film. Also I thank Professor V. Barger and Professor W. Fry for helpful discussions.

*Research supported by the U. S. Atomic Energy Commission.

[1]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); see also W. Galbraith et al., Phys. Rev. Letters 14, 383 (1965); X. De Bourad et al., Phys. Letters 15, 58 (1965).

[2]J. A. Anderson et al., Phys. Rev. Letters 14, 475 (1965); U. Camerini, et al., Phys. Rev. Letters 14, 989 (1965). M. Baldo-Ceolin et al., to be published; B. Aubert et al., to be published.

[3]J. D. Good, Phys. Rev. 113, 352 (1959).

[4]K. M. Watson, Phys. Rev. 95, 228 (1954); E. Fermi, Nuovo Cimento 2, Suppl. 1, 17 (1955).

[5]H. Chew, Nuovo Cimento 25, 1109 (1962).

[6]Assuming that the ρ dominates the $T=1$, $\pi\pi$ interaction we obtain $\delta_1 < 10°$ for the range of invariant mass involved in $\pi^+\pi^0\gamma$ decay. The $T=2$, $\pi\pi$ interaction has been estimated by G. C. Oades, in Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963), Vol. I, p. 338. A reasonable estimate of the value of δ_2 is between 5° and 12° in the range of interest. Thus $\delta_1 - \delta_2$ is expected to be less than 10°.

[7]S. V. Pepper and Y. Ueda, Nuovo Cimento 35, 1614 (1964). These authors have constructed a loop model for the electric-dipole direct-emission process that goes through the $\Delta T = \frac{1}{2}$ weak interaction. γ is related to g_2 defined in this reference by $\gamma = 0.05 g_2$.

[8]D. Cline and W. Fry, Phys. Rev. Letters 13, 101 (1964).

[9]S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. 136, B1064 (1964); K. Tanaka, Phys. Rev. 136, B1813 (1964); Harry J. Lipkin, Phys. Rev. 137, B1561 (1965).

[10]Our value for the $\pi^+\pi^0\gamma$ branching ratio is essentially the same as reported in Ref. 8 and is consistent with two values of γ within the range of γ values deduced from the spectrum: $\gamma \sim -0.8$ and $\gamma \sim -0.2$.