[12] R. H. Dalitz, in Proceedings of the International Conference on Fundamental Aspects of Weak Interactions (Brookhaven National Laboratory, Upton, New York, 1964), p. 378.

[13] The numerical factor 0.97 comes from corrections to phase space due to electromagnetic mass differences. See G. Källén, Elementary Particle Physics (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1964), pp. 445 and 449.

[14] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

[15] The 3π phase space is taken from J. C. Pati, thesis, University of Maryland, 1960 (unpublished).

STUDY OF THE DECAY $\eta^0 \to \pi^+ + \pi^- + \gamma$*

Frank S. Crawford, Jr., and LeRoy R. Price

Lawrence Radiation Laboratory, University of California, Berkeley, California
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We present the results of a study of the decay process

$$\eta \to \pi^+ + \pi^- + \gamma. \tag{1}$$

The establishment of the existence of this decay mode of the eta, and a preliminary value for

$$R \equiv \Gamma(\eta \to \pi^+ + \pi^- + \gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)$$

was given in an earlier paper.[1]

Based on an almost background-free sample of 33 events of type (1), we obtain the following results:

1. The branching ratio is

$$R = 0.30 \pm 0.06. \tag{2}$$

Our result (2) is fairly consistent with those previously reported.[1-4]

2. The charge asymmetry is

$$f_+ - f_- = -0.02 \pm 0.17, \tag{3}$$

where f_+ is the fraction of events with π^+ more energetic than π^- in the eta frame, and $f_- = 1 - f_+$. We conclude that reaction (1) exhibits no large violation of C invariance.[5]

3. The angular distribution of the γ ray in the di-pion rest frame shows that the di-pion has $J = 1$. Other values for J are strongly rejected.

4. The assumption that the rho meson dominates the decay mode gives a good fit to the gamma-ray energy distribution, whereas the assumption of a nonresonant $J = 1$ di-pion fits rather poorly.

5. We find no evidence for the enhancement at low gamma-ray energy (<60 MeV) reported by Pauli and Muller.[6]

The remainder of the paper is devoted to experimental details and a more detailed discussion of the results. Our initial sample of events consists of about 4000 four-pronged events produced by 1170-MeV/c π^+ in the Alvarez 72-in. hydrogen bubble chamber. Protons are identified on the scanning table on the basis of their bubble density. All events are fitted to a number of hypotheses (described below), and then a series of cutoffs is applied. The effect of the cutoffs is estimated using the Monte Carlo program FAKE.[7] The cutoffs are as follows:

(A) Four-constraint (4C) fit. If χ^2 for the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- \tag{4}$$

is less than 35, we reject the event.[8]

(B) $\pi^+\pi^-\pi^0$ production. The events are fitted (1C) to the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- + \pi^0 \tag{5}$$

and are removed if χ^2 is less than 7.

(C) $\pi^+\pi^-\gamma$ production. If χ^2 for the fit (1C) to

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- + \gamma \tag{6}$$

is greater than 8.6, the event is discarded. It is also discarded if χ^2 for Reaction (5) is less than that for Reaction (6).

(D) Coulomb scatter. One of the four final tracks is deleted, and the remaining tracks are then fitted (1C) to reaction (4). Events for which χ^2 is less than χ^2 for reaction (6) are removed, provided that $\psi p\beta$ is less than 35 rad MeV/c, where ψ is the space angle between the fitted and measured momentum vector of the deleted track, and $p\beta$ is the fitted momentum-velocity product for this track. This cutoff removes 12 events, of which 1.7 ± 0.3 are good eta decays, according to FAKE.

Table I. Details of 33 events. θ is the angle between the π^+ and γ in the di-pion rest frame; p is the γ energy in the eta frame.

Event	Cos θ	p(MeV)	Event	Cos θ	p(MeV)
2153458	0.521	146.3	2180249	0.035	116.7
2159233	0.570	80.5	2183369	0.765	140.5
2159366	0.670	62.3	2184040	0.937	132.9
2162397	0.322	79.7	2195383	-0.434	123.1
2163095	-0.492	144.3	2196202	-0.149	107.8
2163288	0.288	117.4	2197247	0.176	193.5
2163466	-0.564	123.1	2197352	-0.444	176.0
2169380	-0.187	185.4	2198452	0.236	76.7
2172460	-0.387	63.3	2199275	-0.739	84.2
2175201	-0.750	99.1	2200242	-0.739	167.8
2175317	0.656	92.3	2202342	-0.167	103.4
2175441	0.229	69.5	2202417	0.251	77.0
2176531	-0.662	155.7	2202520	-0.269	147.1
2177176	0.197	112.7	2205066	-0.453	123.4
2177572	0.500	145.3	2208477	0.670	140.9
2179400	0.548	113.4	2211063	-0.273	166.8
2180190	-0.578	97.6			

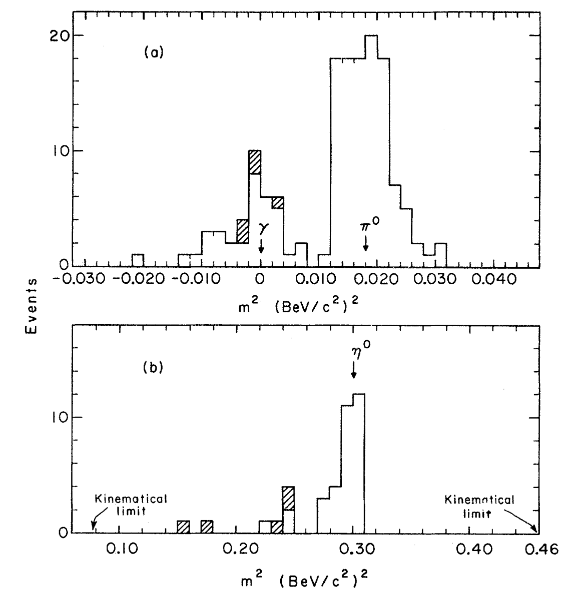

FIG. 1. Mass distributions. (a) Distribution in m^2 (mass squared) of the missing neutral in $\pi^+ + p \rightarrow \pi^+ + p + \pi^+ + \pi^- +$neutral. All events with m^2 less than 0.006 (BeV/c^2)^2 also happen to satisfy our χ^2(1C) criteria for selecting gamma rays. The five shaded gamma rays do not come from eta decay. (b) Distribution in m^2 of $\pi^+\pi^-\gamma$ for $\pi^+ + p \rightarrow \pi^+ + p + \pi^+ + \pi^- + \gamma$. [That π^+ is chosen which gives $m(\pi^+\pi^-\gamma)$ closest to the eta mass, 548 MeV.] The five shaded gamma rays do not satisfy our χ^2(2C) criteria for $\eta \rightarrow \pi^+ + \pi^- + \gamma$. The three "good-eta gamma rays" that lie outside the main eta peak do satisfy our χ^2 criteria and are used. (According to our FAKE calculation, the sample contains an estimated 2.2 spurious gammas arising from neutral pions with large measurement errors.)

(E) Scanning-table examination. The remaining events are examined on the scanning table to search for possible electrons misfit as pions. Two events involve Dalitz-pair electrons and are removed.

We are left with 38 good events of type (6). These are fitted (2C) to

$$\pi^+ + p \rightarrow \pi^+ + p + \eta^0, \quad \eta^0 \rightarrow \pi^+ + \pi^- + \gamma, \tag{7}$$

using an eta mass of 548. Their χ^2 distribution agrees well with the theoretical χ^2(2C) distribution[8] up to χ^2 about 20. Five events have large χ^2(2C) and are believed to be type (6), but not from eta decay (7). We take the 33 events with χ^2(2C) less than 20 as our final sample. In Table I we give details of the 33 events.

From the same sample of 4000 four-pronged events, using a similar method of analysis,[9] we find 113 good events of the type

$$\pi^+ + p \rightarrow \pi^+ + p + \eta, \quad \eta \rightarrow \pi^+ + \pi^- + \pi^0. \tag{8}$$

To illustrate the clean separation between $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\gamma$ production, we show in Fig. 1(a) a plot of the unfit missing-neutral mass (squared) recoiling against the $\pi^+ p \pi^+ \pi^-$ for our $33+5+113$ events. To illustrate the lack of non-eta background for $\pi^+\pi^-\gamma$ production we show in Fig. 1(b) a plot of $m^2(\pi^+\pi^-\gamma)$, using the final π^+ that gives a mass closest to the eta, for our 38 events of type (6). We see that our selection criteria based on χ^2 give essentially the same sample that we would obtain if we selected on missing-neutral mass and on $m(\pi^+\pi^-\gamma)$. The χ^2 method carries less visual appeal than the mass plots, but has the advantages that it takes the measurement errors into account systematically, and that it is easier to calculate (using FAKE) the effects of cutoffs based on χ^2 than of cutoffs based on calculated errors in missing mass.[1] We now turn to the results.

Angular distribution.—In Fig. 2 we plot the (folded) angular distribution in $|\cos\theta|$, where θ is the angle between the π^+ and γ in the di-pion c.m. system. From FAKE we find that our detection efficiency is essentially independent of $\cos\theta$. Angular-momentum conservation and zero eta spin demand that the di-pion have $J_z = \pm 1$ for z along the gamma direction in the di-pion frame. Thus for a pure di-pion state J, the distribution in $\cos\theta$ is given by $|Y_J^{\pm 1}|^2$. Normalizing to 33 counts and calculating[10] χ^2 for curves corresponding to $J=1$, 2, and 3, we find $\chi^2 = 4.5$ for $J=1$, 49.4 for $J=2$, and 102.0 for $J=3$; in each case the "expected" χ^2 is 4. We conclude that the di-pion is dom-

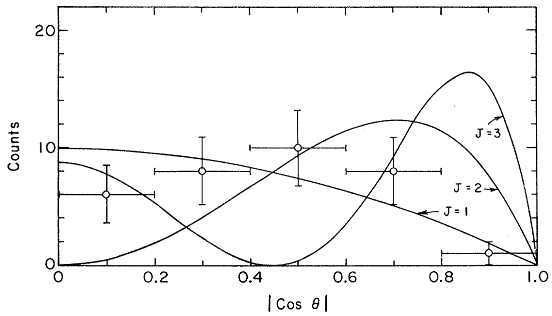

FIG. 2. Folded angular distribution for $\eta \to \pi^+ + \pi^- + \gamma$ Here θ is the angle between the π^+ and the γ in the di-pion rest frame. The three smooth curves correspond to $J=1$, 2, and 3 for the di-pion. ($J=0$ is forbidden by angular-momentum conservation, since the eta spin is zero.) We see that $J=1$ fits well, and $J=2$ and 3 fit poorly.

inated by $J=1$. The dominant decay amplitude is therefore C-conserving (i.e., has odd J),[11] and the di-pion has the spin and isospin of the rho meson.

Charge asymmetry.—If there is a small amount of C-nonconserving amplitude, we may have some $J=2$ (or other even value) amplitude present. The interference between the dominant $J=1$ and any even J leads to odd powers of $\cos\theta$ in the angular distribution, or, equivalently, to a charge asymmetry $f_+ - f_- \neq 0$ in the pion-energy distribution in the eta rest frame.[5] Since we have 17 events with $\cos\theta > 0$ and 16 with $\cos\theta < 0$, the raw data give $f_+ - f_- = -(1/33) \pm 0.17$. Using FAKE we find that the choice of the wrong π^+ in a small fraction of the events leads to a small spurious asymmetry. Correcting for this, we find the result given in Eq. (3).

Energy spectrum of the gamma ray.—In Fig. 3 we plot the distribution of the energy p of the gamma ray in the eta rest frame, for the 33 events. We have no cutoff on gamma-ray energy. The detection efficiency $\epsilon(p)$ depends on the gamma energy and on our χ^2 cutoffs; it is calculated using FAKE,[12] and plotted in Fig. 3. We multiply any theoretical curve by $\epsilon(p)$ before fitting to the data.

The simplest matrix element corresponding to a di-pion with $J=1$ is

$$|M|^2 = p^2 q^2 \sin^2\theta, \tag{9}$$

where q is the momentum of either pion in the di-pion frame, and p and θ are as previously defined. We have already verified in Fig. 2 that the angular distribution fits $\sin^2\theta$. We therefore integrate over $\cos\theta$ and write

$$dN = C\epsilon(p)p^2q^2dp, \tag{10}$$

where C is a normalization constant, and dN is the number of counts expected in the interval dp, taking into account the detection efficiency. This "nonresonant $J=1$" curve is normalized to 33 events in Fig. 3 and gives $\chi^2 = 13.2$ with 6 degrees of freedom for a χ^2 probability[10] of 4%, a rather poor fit.

Next we assume that the $I=1$, $J=1$ di-pion phase shifts are dominated by the rho meson.[13] We then replace the factor q^2 in Eq. (10) by a resonance factor[14]:

$$q^2 \to \frac{m_{\pi\pi}}{q} \frac{\Gamma}{(m_\rho^2 - m_{\pi\pi}^2)^2 + m_\rho^2\Gamma^2},$$

where $\Gamma = (q^3/q_0^3)\gamma$, $m_{\pi\pi}$ is the mass of the di-pion, m_ρ is the rho mass (765 MeV), q_0 is the value of q at resonance (357 MeV), and γ is the "reduced width" of the rho (124 MeV). This "rho-dominant" curve is normalized to 33 events in Fig. 3 and gives $\chi^2 = 5.9$ for a χ^2 probability[10] of 40%, a good fit. Thus we lend some support to the rho-dominant model. (However, it is apparent from Fig. 3 that any other model that shifts the spectrum towards lower gamma-ray energies would also fit.)

The fact that we find evidence for final-state interactions in the di-pion system lends encouragement to the possibility of (eventually) detecting the charge asymmetry, if a small amount of C-nonconserving amplitude is actually present. If there were no final-state interactions,

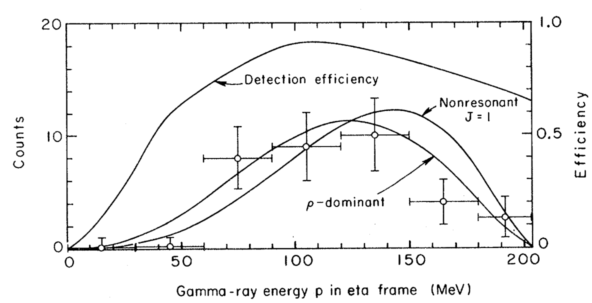

FIG. 3. Energy distribution. The detection efficiency $\epsilon(p)$ is calculated using FAKE. The two theoretical curves correspond to a nonresonant $J=1$ and to a rho-dominant $J=1$ di-pion. Neither curve has any free parameter except for a normalization constant. The curves are multiplied by $\epsilon(p)$ before plotting and comparing with the data. The rho-dominant model fits very well, the nonresonant model not so well.

the interference term would necessarily vanish (by CPT invariance) and there could then be no charge asymmetry even if C invariance were violated.[15]

Branching ratio. – Using the detection efficiency $\epsilon(p)$ and the "rho-dominant" curve, we calculate[16] that our total corrected number of decays $\eta \to \pi^+ + \pi^- + \gamma$ is 40.2 ± 7.1. We calculate using FAKE that this includes 2.2 ± 0.5 events of type (8) that were not removed by the cutoffs. All other corrections are negligible.[17] We also calculate from our 113 good events of type (8) a corrected number 128 ± 11.3. We thus find $R = (40.2-2.2)/128 = 0.30 \pm 0.06$. This is consistent with the prediction of the rho-dominant model.[18]

Low-energy gamma rays. – Pauli and Muller[6] find about as many decays $\eta \to \pi^+ + \pi^- + \gamma$ with gamma energy between 10 and 60 MeV in the eta frame as they do between 60 and the maximum allowed value of 203 MeV; namely, they find about 12 events above estimated background in each of the two regions. If the "true" spectrum (corresponding to 100% detection efficiency) gave equal numbers of counts in these two regions, then, taking into account our detection efficiency $\epsilon(p)$ as plotted in Fig. 3, we would expect to find 20 counts below 60 MeV, whereas we find none. (For our best-fit curve in Fig. 3 we expect two counts below 60 MeV and find none; this is an entirely reasonable statistical fluctuation.) We conclude that there are no anomalous low-energy gamma rays.[19]

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[1]E. C. Fowler, F. S. Crawford, Jr., L. J. Lloyd, R. A. Grossman, and L. R. Price, Phys. Rev. Letters 10, 110 (1963), found $R = 0.26 \pm 0.08$.

[2]Margaret C. Foster, thesis, University of Wisconsin, 1965 (unpublished), finds $R = 0.20 \pm 0.04$.

[3]H. W. J. Foelsche and H. L. Kraybill, Phys. Rev. 134, B1138 (1964), find $R = 0.14 \pm 0.08$.

[4]E. Pauli and A. Muller, Phys. Letters 13, 351 (1964), would find $R = 0.27 \pm 0.10$ if they used only their "Class a" events, which involve unambiguous gamma rays. [When they include "Class b" events (Ref. 6), they find $R = 0.73 \pm 0.25$.]

[5]The possibility of small violations of C invariance with $f_+ - f_-$ perhaps of order 0.1 has been suggested by J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965), and by S. L. Glashow and C. M. Sommerfield, Phys. Rev. Letters 15, 78 (1965).

[6]Pauli and Muller, Ref. 4, and E. Pauli, thesis, University of Strasbourg, 1965 (unpublished), find an enhancement at gamma energies between 10 and 60 MeV in the eta frame. The enhancement arises from their "Class b" events, which are kinematically ambiguous with a zero-energy gamma ray, i.e., with no gamma ray at all. Their Class b peak at the eta mass band amounts to about 2.7 standard deviations above their non-eta background. (See also Ref. 19.)

[7]Gerald R. Lynch, Lawrence Radiation Laboratory Report UCRL-10335, 1962 (unpublished).

[8]Experience has shown that, for a correct hypothesis, the χ^2 distributions for 72-in. chamber events agree with the theoretical distributions for the appropriate constraint class, provided the theoretical values of χ^2 are multiplied by 2. Thus the 4C cutoff at $\chi^2 = 35$ corresponds to a theoretical χ^2 of approximately 18. Our 4C cutoff causes our detection efficiency for gamma rays to be negligibly small below a laboratory gamma energy of about 20 MeV. We make no cutoff on gamma-ray energy.

[9]We reject events with $\chi^2(4C) < 35$, and then demand $\chi^2(1C) < 8.6$ for reaction (5), $\chi^2(2C) < 6$ for reaction (8), and $\chi^2(2C)$ for (8) less than that for (7).

[10]We define $\chi^2 = \sum[(N_i - \overline{N}_i)^2/\overline{N}_i]$, where N_i is the observed number of counts in the ith bin, and $\overline{N}_i$ is the expected number from the smooth theoretical curve. The smooth curve is normalized to the observed 33 counts, and there are no free parameters. Note that since several bins have $\overline{N}_i$ of order one count or less, least-squares theory is not strictly applicable. Therefore our phase "χ^2 probability" is to be taken not literally, but rather as a crude measure of goodness of fit.

[11]The eta and gamma have $C = +1$ and -1, respectively. Thus C conservation demands $C(\pi^+\pi^-) = -1$. But $C(\pi^+\pi^-)$ equals $(-1)^J$; therefore J is odd. Thus if we had found $J = 2$ dominant for the di-pion, it would have indicated a gross violation of C conservation.

[12]We start with about 1000 simulated events of type (7). The momentum distribution of the etas and of the other final particles is taken to be that observed in our larger sample of etas that decay via mode (8). The simulated events are analyzed by the same kinematics programs and subjected to the same cutoffs as those used for the real candidates.

[13]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[14]J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[15]J. Bernstein et al., Ref. 5.

[16]We divide our 33 events by the average detection efficiency $\overline{\epsilon}$, where $\overline{\epsilon} \equiv [\int \epsilon(p)(dN/dp)dp]/[\int(dN/dp)dp]$, where the integral extends over the entire range $p = 0$ to 203 MeV. If dN/dp is given by the rho-dominant curve, we find $\overline{\epsilon} = 0.821$. The nonresonant spectrum (10) would give $\overline{\epsilon} = 0.820$, and thus the same branching ratio R.

[17]For example, we estimate that our 33 events include less than 0.1 events of type (4).

[18]M. Gell-Mann et al. (Ref. 13) predict $r \equiv \Gamma(\eta \to \pi^+ + \pi^- + \gamma)/\Gamma(\eta \to \gamma + \gamma) = \frac{1}{2}(\gamma_{\rho\pi\pi}^2/4\pi)(\gamma_\rho^2/4\pi)(9/4) \approx 0.28$, for $\gamma_{\rho\pi\pi}^2/4\pi \approx \gamma_\rho^2/4\pi \approx \frac{1}{2}$. If we take $\Gamma(\eta \to \gamma+\gamma)/\Gamma(\eta \to \text{charged}) \approx 1.26$, from A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965), then our result (2) gives $r \approx 0.18$.

[19]In this experiment we would not detect a low-energy gamma-ray peak if it lay below 20 MeV in the eta frame. Neither would we detect the decay $\eta \to \pi^+ + \pi^-$, which is forbidden by parity conservation. Neither would we detect $\zeta(560) \to \pi^+ + \pi^-$, which would be nearly indistinguishable from $\eta(548) \to \pi^+ + \pi^-$, and is forbidden only by the fact that the $\zeta(560)$ apparently does not exist. [These processes are not detected because events with zero or very low γ-ray energy are removed by the same cutoffs which remove events of type (4). When we examine the distribution in $m^2(\pi^+\pi^-)$ of these cut-off events, we see no enhancement near the eta mass. However, an enhancement of order 20 or 30 counts would be unresolvable against the large non-eta background of events of type (4).] All of these possible processes would (if they occurred) be included in the Class b events of Pauli and Muller.[6]

GENERALIZED DEFINITION OF P AND C

N. P. Chang

The City College of the City University of New York, New York, New York
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Since the discovery of CP nonconservation over a year ago,[1] many attempts have been made toward a reformulation of the discrete symmetry operations in fundamental physics.[2] In this paper we present a new definition of the transformation properties corresponding to space inversion and antiparticle conjugation. This new concept leaves invariant the interaction Hamiltonian involving the hadrons, including the strong and electromagnetic as well as weak interactions of the hadrons.

Parity invariance in this generalized sense implies that a measurement of the hyperon nonleptonic decays yields, at the same time, information on the structure of the baryon-meson strong interactions. This unification of strong with weak interactions will be particularly interesting when accurate data on the hyperon nonleptonic decays become available.

The situation becomes even more interesting when we extend the generalized concept of parity and antiparticle conjugation to the leptonic decays of the hadrons. $\mathcal{P}$ and $\mathcal{C}$ invariance now lead to a new form of μ-e universality, and could, in principle, explain the μ-e mass difference, if we assume zero bare mass for the leptons. The new form of μ-e universality is, stated simply, that if $e\bar{\nu}$ is coupled to $(V_\lambda + A_\lambda)$ baryon current, then $\mu\nu'$ is coupled to $(V_\lambda - A_\lambda)$. As discussed below, we believe that the μ-capture data in hydrogen do not preclude this possibility provided we allow for a large induced pseudoscalar coupling constant.

I. Hadron interactions.—We assume the group SU(3) to be the fundamental symmetry of particle physics. We study the properties of the interaction Hamiltonian within this framework of SU(3). The free part of the Hamiltonian contains the mass term for baryons and mesons. This mass term, which transforms like a pure λ_8, does not commute with the generalized parity that is discussed below.[3] This is desirable since otherwise the physical particle states can be made eigenstates of $\mathcal{P}$, the generalized parity, and no asymmetry parameters can ensue for hyperon decays.[4]

Let $[B_\alpha(x)]_j{}^i$ be a four-component field operator representing an SU(3) baryon octet (i.e., $i,j = 1,2,3$; $\alpha = 1, \cdots, 4$). Under a space inversion, we postulate that

$$\mathcal{P}[B_\alpha(\vec{x},t)]_j{}^i\mathcal{P}^{-1} = -\Pi_l{}^i[(\gamma_4)_{\alpha\beta}B_\beta(-\vec{x},t)]_k{}^l\Pi_j{}^k, \tag{I.1}$$

where Π is a Hermitian 3×3 matrix satisfying the properties

$$\Pi^2 = 1, \quad \det\Pi = -1, \tag{I.2a}$$

$$[\Pi, Q] = 0. \tag{I.2b}$$

Q is the charge matrix, taken to be

$$Q = \frac{1}{3}\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}. \tag{I.3}$$

An explicit representation for Π is

$$\Pi = \begin{pmatrix} 1 & & \\ & \sin\varphi & \cos\varphi \\ & \cos\varphi & -\sin\varphi \end{pmatrix}. \tag{I.4}$$

For $\varphi = \pi/2$, it is clear that Eq. (I.1) implies that the intrinsic parities of the baryon octet