CURRENT ALGEBRAS AND NONLEPTONIC K-MESON DECAYS

S. K. Bose* and S. N. Biswas

Centre for Advanced Study in Theoretical Physics and Astrophysics, University of Delhi, Delhi, India
(Received 3 January 1966)

Recently the commutation relations of the vector and axial-vector currents proposed by Gell-Mann[1] have found several[2,3] important applications. In particular, Sugawara[3] and Suzuki[3] have given a model for the s-wave part of the nonleptonic hyperon decays based on the following assumptions: (1) The weak Hamiltonian is a product of CP-even Cabibbo currents, (2) the strangeness-conserving axial-vector current is partially conserved, (3) the commutation relations of the fourth component of axial-vector current $A_0(x)$ with the weak Hamiltonian is that derived from a quark model, and (4) an unsubtracted dispersion relation in momentum transfer is valid for the decay amplitudes. In this note, we apply the above model to the nonleptonic K-meson decays together with one further assumption; namely, that (5) the final 3π state in these decays is completely symmetric with respect to the space coordinates, i.e., each pair of pions is in relative s wave. This last assumption is the simplest one consistent with Dalitz's analysis of the τ spectrum.[4] We shall throughout neglect the CP-nonconserving effects in K-meson decays.

We first consider $K\to 3\pi$ modes. From assumptions (1)-(4) and following the procedure of Ref. 3, we obtain the decay-matrix elements[5]

$$\langle\pi^{\nu_0}\pi^{\nu_1}\pi^{\nu_3}|H^{\nu_4\nu_5}|K^{\nu_2}\rangle = i\frac{g_r}{Mg_A}\begin{pmatrix}8 & 8 & 8_a\\ \nu_3 & \nu_4 & \nu_6\end{pmatrix}\langle\pi^{\nu_0}\pi^{\nu_1}|H^{\nu_5\nu_6}|K^{\nu_2}\rangle + \nu_4 \longleftrightarrow \nu_5. \quad (1)$$

In (1), g_r is the pion-nucleon coupling constant $g_r^2/4\pi\simeq 14.6$, g_A the axial-vector renormalization constant, and M the nucleon mass. H is the weak Hamiltonian and ν stands for (I, I_3, Y). For K decays $\nu_4=(1,1,0)$, $\nu_5=(\frac{1}{2},-\frac{1}{2},-1)$, while for $\bar{K}$ decays $\nu_4=(1,-1,0)$ and $\nu_5=(\frac{1}{2},\frac{1}{2},1)$. Because of assumption (5) the 2π states occurring in the right-hand side of Eq. (1) are in s wave and hence directly related to the $K\to 2\pi$ amplitude. Now it is a well-known consequence of CP invariance and the current×current picture that H is a linear combination of octet and 27-plet tensors and that this Hamiltonian rigorously forbids the $K\to 2\pi$ transitions[6] in the limit of exact SU(3). Equation (1) now says that the $K\to 3\pi$ transitions are forbidden as well. We thus conclude that, in the limit of exact SU(3), assumptions (1)-(5) forbid all nonleptonic K-meson decays.

To discuss these decays, we have thus to consider departures from exact SU(3) brought about by medium-strong interaction (mass differences). We do this in the usual way, e.g., by introducing a spurion η and treating the decay process as $\eta+K\to 3\pi$. From the Okubo[7] hypothesis η transforms as the $I=0$, $Y=0$ member of an octet, all other properties of η being those of vacuum. The initial state can be decomposed into a linear combination of $\underline{8}_s$, $\underline{8}_a$, $\underline{27}$, $\underline{10}$, and $\underline{10}^*$ states. Once again, from CP invariance it follows that[8] out of these states only the $\underline{8}_a$ will make a nonvanishing contribution to the decay matrix elements. Thus we obtain the final expression for the decay amplitude:

$$\langle\pi^{\nu_0}\pi^{\nu_1}\pi^{\nu_3}|H^{\nu_4\nu_5}|K\eta^{\nu_2}\rangle = i\frac{g_r}{Mg_A}\begin{pmatrix}8 & 8 & 8_a\\ \nu_2 & 0 & \nu_2\end{pmatrix}\begin{pmatrix}8 & 8 & 8_a\\ \nu_3 & \nu_4 & \nu_6\end{pmatrix} \times\langle\pi^{\nu_0}\pi^{\nu_1}|H^{\nu_5\nu_6}|K\eta^{\nu_2}, 8_a\rangle + \nu_4 \longrightarrow \nu_5. \quad (2)$$

It is understood that the above expression is to be properly symmetrized with respect to the final 3π system. From assumption (5) and the Bose principle it follows that the 2π states occurring on the right-hand side of Eq. (2) exist in $\underline{8}_s$ and $\underline{27}$ only:

$$\langle\pi^{\nu_0}\pi^{\nu_1}| = \sum_{\nu'}\left\{\begin{pmatrix}8 & 8 & 8_s\\ \nu_0 & \nu_1 & \nu'\end{pmatrix}\left\langle\begin{matrix}8_s\\ \nu'\end{matrix}\right| + \begin{pmatrix}8 & 8 & 27\\ \nu_0 & \nu_1 & \nu'\end{pmatrix}\left\langle\begin{matrix}27\\ \nu'\end{matrix}\right| + \begin{pmatrix}8 & 8 & 1\\ \nu_0 & \nu_1 & \nu'\end{pmatrix}\left\langle\begin{matrix}1\\ \nu'\end{matrix}\right|\right\}. \quad (3)$$

We express $H^{\nu_5\nu_6}$ in terms of $\underline{8}$ and $\underline{27}$ tensors:

$$H^{\nu_5\nu_6}=\sum_\nu \begin{pmatrix} 8 & 8 & | & 8_S \\ \nu_5 & \nu_6 & | & \nu \end{pmatrix} T_\nu^{\ 8} + \sum_\nu \begin{pmatrix} 8 & 8 & | & 27 \\ \nu_5 & \nu_6 & | & \nu \end{pmatrix} T_\nu^{\ 27}. \tag{4}$$

From Eqs. (2)-(4) and using the Wigner-Eckart theorem we express the decay amplitudes in terms of seven reduced matrix elements. Notice that in the same process we have picked up the $K\to 2\pi$ amplitudes as well. We evaluate the relevant Clebsch-Gordan coefficients[9] and summarize our results in Table I. A closer examination of this table reveals that the decay amplitudes are given actually in terms of five parameters, as the reduced amplitudes $\langle 8\,\|\,T^{27}\,\|\,8\rangle$, $\langle 8\,\|\,T^8\,\|\,8\rangle_S$, and $\langle 27\,\|\,T^8\,\|\,8\rangle$ occur always in a fixed linear combination and hence act effectively as a single parameter. It is further evident from Table I that the requirement of CP invariance has not yet been exhausted. Imposing this requirement, i.e., $\langle 2\pi\,|H\,|K_2^{\,0}\rangle=0$, $\langle 3\pi\,|H\,|K_1^{\,0}\rangle=0$, we get three constraints:

$$\langle 27\,\|\,T^{27}\,\|\,8\rangle_1=0;\quad \langle 1\,\|\,T^8\,\|\,8\rangle=0; \tag{5}$$

$$\tfrac{2}{3}\langle 8\,\|\,T^{27}\,\|\,8\rangle+(\tfrac{3}{2})^{1/2}[\langle 8\,\|\,T^8\,\|\,8\rangle_S+\tfrac{1}{4}\langle 27\,\|\,T^8\,\|\,8\rangle]=0. \tag{6}$$

Thus, finally, we obtain a two-parameter formula for each of the seven nonvanishing decay amplitudes. Hence there exist five relations connecting these amplitudes. These are

$$\mathfrak{M}(K_1^{\,0}\to\pi^++\pi^-)-\sqrt{2}\,\mathfrak{M}(K_1^{\,0}\to\pi^0+\pi^0) = 2\mathfrak{M}(K^+\to\pi^++\pi^0), \tag{7}$$

$$\mathfrak{M}(K^+\to\pi^++\pi^0\pi^0)=\tfrac{1}{2}\mathfrak{M}(K^+\to\pi^++\pi^-+\pi^+), \tag{8}$$

$$\mathfrak{M}(K_2^{\,0}\to\pi^0+\pi^0+\pi^0)=(\tfrac{3}{2})^{1/2}\mathfrak{M}(K_2^{\,0}\to\pi^++\pi^-+\pi^0), \tag{9}$$

$$\frac{\mathfrak{M}(K_1^{\,0}\to\pi^0+\pi^0)}{\mathfrak{M}(K_1^{\,0}\to\pi^++\pi^-)}=-\frac{1}{2}\,\frac{\mathfrak{M}(K_2^{\,0}\to\pi^++\pi^-+\pi^0)}{\mathfrak{M}(K\ \ \to\pi^++\pi^0+\pi^0)}, \tag{10}$$

$$\mathfrak{M}(K_2^{\,0}\to\pi^+\pi^-+\pi^0) =-(ig_\gamma/Mg_A)(1/3\sqrt{2})\mathfrak{M}(K_1^{\,0}\to\pi^0+\pi^0). \tag{11}$$

Table I. Expressions for decay amplitudes.

Decay process \ Coefficient of	$\langle 27\|T^{27}\|8\rangle_1$	$\langle 27\|T^{27}\|8\rangle_2$	$\langle 8\|T^{27}\|8\rangle$	$\langle 8\|T^8\|8\rangle_S$	$\langle 8\|T^8\|8\rangle_a$	$\langle 27\|T^8\|8\rangle$	$\langle 1\|T^8\|8\rangle$
$K^+\to\pi^++\pi^0$	$\frac{3}{8}\sqrt{\frac{5}{84}}$	$-\frac{1}{16}$	0	0	0	0	0
$K_1^{\,0}\to\pi^++\pi^-$	$-\frac{6}{25}\sqrt{\frac{5}{84}}$	$-\frac{1}{20}$	0	0	$-\frac{1}{10}\sqrt{\frac{3}{2}}$	0	$-\frac{1}{8}\sqrt{\frac{3}{10}}$
$K_1^{\,0}\to\pi^0+\pi^0$	0	$\frac{3\sqrt{2}}{80}$	0	0	$-\frac{\sqrt{3}}{20}$	0	$-\frac{1}{8}\sqrt{\frac{3}{20}}$
$K_2^{\,0}\to\pi^++\pi^-$	0	0	$-\frac{\sqrt{5}}{75}$	$-\frac{1}{10}\sqrt{\frac{3}{10}}$	0	$-\frac{1}{40}\sqrt{\frac{3}{10}}$	0
$K_2^{\,0}\to\pi^0+\pi^0$	$-\frac{17}{100}\sqrt{\frac{15}{56}}$	0	$-\frac{1}{75}\sqrt{\frac{5}{2}}$	$-\frac{1}{10}\sqrt{\frac{3}{20}}$	0	$-\frac{1}{40}\sqrt{\frac{3}{20}}$	0
$K^+\to\pi^++\pi^0+\pi^0$	$\frac{1}{50}\sqrt{\frac{5}{42}}$	$-\frac{\sqrt{2}}{240}$	$-\frac{1}{450}\sqrt{\frac{5}{2}}$	$-\frac{1}{60}\sqrt{\frac{3}{20}}$	$-\frac{\sqrt{3}}{120}$	$-\frac{1}{240}\sqrt{\frac{3}{20}}$	$\frac{1}{16}\sqrt{\frac{1}{60}}$
$K^+\to\pi^++\pi^-+\pi^+$	$\frac{1}{25}\sqrt{\frac{5}{42}}$	$-\frac{\sqrt{2}}{120}$	$-\frac{1}{225}\sqrt{\frac{5}{2}}$	$-\frac{1}{30}\sqrt{\frac{3}{20}}$	$-\frac{\sqrt{3}}{60}$	$-\frac{1}{120}\sqrt{\frac{3}{20}}$	$\frac{1}{8}\sqrt{\frac{1}{60}}$
$K_1^{\,0}\to\pi^++\pi^-+\pi^0$	$\frac{17}{200}\sqrt{\frac{5}{84}}$	0	$\frac{\sqrt{5}}{450}$	$\frac{1}{10}\sqrt{\frac{1}{120}}$	0	$\frac{1}{40}\sqrt{\frac{1}{120}}$	$-\frac{1}{8}\sqrt{\frac{1}{120}}$
$K_1^{\,0}\to\pi^0+\pi^0+\pi^0$	$\frac{17}{200}\sqrt{\frac{5}{56}}$	0	$\frac{1}{150}\sqrt{\frac{5}{6}}$	$\frac{1}{20}\sqrt{\frac{1}{20}}$	0	$\frac{1}{80}\sqrt{\frac{1}{20}}$	$-\frac{1}{16}\sqrt{\frac{1}{20}}$
$K_2^{\,0}\to\pi^++\pi^-+\pi^0$	0	$-\frac{1}{80}$	0	0	$\frac{\sqrt{6}}{120}$	0	0
$K_2^{\,0}\to\pi^0+\pi^0+\pi^0$	0	$-\frac{1}{80}\sqrt{\frac{3}{2}}$	0	0	$\frac{1}{40}$	0	0

We now discuss relations (7)-(11). First, we must emphasize the general validity of these results notwithstanding our restrictive derivation, e.g., use of first-order symmetry breaking. To see this, consider second-order symmetry breaking. By repeating the arguments used before we conclude that the only "initial" state which now contributes to the decay process is an octet constructed symmetrically out of η and $|K\eta\rangle_{8_a}$. Clearly, (7)-(11) are reproduced to this order. Proceeding in the same spirit we may confirm the validity of (7)-(11) to any order in symmetry breaking.[10]

In our derivation, the current commutation rules have not been used at all in obtaining relation (7). In fact, it follows from isospin formalism and assumption (1) alone.[11] Equations (8) and (9) are well known. They can be derived[12] from isospin formalism and assumptions (1) and (5). Relations (7)-(9) thus do not test the validity of the present model. The results that do depend on the reduction of the pion and the use of current commutation relations are Eqs. (10) and (11). From (10), which may be viewed as a generalization of the usual $T=\frac{1}{2}$ rule, we obtain[13]

$$\frac{\mathcal{R}(K_1{}^0\to\pi^0+\pi^0)}{\mathcal{R}(K_1{}^0\to\pi^++\pi^-)}=\frac{0.97}{4}\,\frac{\mathcal{R}(K_2{}^0\to\pi^++\pi^-+\pi^0)}{\mathcal{R}(K^+\to\pi^++\pi^0+\pi^0)}. \quad (12)$$

Experimentally,[14] the left-hand side of (12) is 0.46 ± 0.02 while the right-hand side is $0.43^{+0.10}_{-0.08}$. The agreement between theory and experiment is thus excellent. From Eq. (11) we obtain the ratio of partial rates[15]

$$R\equiv\frac{\mathcal{R}(K_2{}^0\to\pi^++\pi^-+\pi^0)}{\mathcal{R}(K_1{}^0\to\pi^0+\pi^0)}$$
$$=\frac{0.04}{36\pi^2}\frac{g_\gamma{}^2}{g_A{}^2}\left(\frac{\mu}{M}\right)^2\frac{1}{[1-4\mu^2/m_K{}^2]^{1/2}}. \quad (13)$$

In (13), μ is the pion mass and m_K that of the K meson. Putting in numbers we find

$$R=4.0\times10^{-4}. \quad (14)$$

Experimental[14] value of this ratio is

$$R_{\text{exp}}=(6.6^{+1.6}_{-1.3})\times10^{-4} \quad (15)$$

so that (11) is in fairly good agreement with experiment. We thus conclude that the present model provides a good description of the nonleptonic K-meson decays.

The vanishing of the parity-conserving part of the nonleptonic hyperon decays is a serious drawback of the Sugawara-Suzuki model. It is nonetheless gratifying to note that we have at least a partially successful theory which provides a unified picture of all s-wave nonleptonic decay processes.

We acknowledge the benefit of a stimulating conversation with Dr. V. Gupta.

---

*Address from 1 March 1966: International Centre for Theoretical Physics, Trieste, Italy.

[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

[2]S. Fubini and G. Furlan, Physics 4, 229 (1965); S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); L. K. Pandit and J. Schechter, Phys. Letters 19, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, Phys. Letters 19, 59 (1963).

[3]H. Sugawara, Phys. Rev. Letters 15, 870 (1965); M. Suzuki, Phys. Rev. Letters 15, 986 (1965).

[4]R. H. Dalitz, Phil. Mag. 44, 1068 (1953); Phys. Rev. 94, 1046 (1965).

[5]The right-hand side of Eq. (1) is not exactly the decay amplitude, but its analytic continuation. We assume that this continuation exists. In writing Eq. (1) we have suppressed a factor of $(2k_0)^{1/2}$ (k_0 = energy of the pion dispersed) and absorbed it instead in the phase space. This is a matter of convention.

[6]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964). The vanishing of $K\to2\pi$ transition through an octet Hamiltonian is shown in this paper. See also N. Cabibbo, Phys. Rev. Letters 12, 62 (1964). The same argument may be extended to a 27-plet Hamiltonian. See K. Itabashi, Phys. Rev. 136, B221 (1964).

[7]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[8]The validity of this assertion is as follows. For the 8_S part of $K\eta$ state this is essentially proven in Ref. 6. The same argument evidently extends to the 27 part of $K\eta$. As for the rest, we may note that for K ($\bar{K}$) decays only the 10* (10) states of $K\eta$ contributes. Since 10* (or 10) is not an eigenstate of CP, these transitions connect initial states of ill-defined CP to final states of well-defined CP and are hence CP nonconserving.

[9]These are taken from P. McNamee and F. Chilton, Rev. Mod. Phys. 36, 1005 (1964).

[10]Alternately, we may note that from assumption (1) the weak Hamiltonian contains only $I=\frac{3}{2}$ and $I=\frac{1}{2}$ tensors. The reduction of the $K\to2\pi$ amplitude into two independent irreducible matrix elements is thus valid to all orders of isospin-conserving, SU(3)-breaking interaction. From Eq. (1) the same is true for $K\to3\pi$ amplitudes.

[11]Sum rule (7) has been noted previously by T. Das and K. T. Mahantappa, to be published; and E. C. G. Sudarshan, to be published.

[12]R. H. Dalitz, in Proceedings of the International Conference on Fundamental Aspects of Weak Interactions (Brookhaven National Laboratory, Upton, New York, 1964), p. 378.

[13]The numerical factor 0.97 comes from corrections to phase space due to electromagnetic mass differences. See G. Källén, Elementary Particle Physics (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1964), pp. 445 and 449.

[14]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

[15]The 3π phase space is taken from J. C. Pati, thesis, University of Maryland, 1960 (unpublished).

STUDY OF THE DECAY $\eta^0 \to \pi^+ + \pi^- + \gamma$*

Frank S. Crawford, Jr., and LeRoy R. Price

Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 24 January 1966)

We present the results of a study of the decay process

$$\eta \to \pi^+ + \pi^- + \gamma. \tag{1}$$

The establishment of the existence of this decay mode of the eta, and a preliminary value for

$$R \equiv \Gamma(\eta \to \pi^+ + \pi^- + \gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)$$

was given in an earlier paper.[1]

Based on an almost background-free sample of 33 events of type (1), we obtain the following results:

1. The branching ratio is

$$R = 0.30 \pm 0.06. \tag{2}$$

Our result (2) is fairly consistent with those previously reported.[1-4]

2. The charge asymmetry is

$$f_+ - f_- = -0.02 \pm 0.17, \tag{3}$$

where f_+ is the fraction of events with π^+ more energetic than π^- in the eta frame, and $f_- = 1 - f_+$. We conclude that reaction (1) exhibits no large violation of C invariance.[5]

3. The angular distribution of the γ ray in the di-pion rest frame shows that the di-pion has $J = 1$. Other values for J are strongly rejected.

4. The assumption that the rho meson dominates the decay mode gives a good fit to the gamma-ray energy distribution, whereas the assumption of a nonresonant $J = 1$ di-pion fits rather poorly.

5. We find no evidence for the enhancement at low gamma-ray energy (<60 MeV) reported by Pauli and Muller.[6]

The remainder of the paper is devoted to experimental details and a more detailed discussion of the results. Our initial sample of events consists of about 4000 four-pronged events produced by 1170-MeV/c π^+ in the Alvarez 72-in. hydrogen bubble chamber. Protons are identified on the scanning table on the basis of their bubble density. All events are fitted to a number of hypotheses (described below), and then a series of cutoffs is applied. The effect of the cutoffs is estimated using the Monte Carlo program FAKE.[7] The cutoffs are as follows:

(A) Four-constraint (4C) fit. If χ^2 for the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- \tag{4}$$

is less than 35, we reject the event.[8]

(B) $\pi^+\pi^-\pi^0$ production. The events are fitted (1C) to the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- + \pi^0 \tag{5}$$

and are removed if χ^2 is less than 7.

(C) $\pi^+\pi^-\gamma$ production. If χ^2 for the fit (1C) to

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- + \gamma \tag{6}$$

is greater than 8.6, the event is discarded. It is also discarded if χ^2 for Reaction (5) is less than that for Reaction (6).

(D) Coulomb scatter. One of the four final tracks is deleted, and the remaining tracks are then fitted (1C) to reaction (4). Events for which χ^2 is less than χ^2 for reaction (6) are removed, provided that $\psi p\beta$ is less than 35 rad MeV/c, where ψ is the space angle between the fitted and measured momentum vector of the deleted track, and $p\beta$ is the fitted momentum-velocity product for this track. This cutoff removes 12 events, of which 1.7 ± 0.3 are good eta decays, according to FAKE.