±0.082, where we have made the comparison at the same incident momentum. However, it can be argued that one should really compare the cross sections at the same $Q$ value.[14] Assuming the energy dependence of Reaction (2) to be the same as that observed for Reaction (4), the above experimental ratio is reduced to 0.21. The good agreement between this and the predicted ratio provides some quantitative confirmation of the suggestion that Reactions (1) and (2) may be described by the same Reggeized $\rho$-exchange model.

We wish to thank many members of the Rutherford High Energy Laboratory for their contributions to the experiment. In particular, we acknowledge our indebtedness to Dr. G. H. Stafford, to Dr. L. C. W. Hobbis and all the Nimrod Division, to the Computing Division, and to Mrs. J. Huxtable and her scanners in Oxford. We are grateful for valuable discussions with Dr. G. Höhler, Dr. G. Moorhouse, and Dr. R. J. N. Phillips.

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# UNIVERSALITY: A DYNAMICAL PRINCIPLE FOR THE THEORY OF HADRONS*

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(Received 27 December 1965)

If hadrons were bound states of fundamental quarks[1] then a problem similar to that of nuclear spectroscopy would be the ultimate goal of their theory. Besides the experimental difficulty of the possible inexistence of quarks, this problem raises formidable theoretical difficulties due to the large binding energies involved. Since it has so far proved impossible to solve exactly, or in a meaningful approximation, the essentially dynamical many-body problem of relativistic quantum theory, it is interesting to see whether this problem could be circumvented when calculating at least some of the physically interesting quantities, such as the masses of hadrons, their couplings, etc.

We shall show here that a property of strong interactions which we shall refer to as their universality, when taken together with the large $M(12)$ symmetry of strong interactions, does indeed provide the necessary technique to achieve this goal. By the universality of strong interactions, we mean the usual assumption that (A) the matrix elements, between one-hadron states, of the vector current and of the divergence of the axial-vector current are dominated, respectively, by single $1^-$- and $0^-$-meson intermediate states[2] (pole terms) belonging to the same ($\underline{6}$, $\underline{6}^*$; $L=0$) representation of the group U(6)⊗U(6)⊗O(3), and the new assumption that (B) the matrix elements of the axial-vector current,[3] of the energy-momentum tensors, and of the trace of the energy momentum tensor[4] are dominated, respectively, by single $J^{PC}=1^{++}$-, $2^{++}$-, and $0^{++}$-meson intermediate states (pole terms) belonging to the same (6, 6*; $L=1$) representation of U(6)⊗U(6)⊗O(3). What we intend to show is that under these assumptions, the central masses and the Gell-Mann–Okubo splittings of hadrons, as well as their electromagnetic and weak interaction properties, can be accounted for.

The consequences of assumption (A) have been investigated elsewhere[5,6] with the well-known results

$$\mu_p=-\tfrac{3}{2}\mu_n=\frac{2M_B}{M_M},\quad \mu_{\rho^+}=2\mu_Q=\frac{e_QM_Q}{e_pM_p}\mu_p,\qquad(1)$$

and

$$G_{Mp}/\mu_p=G_{Mn}/\mu_n=G_{Ep},\quad G_{En}\equiv 0,\qquad(2)$$

where we used the following notations: $\mu_x$ = total magnetic moment of particle $x$ in $x$ magnetons, $M_M$ ($M_B$) = central mass of the ($\underline{6}$, $\underline{6}^*$; $L=0$) [($\underline{56}$, $\underline{1}$; $L=0$)] representation of U(6)⊗U(6)⊗O(3).

We shall here deal specifically with assumption (B) and as an illustration of our method we shall use the representations $Q_\alpha=(\underline{6},\underline{1};$ $L=0)$ and $B_{\alpha\beta\gamma}$ ($\underline{56}$, $\underline{1}$; $L=0$), i.e., quarks and baryons, and then state our results for mesons ($\underline{6}$, $\underline{6}^*$; $L=0$). The representation ($\underline{6}$, $\underline{6}^*$; $L=1$) that appears in assumption (B) will be relativistically described by a kinetic supermultiplet[7]

$$R_\mu(q)=(1+\gamma q/m)(\gamma^2V_{\lambda\mu}+\gamma_5P_\mu),\qquad(3)$$

with

$$V_{\lambda\mu}=T_{\lambda\mu}+A_{\lambda\mu}+(\tfrac{2}{3})^{1/2}(g_{\lambda\mu}-q_\lambda q_\mu/m^2)S,$$

$$T_{\lambda\mu}=T_{\mu\lambda},\quad g_{\lambda\mu}T^{\lambda\mu}=0,\quad q_\lambda T^{\lambda\mu}=0,$$

$$A_{\lambda\mu}=(1/m)\epsilon_{\lambda\mu\nu\rho}q^\nu C^\rho,$$

$$P_\mu=\sqrt{2}B_\mu,\quad q_\mu B^\mu=0,\qquad(4)$$

where $m$ is the central mass of the kinetic supermultiplet.

The coupling of $R_\mu$ to quarks is then

$$\Gamma_{RQQ}=q_Q\bar{Q}^\alpha(p')Q_\beta(p)K^\mu R_{\mu\alpha}{}^\beta(q)$$
$$+h_Q\bar{Q}^\alpha(p')Q_\alpha(p)K^\mu R_{\mu\beta}{}^\gamma(q)(\gamma_\nu K^\nu)_\gamma{}^\beta,\qquad(5\text{a})$$

with $q=p'-p$, $K=p+p'$. The terms in the parenthesis are those obtained from the first term by exhaustive use of the kinetons $\gamma q$ and $\gamma K$.[8] We have not included further kinetons of the type, say, $\gamma_5\otimes\gamma_5$,[8] and as a matter of fact we shall also set $h_Q=0$ and refer to this assumption as (C) the "exact" symmetry limit.[9]

Similarly,

$$\Gamma_{RBB}=g_B\bar{B}^{\alpha\beta\gamma}(p')B_{\alpha\beta\delta}(p)K^\mu R_{\mu\gamma}{}^\delta(q)$$
$$+\text{kineton terms},\qquad(5\text{b})$$

and again we shall ignore kinetic emission. The couplings of the $1^{++}$ nonet $C_\alpha$ to quarks and the eight $J^P=\frac{1}{2}^+$ baryons are then

$$\Gamma_{CQQ}(q^2)=g_Q\frac{q^2}{m^2}\left(1+\frac{2M_Q}{m}\right)\bar{u}_Q i\gamma_S\gamma_\mu C^\mu u_Q$$
$$+\text{induced pseudoscalar term},\qquad(6\text{a})$$

and

$$\Gamma_{CB_{1/2}B_{1/2}}(q^2)=g_B\frac{q^2}{m^2}\left(1+\frac{2M_B}{m}\right)\left(1-\frac{q^2}{4M_B{}^2}\right)$$
$$\times(\bar{u}_Bi\gamma_5\gamma_\mu u_B)_{D+\frac{2}{3}F-\frac{1}{3}S}$$
$$+\text{induced pseudoscalar term},\qquad(6\text{b})$$

with

$$F=\mathrm{Tr}(\bar{B}[C,B]),\quad D=\mathrm{Tr}(\bar{B}\{C,B\}),\quad S=\mathrm{Tr}C\,\mathrm{Tr}(\bar{B}B).$$

Because of the Bargmann-Wigner (BW) equations imposed on $R_\mu$ these equations are only valid near $q^2=m^2$. Since we will be interested in the extrapolation of (6) to $q^2=0$, it is important to specify that we will use the procedure of Ref. 5 to do this. This amounts to replacing $q^2/m^2$ in (6) by its value (=1) at the pole $q^2=m^2$,[10] but keeping the $q^2$ dependence due to $(1-q^2/4M_B{}^2)$ which is due to the baryons rather than mesons being on their mass shell. Requiring the $1^{++}$ $C_\rho$-meson pole to dominate the matrix elements of the axial-vector current of weak interactions then means

$$\frac{g_Q(1+2M_Q/m)}{g_B(1+2M_B/m)}=1, \tag{7}$$

so that we have[11]

$$(G_A/G_V)_{n\to p}/(G_A/G_V)_{Q\to Q}=5/3. \tag{8}$$

The couplings of the scalar nonet to quarks and baryons are

$$\Gamma_{SQQ}(q^2)=\left(\frac{2}{3}\right)^{1/2} g_Q\frac{2M_Q}{m}\left(1+\frac{q^2}{2mM_Q}\right)\bar{u}_Q S u_Q \tag{9a}$$

and

$$\Gamma_{SB_{1/2}B_{1/2}}(q^2)=\left(\frac{2}{3}\right)^{1/2} g_B\frac{2M_B}{m}\left(1+\frac{q^2}{2mM_B}\right)\times\left(1-\frac{q^2}{4M_B{}^2}\right)(\bar{u}_B S u_B)_{F+S}. \tag{9b}$$

Here again we replace $1+q^2/2mM_x\to 1+m/2M_x$ $(x=Q,B)$ and $1-q^2/4M_B{}^2\to 1-q^2/4M_B{}^2$ before extrapolating away from $q^2=m^2$. Now according to (B) we assume the SU(3)-singlet ($I=Y=0$ member of the octet) $S$ meson to dominate, at $q^2=0$, the central [SU(3)-breaking] part of the matrix element of the mass–i.e., trace of the energy-momentum tensor–operator. With the ratio of $g_Q/g_B$ given by (7), we then find

$$M_B/M_Q=3, \tag{10a}$$

$$(\Xi-N)/(S-d)=2, \tag{10b}$$

where $M_Q$ is the central quark mass and $S$ ($d$) is the mass of the $I=0$ ($I=\frac{1}{2}$) quark. Relation (10b) is trivial to understand with a quark model of baryons.[1] The factor 3 in (10a) is, of course, due to the fact that there are three quarks in a baryon. It disagrees with present experimental "evidence" but this, of course, is of no serious consequence to our theory for either (i) there may not exist quarks at all in which case our argument so far is of merely academic interest or (ii) even if there exist quarks there might appear new interquark interactions due to the nonvanishing quark triality. Such interactions would, of course, upset (10a) without affecting (10b).

We have first compared quarks and baryons merely because in this case the algebra is essentially simplified and the physical ideas can be clearly exhibited. The nontrivial case is, of course, that of applying universality ideas to mesons ($\underline{6}$, $\underline{6}^*$; $L=0$) and baryons ($\underline{56}$, $\underline{1}$; $L=0$). Again we fix the $g_B/g_M$ ratio from the condition

$$(G_A/G_V)_{n\to p}/(G_A/G_V)_{K^{*0}\to K^{*+}}=5/3, \tag{11}$$

obtained in exactly the same way as (8). We then obtain from assumption (B) applied as above[12]

$$M_B/M_M=\tfrac{3}{2}, \tag{12a}$$

$$(K^2-\pi^2)/(\Xi^2-N^2)=\tfrac{1}{3}. \tag{12b}$$

Of course, from (1) and (12a) we also find[12]

$$\mu_p=3\mu_N. \tag{12c}$$

Relations (12) have been obtained previously by the author[13] under more dynamically specialized assumptions. Both relations (12) are in good (error $\lesssim 20\%$) agreement with experiment.

One can discuss the universality of the $2^{++}$ meson couplings along the same lines. One then finds the relation[14]

$$\sigma_{\rm tot}(MB)\cong\tfrac{2}{3}\sigma_{\rm tot}(BB). \tag{13}$$

Similarly, as is well known, (A) and $M(12)$ invariance also lead to the relation[14]

$$\sigma_{\rm tot}(\bar{p}p)-\sigma_{\rm tot}(pp)=5[\sigma_{\rm tot}(\pi^-p)-\sigma_{\rm tot}(\pi^+p)]. \tag{14}$$

We now wish to return to the problem mentioned at the beginning of this paper, namely that of hadron spectroscopy. The relations (1), (2), (10), (12), (13), and (14) could be also obtained from an independent-particle nonrelativistic model of hadrons in terms of quarks.[15] The advantage in using universality is that one has an explicit relativistic dynamical principle from which to derive these results. Presumably, a detailed solution (if it were possible) of a relativistic many-body problem would automatically lead to these relations. In particular (A) is an automatic consequence of any bound-state model.[13] It is an open question, however, whether (B) can be derived from bound-state equations without an explicit solution of these equations. Nevertheless (A) and (B) are very simple quantum-theoretical statements which hold independently of any quark substructure of hadrons.

It is also interesting to speculate on the unification of (A) and (B) by extending the group $U(6)\otimes U(6)\otimes O(3)$ to[16] $U(6)\otimes U(6)\otimes O(3,1)$ or $U(6)\otimes U(6)\otimes O(4)$. In the case of $U(6)\otimes U(6)\otimes O(3,1)$, both $(\underline{6},\underline{6}^*;L=0)$ and $(\underline{6},\underline{6}^*;L=1)$ would be members of an infinite-dimensional unitary representation $D_\alpha=(\underline{6},\underline{6}^*;L=0,1,2,\cdots)$. In the case of $U(6)\otimes U(6)\otimes O(4)$, the finite-dimensional unitary representation $(\underline{6},\underline{6}^*;\frac{1}{2},\frac{1}{2})$ would relate precisely $(\underline{6},\underline{6}^*;L=0)$ and $(\underline{6},\underline{6}^*;L=1)$.[17]

To conclude, let us emphasize once more that along with $M(12)$ invariance, universality is a basic ingredient in the derivation of Eqs. (1), (2), (10), (12), (13), and (14), all of which agree with experiment and significantly reduce the number of independent parameters in the theory of hadrons.

The author wishes to express his sincere thanks to Reinhard Oehme for many valuable discussions.

Note added in proof.—One can apply the techniques expounded above to electromagnetic mass splittings of hadrons. One then has to take into account the self-energy corrections due to hadron-photon intermediate states, the contribution of which contains an SU(3) 27-plet term. After subtracting this, we find

$$\frac{(K^{02}-K^{+2})-(\pi^{02}-\pi^{+2})}{\Sigma^{-2}-\Sigma^{+2}}=\frac{1}{3}, \qquad (12c)$$

to be compared with the experimental value 1/3.4. Eq. (12c) is the electromagnetic analog of Eq. (12b).

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*This work is supported in part by the U. S. Atomic Energy Commission.

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[4]P. G. O. Freund, Phys. Letters 2, 136 (1962); M. Gell-Mann, Phys. Rev. 125, 1067 (1962); J. Schwinger, Phys. Rev. 140, B158 (1965); S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964). This essentially means that the $2^{++}$-meson pole dominates the gravitational form factors much as the $1^{--}$-meson pole dominates the electromagnetic form factors.

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[9]By this we mean the symmetry describing a $U(6)\otimes U(6)\otimes O(3)$ multiplet in motion. See K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 405 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); R. E. Marshak and S. Okubo, Phys. Rev. Letters 13, 818 (1964), for the $L=0$ case, and Ref. 7 for the $L=1$ case.

We also wish to emphasize that our assumption (C) is contrary to the spirit of the collinear SU(6) group [see Ref. 6; R. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965); H. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965)]. In these papers it is argued that the $\gamma K$ kineton terms can be arbitrarily strong. Such an approach runs into severe trouble when SU(3)-breaking effects are included. Thus without kineton terms the $D$:$F$:$S$ ratio of electric couplings of vector mesons to baryons is 0:1:1 whereas upon inclusion of kineton terms it becomes 0:1:$X$ with $X$ arbitrary. Now in a usual meson-pole model including SU(3) breaking, the relations (2) of the main text can only be maintained if the $\varphi$-meson contribution cancels out ($m_\varphi > m_\rho \approx m_\omega$). This is only the case if $X=1$. Similarly, Eqs. (12a), (13), and (14) are lost in the collinear SU(6)-group scheme.

[10]If we should keep the kinematical factor $q^2/m^2$ intact, we would end up with the situation of the $1^{++}$ mesons not contributing at all to $G_A/G_V$. The extrapolation procedure for vector form factors which corresponds to keeping $q^2/m^2$ rather than replacing it by 1 is anyway known (see Ref. 5) to lead to serious difficulties. So, the only procedure consistent with pole models of electromagnetic and weak-vector form factors [Eqs. (1)] is to replace $q^2/m^2\rightarrow 1$. I am indebted to R. Oehme for a very useful discussion on this point.

[11]Note that we do not specify $(G_A/G_V)_{n\rightarrow p}=-5/3$ but only require the ratio in (B) to equal 5/3. Thus from $(G_A/G_V)_{n\rightarrow p,\,\text{expt.}}=-1.18$, we would predict $(G_A/G_V)_{Q\rightarrow Q}=-1.18/(5/3=-0.7$ [rather than $(G_A/G_V)_{Q\rightarrow Q}=-1$] if quarks exist and universality encompasses them.

[12]Here, again, the extrapolation to $q^2=0$ has to be done so that $G_A/G_V\neq 0$ in keeping with our above discussion (see Ref. 9). We further wish to make some remarks about the normalization procedure. Take, e.g., the matrix element of $j_0^{\text{el}}(0)$ between two "single proton at rest" states: $\langle p|j_0^{\text{el}}(0)|p\rangle = a\bar{u}(0)\gamma_0 u(0)M/(M^2)^{1/2}=a$. Since we want to normalize this to the charge of the proton we have $a=1$. Similarly $\langle p|\tilde{T}_\mu{}^\mu(0)|p\rangle = x\bar{u}uM/(M^2)^{1/2}=X$, and we have to normalize this to $x=M_B$ [we assume in this example, for simplicity, that $\tilde{T}_\mu{}^\mu$ is the SU(3) singlet part of $T_\mu{}^\mu$].

Now similarly, e.g., $\langle\pi^+|j_0{}^{el}(0)|\pi^+\rangle = a\times 2(M^2)^{1/2}\times\frac{1}{2}(M^2)^{-1/2} = 1$ and $\langle\pi^+|T_\mu{}^\mu(0)|\pi^+\rangle = y\times 2(M^2)^{1/2}\times\frac{1}{2}(M^2)^{-1/2} = y$, $y = M_M$. Now from (8) and (11) and our assumption (B), we find $y=\frac{2}{3}x$ and hence (12a). Similarly one obtains $K^*-\rho = \Xi^*-Y_1{}^*$. This can then be rewritten in the form (12b) in which form it has been earlier obtained by the author.[13] I wish to thank Y. Nambu for a very illuminating discussion on these points.

[13]P. G. O. Freund, Phys. Rev. Letters 14, 1088 (1965); 15, 176(E) (1965); Nuovo Cimento 39, 769 (1965). It is worthwhile to emphasize that the kinematical factor $\frac{2}{3}(1+M_M/2M_B)\approx 0.9$ in the third relation of the latter paper naturally disappears in the present derivation so that there is no need for an approximation in this respect. As already emphasized in these papers, along with (12) [and for that matter (10)], one also obtains $\Sigma=\Lambda$ which of course estimates the error involved in a single-pole model to be $\approx 20\%$. It appears that the $\Sigma-\Lambda$ mass difference has its origin in the SU(3)-noninvariant part of the interaction that produces the mass splitting between the decuplet and the octet of the 56. We hope to return to this point elsewhere.

[14]P. G. O. Freund, Phys. Rev. Letters 15, 930 (1965). One can include the octet parts of the $2^{++}$ mesons in universality considerations and obtain more detailed relations than (13). At high energies this, however, does not appear to be very important since (13) itself is reasonably well obeyed.

[15]For Eq. (12a), see Y. Nambu, to be published. Eqs. (13) and (14) have been rederived in the context of a quark model by H. Lipkin and F. Scheck, to be published.

[16]M. Gell-Mann, Y. Dothan, and Y. Ne'eman, Phys. Letters 17, 148 (1965).

[17]Experimentally, nine $2^{++}$ mesons are known and it is an important problem to discover the missing $0^{++}$, $1^{++}(A_1?)$, $1^{+-}(B?)$ states. For the alternative possibility ($\underline{21}, \underline{21}^*$; $L=0$) of classifying the observed $2^{++}$ nonet, see P. G. O. Freund and B. R. Desai, to be published. Our present theory favors the assignment ($\underline{6}, \underline{6}^*$; $L=1$) by providing it with a raison d'être.