# POLARIZATION AS A TEST FOR REGGE BEHAVIOR IN BACKWARD $\pi^{\pm}p$ SCATTERING

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Recent experiments have shown a sharp peak in the backward direction in high-energy elastic $\pi^{\pm}p$ scattering.[1,2] The Regge-pole hypothesis ascribes this peak to exchange of the crossed-channel baryon trajectories. The purpose of this note is to point out that exchange of these trajectories, since they are associated with fermions, gives rise to amplitudes which are highly spin dependent. In particular, the measurement of the polarization of the recoil proton can provide an important test for Regge behavior in these reactions.

In the $\pi^{-}p$ reaction, the crossed baryon channel is in a pure $I=\frac{3}{2}$ state. The only known trajectory which can be exchanged is the $\Delta$. In the $\pi^{+}p$ reaction, exchange of both $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states is allowed. If the exchange of the $\Delta$ were to dominate here as well, the relation $d\sigma(\pi^{-}p)/d\Omega = 9d\sigma(\pi^{+}p)/d\Omega$ would hold. The experimental data show, however, that the differential cross section for $\pi^{+}p$ is several times that for $\pi^{-}p$, with both having roughly the same energy dependence. This difference can only be explained if $I=\frac{1}{2}$ exchange is dominant in $\pi^{+}p$. Consequently, in what follows, the $\pi^{+}p$ reaction is assumed to be dominated by exchange of $N$, the leading $I=\frac{1}{2}$ trajectory, with $\alpha_N(0) \cong \alpha_\Delta(0)$.[3]

The scattering amplitude for $\pi^{\pm}p$ scattering can be written[4]

$$f^{\pm}(\sqrt{s},u) = f_1^{\pm}(\sqrt{s},u) - \cos\theta f_1^{\pm}(-\sqrt{s},u) + i\sin\theta\vec{\sigma}\cdot\hat{n} f_1^{\pm}(-\sqrt{s},u). \quad (1)$$

The contributions to $f_1^{\pm}(\sqrt{s},u)$ of the baryon trajectories in the crossed channel are easily calculated using crossing symmetry and the Sommerfeld-Watson transformation[5]:

$$f_1^{\pm}(\sqrt{s},u) \underset{\substack{s\to\infty,\\ u \text{ fixed}}}{\to} \frac{1}{4}\left[1-\frac{\sqrt{s}}{\sqrt{u}}\right]\frac{\Gamma_{\mp}(\sqrt{u})}{\cos\pi\alpha_{\mp}(\sqrt{u})}\left(\frac{s}{s_0}\right)^{a\pm}\left[1\pm e^{-i\pi a\pm}\right] + \frac{1}{4}\left[1+\frac{\sqrt{s}}{\sqrt{u}}\right] \times\frac{\Gamma_{\mp}(-\sqrt{u})}{\cos\pi\alpha_{\mp}(-\sqrt{u})}\left(\frac{s}{s_0}\right)^{b\pm}\left[1\pm e^{-i\pi b\pm}\right], \quad (2)$$

with $a\pm = \alpha_{\mp}(\sqrt{u})-\frac{1}{2}$, $b\pm = \alpha_{\mp}(-\sqrt{u})-\frac{1}{2}$, where $\alpha_{\mp}(\sqrt{u})$ and $\Gamma_{\mp}(\sqrt{u})$ are pole positions and modified reduced residues in the crossed $\pi^{\mp}p$ channel. Under the assumptions made above, the relations

$$\alpha_{+}(\sqrt{u}) = \alpha_{\Delta}(\sqrt{u}), \quad \alpha_{-}(\sqrt{u}) = \alpha_{N}(\sqrt{u}),$$

$$\Gamma_{+}(\sqrt{u}) = \Gamma_{\Delta}(\sqrt{u}),$$

and

$$\Gamma_{-}(\sqrt{u}) = \tfrac{2}{3}\Gamma_{N}(\sqrt{u})$$

hold. The quantity $s_0$ is an arbitrary scale energy taken to be 1 GeV. These expressions represent leading asymptotic terms in $s$ and come from the poles in the $l=J-\frac{1}{2}$ amplitude in the crossed channel. This means that $\alpha_\Delta(1238\text{ MeV})=\frac{3}{2}$, $\alpha_\Delta(1920\text{ MeV})=\frac{7}{2}$, etc., but that $\alpha_N(-938\text{ MeV})=\frac{1}{2}$, $\alpha_N(-1688\text{ MeV})=\frac{5}{2}$, etc. This difference occurs because the $N$ trajectory appears in the right-half $J$ plane in the $l=J+\frac{1}{2}$ amplitude. By the MacDowell symmetry,[6] this amplitude is the continuation to negative energies of the $l=J-\frac{1}{2}$ amplitude.

The contrast with the exchange of a boson trajectory should be noted. As was first shown by Gribov,[7] the positions and residues of fermion Regge poles are real analytic functions of the c.m. energy in the cut plane of the channel in which the pole appears. To achieve an amplitude $f_1^{\pm}(\sqrt{s},u)$ analytic in the cut $u$ plane and having no singularity near $u=0$, each trajectory in the crossed baryon channel contributes two terms, resulting in an expression of the form $g(\sqrt{s},\sqrt{u})+g(\sqrt{s},-\sqrt{u})$. The two terms contributed by any one trajectory have different phases and produce a phase difference between the spin-flip and spin-nonflip amplitudes which allows a nonvanishing polarization for the recoil proton. In the case of one boson trajectory exchanged, the position and residue are real analytic functions of the square of the c.m. energy in the relevant crossed channel. In this case, the spin-flip and spin-nonflip amplitudes in the direct channel are always relatively real and no polarization can occur.

Using the expression of Eq. (2) to calculate the c.m. polarization of the recoil proton for an initially unpolarized target leads to the following result:

$$\vec{\mathrm{P}}_{\pm}(\sqrt{s},u)=\hat{n}\left(\frac{\sqrt{s}\sin\theta}{-i2\sqrt{u}}\right)\tanh\left\{\frac{-i\pi}{2}[\alpha_{\mp}(\sqrt{u})-\alpha_{\mp}(-\sqrt{u})]\right\}\times\frac{F_{\mp}(\sqrt{s},u)}{F_{\mp}(\sqrt{s},u)+G_{\mp}(\sqrt{s},u)}. \quad (3)$$

$F_{\mp}(\sqrt{s},u)$ and $G_{\mp}(\sqrt{s},u)$ are functions which are expressible in terms of $\Gamma_{\mp}(\sqrt{u})$ and $\alpha_{\mp}(\sqrt{u})$. The precise forms for $F_{\mp}(\sqrt{s},u)$ and $G_{\mp}(\sqrt{s},u)$ are irrelevant to the argument which follows and will not be given here. However, it is easily shown that $F_{\mp}(\sqrt{s},u)\gg G_{\mp}(\sqrt{s},u)$ if $s$ is sufficiently large and that $F_{\mp}(\sqrt{s},u)$ and $G_{\mp}(\sqrt{s},u)$ are real and positive in the direct-channel physical region. Therefore, the sign of the polarization is determined by the sign of the term

$$(1/-i\sqrt{u})\tanh\{-\tfrac{1}{2}i\pi[\alpha_{\mp}(\sqrt{u})-\alpha_{\mp}(-\sqrt{u})]\}.$$

The sign of this term can be obtained using the fact that the functions $\alpha_{\mp}(\sqrt{u})$ are real analytic in the neighborhood of $\sqrt{u}=0$, with nearest cuts starting at $\sqrt{u}=\pm(M_N+M_\pi)$. Expanding the $\alpha_{\mp}(\sqrt{u})$ around $\sqrt{u}=0$ and keeping terms up to cubic order, the above term simplifies to

$$(1/-i\sqrt{u})\tanh[-i\pi\alpha_{\mp}'(0)\sqrt{u}].$$

As noted above, $\alpha_+(\sqrt{u})=\alpha_\Delta(\sqrt{u})$, and $\alpha_\Delta(1238\ \mathrm{MeV})=\frac{3}{2}$, etc. Barring exceptional behavior, $\alpha_\Delta(\sqrt{u})$ is expected to increase steadily as $\sqrt{u}$ increases through real values from zero. This implies $\alpha_+'(0)>0$. On the other hand, $\alpha_-(\sqrt{u})=\alpha_N(\sqrt{u})$ and $\alpha_N(-938\ \mathrm{MeV})=\frac{1}{2}$, etc. Here $\alpha_N(\sqrt{u})$ is expected to increase steadily as $\sqrt{u}$ decreases through real values from zero, which implies $\alpha_-'(0)<0$. Therefore, under the stated assumptions, the polarization is predicted to be positive (i.e., along $\hat{n}$) for $\pi^-p$ and negative for $\pi^+p$.

The magnitude of the polarization can also be predicted if $s$ is sufficiently large. The criterion for this is that $|\pi M_N^2\alpha_{\mp}'(0)/\sqrt{s}|\ll 1$. If this inequality is satisfied, $F_{\mp}(\sqrt{s},u)\gg G_{\mp}(\sqrt{s},u)$, and Eq. (3) for $\vec{\mathrm{P}}_{\pm}(\sqrt{s},u)$ simplifies to[8]

$$\vec{\mathrm{P}}_{\pm}(\sqrt{s},u)=\hat{n}\frac{[(M_N^2-M_\pi^2)^2-us]^{1/2}}{\sqrt{s}}\times\frac{\tanh[-i\pi\alpha_{\mp}'(0)\sqrt{u}]}{-i\sqrt{u}}, \quad (4)$$

where the explicit high-energy form for $\sin\theta$ has been used and $\alpha_{\mp}(\sqrt{u})$ has been expanded around $\sqrt{u}=0$ as above. If $|\alpha_{\mp}'(0)|\cong O(1/\mathrm{GeV})$, as would be expected from the known points on the trajectories, it is clear that appreciable polarization can occur throughout the region of the backward peak, except near the exact backward direction where the polarization vanishes. At lower energies, where the inequality above is not satisfied, Eq. (4) provides an order-of-magnitude estimate, since in this region $F_{\mp}(\sqrt{s},u)\cong O(G_{\mp}(\sqrt{s},u))$. As an example, taking values for $\alpha_{\mp}'(0)$ estimated from the known points on the trajectories, Eq. (4) predicts that the polarization will rise from zero to essentially unit magnitude as the scattering angle decreases from 180° to ≲170°, at 9 GeV/$c$ laboratory momentum. For higher energies, the interval in angle over which the magnitude of the polarization rises from zero to near unity decreases.

To sum up, the assumptions are made that Δ exchange dominates $\pi^-p$, that $N$ exchange dominates $\pi^+p$, and that the trajectories have reasonable shapes.[9] Under these assumptions, large positive polarizations are predicted in $\pi^-p$ and large negative polarizations in $\pi^+p$ at high $s$ and fixed $u$, the magnitudes being a measure of the trajectory slopes. Since almost all other theories of high-energy scattering would predict no polarization in the asymptotic region, the observation of the predicted polarizations in backward $\pi^\pm p$ scattering would provide strong evidence for Regge behavior in these reactions.

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[1]W. R. Frisken, A. L. Read, H. Ruderman, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and D. H. White, Phys. Rev. Letters 15, 313 (1965).

[2]C. Coffin, N. Dikeman, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, to be published.

[3]This implies that the residue of the $N$ trajectory is considerably larger than that of the $\Delta$. If the residues are parametrized in a way analogous to that used in forward scattering and constrained to have the correct values at the $N$ and $\Delta$ masses, this difference arises naturally. (Private communication from Geoffrey F. Chew.)

[4]Here $\sqrt{s}$ and $\theta$ are the c.m. energy and scattering angle, respectively; $u$ is the square of the c.m. energy in the crossed baryon channel. In the direct-channel physical region, $u$ is given by $u=2(M_N^2+M_\pi^2)-s+2q^2(1-\cos\theta)$, where $q$ is the c.m. momentum. The quantity $\hat{n}$ is a unit vector, given by $\hat{n}=(\vec{q}_i\times\vec{q}_f)/|\vec{q}_i\times\vec{q}_f|$ where $\vec{q}_i$ and $\vec{q}_f$ are the initial and final c.m. proton momenta. The quantity $f_1(\sqrt{s},u)$ is defined in the paper by V. Singh, Phys. Rev. 129, 1889 (1963).

[5]There has been uncertainty in some of the literature about whether Regge behavior should be expected for backward $\pi^\pm p$ scattering near the point $u=0$, because the cosine of the crossed-baryon-channel angle may be small here even when $s$ is large. However, it can be shown that the Regge behavior continues to hold near $u=0$, if it holds outside of a neighborhood of $u=0$, by making use of the analyticity of $f_1^\pm(\sqrt{s},u)$ in $u$ inside this neighborhood.

[6]S. W. MacDowell, Phys. Rev. 116, 774 (1959).

[7]V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 43, 1529 (1962) [translation: Soviet Phys.—JETP 16, 1080 (1963)].

[8]A result equivalent to this has been previously obtained by V. Gribov, L. Okun', and I. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. 45, 1114 (1963) [translation: Soviet Phys.—JETP 18, 769 (1964)].

[9]It is clear that the first of these assumptions is somewhat weaker than the second. Even if $I=\frac{1}{2}$ exchange dominates in $\pi^+p$, the greater complexity of $I=\frac{1}{2}$ interactions in the low-energy region could mean that other singularities besides the $N$ trajectory contribute.

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# $\pi$-$p$ CHARGE-EXCHANGE PROCESSES IN THE REGION OF 2 GeV/$c$

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A study of the processes

$$\pi^- + p \to n + \pi^0, \tag{1}$$

$$\pi^- + p \to (N_{1238}{}^{*0} \to n + \pi^0) + \pi^0, \tag{2}$$

has been made at incident pion momenta of 1.715, 1.889, 2.071, 2.265, and 2.460 GeV/$c$ using a system of thick-plate spark chambers for detecting the gamma rays. Details of the experiment and its analysis will be reported elsewhere.[1] The differential cross sections for Reaction (1) at higher momenta (4.8-18.2 GeV/$c$) fit well with a Regge-pole model in which a Reggeized $\rho$ meson is exchanged. We show that (rather surprisingly, at such low energies) our measurements of Reaction (1) fit with a model in which this same amplitude is dominant, interfering with weaker amplitudes due to pion-nucleon resonances. This analysis suggests that the 2190-MeV resonance has $J=l-\frac{1}{2}$. We further show that Reaction (2) can also be described by this Regge-pole model.

The cross sections for Reaction (1) at each momentum and the mean cross section for Reaction (2) are shown in Fig. 1. The errors indicated are statistical and do not include an uncertainty in normalization of up to 15%.

The cross sections for Reaction (1) have been fitted with a nonresonant amplitude from the Regge-pole model,[2] together with three resonant amplitudes. For pion momenta from 4.8 to 18.2 GeV/$c$, Reaction (1) has been shown[3] to fit well with the Regge form

$$d\sigma/dt = f(t)\omega^{2\alpha(t)-2}, \tag{3}$$

where $\omega$ is the total pion laboratory energy and $f(t)$, $\alpha(t)$ are energy independent. The data in the range $t=0$ to $t=-1.6$ (GeV/$c$)$^2$ imply strong shrinkage:

$$\alpha(t) = 0.55 + 0.9t.$$

The rise in cross section from $t=0$ to $t=-0.15$ (GeV/$c$)$^2$ suggests a strong spin-flip amplitude. The crossover in $\pi^+$-$p$ and $\pi^-$-$p$ elastic cross sections at small $|t|$ suggests[4] a rapidly fall-