imum in the time constant reflects a minimum in the thermal diffusivity which in turn leads to a maximum in the fraction of the applied heat which escapes due to the heat-loss defect. This leads to an erroneous relative increase in calculated heat capacity near the maximum of τ . The details of the τ -vs- T^* curve will be different for different apparatus. A further discussion is given in Ref. 6.

It is tempting to regard the resistance plotted in Fig. 2 as that of He^3 . If this is done then the data of Fig. 2 show that the temperature dependence of the thermal conductivity of He^3 does not depart strongly, if at all, from that expected for a normal Fermi liquid down to temperatures of a few millidegrees. This cannot be regarded as a strong conclusion, however.

Further experimental work on this point is required.

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FINITE-WELL, SINGLE-PARTICLE WAVE FUNCTIONS IN TWO-NUCLEON STRIPPING*

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Much interest centers on the multinucleontransfer reaction as a possible testing ground¹ for nuclear-structure models, because the probability amplitude is a coherent sum of terms, each involving a product of single-particle wave functions.²⁻⁵ A distorted-wave Born-approximation (DWBA) theory suitable for calculating the coherent sum has been developed in detail.⁶⁻¹¹ In all applications of this theory, harmonicoscillator single-particle functions have been used.⁸⁻¹⁵ Not only are oscillator wave functions particularly convenient for separating the relative and center-of-mass motions¹⁶ of the several nucleons that are transferred, but they are the wave functions normally used in structure calculations, for example those which predict the coefficients in the coherent sum.

An oscillator wave function has, of course, a (Gaussian) tail which decays too rapidly to approximate the (exponential) tail of a wave function for a finite well, say one of Woods-Saxon shape. The shape of this tail is important in nuclear reaction applications; in all (d,p) calculations, for example, it is necessary to use finite-well wave functions to avoid gross error. Fortunately, finite-well singleparticle (FWSP) eigenfunctions have excellent overlap with oscillator single-particle (OSP) eigenfunctions within the nuclear interior and, therefore, it is not difficult to carry over into correct reaction calculations the already available results of typical structure calculations.

It is the purpose of this Letter to point out that there are marked changes in reaction calculations of multinucleon transfer if eigenfunctions of a finite well are used instead of oscillator eigenfunctions. Further, it is easy thus to dispense with the use of oscillator eigenfunctions.

Figure 1 shows the results of zero-range DWBA calculations for the reactions ⁴⁰Ca(³He, $(p)^{42}$ Sc and 54 Fe $(t, p)^{56}$ Fe, along with the corresponding experimental data.^{17,18} The curves labeled OSP and FWSP were calculated by using oscillator and finite-well single-particle wave functions, respectively. A configuration of $(f_{7/2})^2$ was assumed for the ground state of ⁴²Sc and $(p_{3/2})^2$ for the ground state of ⁵⁶Fe, although additional admixtures are to be expected. No attempt to fit data by varying parameters was made in the FWSP calculations, except to consider different families of potentials for the strongly absorbed A = 3 particles. Nonetheless, the FWSP angular distributions, calculated without any radial cutoff in the matrix element, are in reasonable agreement with

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experiment.

The OSP curves illustrate the manner in which calculated angular distributions disagree with experiment if wave functions with too short tails are used. The first minimum of an l = 0 transition is at much too large an angle, and



FIG. 1. Comparison between DWBA predictions and certain two-nucleon transfer experiments. For the ${}^{40}\text{Ca}({}^{3}\text{He},p)$ data see Ref. 17, for the ${}^{54}\text{Fe}(t,p)$ data see Ref. 18. The DWBA curves were calculated with oscillator single-particle functions (OSP), with oscillator quanta (in MeV) of 12.2 (Ca) and 10.2 (Fe), and with Woods-Saxon single-particle functions (FWSP). The departure of each FWSP from the corresponding OSP curve is due to a total admixture of less than 0.9% of oscillator single-particle states two or more major shells away.

the secondary peaks are shifted to larger angles. The shift in the Ca case is a rather striking one of 15° ; in the case of Fe it is 5° . The angular distributions for higher angular-momentum transfers (not shown) are also shifted by the same amounts. In general, the approximation of a finite-well single-particle wave function by a realistic oscillator single-particle wave function reduces the effective interaction radius, and thereby expands the diffraction pattern. On the whole, this effect could be offset by the use of unrealistic oscillator parameters, that would grossly extend the wave functions to larger radii. Such a procedure would disagree, however, with other information¹⁹ about the structure of the nuclei. It is perhaps noteworthy that in the case of Ca no amount of variation of the oscillator frequency produces any significant improvement in the OSP angular distribution over the first 70°. a result somewhat contrary to the expectations of Ref. 8.

The OSP calculations were made along the lines of Refs. 7-9; the FWSP calculations differ only in the use of FWSP wave functions. The FWSP calculational procedure we follow was independently suggested by Glendenning.²⁰ To treat center-of-mass effects exactly, we expand each finite-well single-particle wave function in an orthonormal basis of oscillator single-particle functions. Typically, only a few terms (see Fig. 2) are required to represent the FWSP function accurately over the range conceivably required in DWBA calculations. Each product of FWSP functions is then a sum of terms, of which the dominant term is the OSP term previously⁶⁻¹⁵ considered. The resulting radial form factor is a sum, over principal quantum number, of oscillator wave functions in the center-of-mass coordinates of the transferred cluster. The convergence of this sum is similar to that shown in Fig. 2, so that only a few additional terms (8-15) are normally required to give the form factor accurately over the range in which it sensibly contributes to the matrix element.

The FWSP wave functions were obtained by adjusting the well depth of a Woods-Saxon potential to give an eigenstate with correct quantum numbers and a specified energy. The energies used were the neutron-separation energy of 41 Ca, the proton-separation energy of 41 Sc, and half the difference in binding energy of 54 Fe and 56 Fe. The geometries of the wells



FIG. 2. Woods–Saxon eigenfunction $(1f_{7/2})$ for ⁴¹Ca and successive approximations using 1, 3, 5, and 7 oscillator terms. The function is accurately represented out to 10 F by 9 terms.

were based on successful studies of (d, p) reactions. A spin-orbit strength 25 times the Thomas term was used.

The optical model parameters used for ³He give a best fit to 12-MeV ³He-⁴⁰Ca elastic scattering data¹⁷; they differ from those of Yntema, Zeidman, and Bassel²¹ only in that V = 181 MeV and W = 11.5 MeV, and are consistent²² with data at higher and lower energies. The shallow well of Ref. 21 fits the same elastic data reasonably well; but the corresponding FWSP angular distribution is 12° out of phase with experiment²³ The proton parameters used for ⁴²Sc give a best fit²⁴ to 12-MeV p-⁴²Ca data. There is some sensitivity to these proton parameters; more work remains to be done. The triton parameters are the ³He-⁴⁰Ca parameters but with the real well decreased to 177.4 MeV to allow for the expected symmetry dependence. The proton parameters for ⁵⁶Fe are from the work of Perey.²⁵

A possible alternative to the FWSP approach is to match smoothly to the OSP form factor (including Talmi coefficient) a tail that is an appropriate Hankel or Coulomb function. For a wide variety of two-nucleon transfers, we find the resulting angular distributions agree reasonably well in shape and to within 40% in magnitude with the FWSP angular distributions, provided that a realistic oscillator frequency is used. The mishandling of center-of-mass effects inherent in this latter procedure is perhaps not large.

It is clear that subsequent DWBA calculations of two-nucleon transfers should use finite-well single-particle functions, or some equivalent, such as that discussed in the preceding paragraph. Details of this work will be published elsewhere. Investigations are underway to study the effects of nonlocality and finite range; preliminary calculations indicate that these effects do not vitiate the effects discussed here.

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MAGNITUDE OF THE $K_1^{\ 0}$ - $K_2^{\ 0}$ MASS DIFFERENCE USING STRONG INTERACTIONS*

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Experiments to determine the magnitude δ of the $K_1^{\ 0}-K_2^{\ 0}$ mass difference have used two essentially different methods. These are as follows:

(I) <u>Strong interactions</u>.^{1,2}-Starting with a sample of K^0 at time t=0, one detects the subsequent time development of \overline{K}^0 by means of secondary strong interactions. Three published strong-interaction experiments³⁻⁵ give the following results (the units are inverse K_1^0 life-time): $\delta < 10$, $\delta = 1.9 \pm 0.3$, and $\delta = 1.5 \pm 0.2$.

(II) <u>Coherent regeneration</u>.^{1,6}-Starting with a K_2^{0} beam, one detects $\pi^+\pi^-$ decays from K_1^{0} coherently regenerated in matter. Five published coherent-regeneration experiments⁷⁻¹¹ give the following results: $\delta = 0.84^{+0.22}_{-0.22}$, 0.55 ± 0.10 , 0.82 ± 0.12 , 0.82 ± 0.14 , and 0.50 ± 0.10 . Thus there has been a discrepancy of more than a factor of two between the average of the values of δ obtained through strong interactions and those obtained through coherent regeneration.¹²

In this paper we report a new measurement of δ using the strong-interaction method. The K^0 are produced in the Alvarez 72-inch hydrogen bubble chamber by associated-production reactions involving a visible Λ decay:

$$\pi^- + p \rightarrow \Lambda + K^0, \quad \Lambda \rightarrow p + \pi^-$$
 (5860 events), (1a)

and

π

$$- + p \rightarrow \Sigma^{0} + K^{0}, \quad \Sigma^{0} \rightarrow \gamma + \Lambda, \quad \Lambda \rightarrow p + \pi^{-}$$
(1360 events). (1b)

The time development of \overline{K}^0 intensity is detected through the secondary interactions

 $\overline{K}^{0} + p \rightarrow \Lambda + \pi^{+}$ (25 events), (2a)

$$-\Sigma^{0} + \pi^{+}$$
 (19 events), (2b)

$$\rightarrow \Sigma^+ + \pi^0 \qquad (9 \text{ events}), \qquad (2c)$$

$$- \Lambda + \pi^{+} + \pi^{0} \quad (4 \text{ events}), \qquad (2d)$$

$$\rightarrow \Sigma^{+} + \pi^{+} + \pi^{-}$$
 (1 event), (2e)

$$\rightarrow \Lambda + \pi^+ + \gamma \quad (1 \text{ event}). \quad (2f)$$

Our statistics are limited (59 events), but we believe that the experiment is free of sources of systematic bias. We find (in units τ_1^{-1} , with $\tau_1 = 0.88 \times 10^{-10}$ sec)

$$\delta = 0.65 \pm 0.30.$$
 (3)

Our result (3) is in poor agreement with previous determinations of δ using strong interactions,^{4,5} and in good agreement with determinations using coherent regeneration.⁷⁻¹¹

We conclude that the strong-interaction and coherent-regeneration methods give compatible results. A least-squares average of our result (3) and those of the five coherent-regeneration experiments gives $\delta = 0.64 \pm 0.06$, with $\chi^2 = 7.3$ giving a χ^2 probability of 0.20.

The events are described in Table I. Their time distribution is shown in Fig. 1. Our like-lihood function for δ is shown in Fig. 2, together with the results of other determinations.