OBSERVATION OF NEGATIVE-MASS INSTABILITY IN AN ENERGETIC PROTON PLASMA*

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Microinstabilities at and above ion gyrofrequency are present in almost all hot ion plasmas, even those stabilized against hydromagnetic instabilities by magnetic wells.[1-3] Microinstabilities are capable of producing hot-ion losses,[4] and therefore are a major threat to progress in generation and confinement of thermonuclear plasmas in many of the present experiments. Since the design of future experiments presumably will rely heavily upon microinstability theory, experimental studies of microinstabilities are of particular importance when the results can be identified with predictions of theory.

In this Letter we briefly describe experimentally observed threshold properties of a microinstability in an energetic proton plasma, and we identify these properties with those of a theoretically recognized instability. We have measured the threshold density and instability growth times for a "standard" proton distribution (to be described), and the response of threshold to changes in (1) the magnetic field shape, (2) the energy spread of trapped protons, and (3) the proton distribution in radial oscillation amplitude. We have also measured the dependence of the fundamental mode frequency on changes in radial oscillation amplitude. We show that the observed properties are those of the negative-mass instability[5,6] familiar from accelerator experience, and we exclude interpretation in terms of the drift-cyclotron instability,[7-9] the other likely candidate.[4,10] A detailed report of these studies is in preparation.

The plasma is created in the DCX-1 facility by dissociation of 600-keV H_2^+ injected into a magnetic mirror field with a central field value of 10 kG. The experimental instability is the "gyrofrequency mode" of earlier papers.[4,10] It is characterized by reception on electrostatic and magnetic (B_z polarization) probes of rf signals near harmonics of the ion gyrofrequency. Descriptions of the experimental facility, details of the proton losses associated with this mode under certain operating conditions, data on the rf spectrum, and previous considerations of mode assignment for this instability are given in the earlier papers.

The gyrofrequency mode of instability can be isolated from other unstable modes of this plasma by using gas-collisional dissociation of the molecular-ion beam at fairly high gas pressures (of order 10^{-6} Torr), and this was done in these experiments. Helium was used as the background gas, and trapped-proton lifetimes for most of the experiments were 50-75 μsec.

The threshold measurements were made in the period of rising density just after the molecular ion beam was turned on. For the energy-spreading experiments, the measurements were also made in steady state, with the density controlled by varying the beam current. For the threshold density, we take values inferred from measurements of the escaping charge-exchange neutrals at the time of first appearance of the rf signals.

The "standard" plasma distribution is that established using the normal plasma radius (20 cm) and H_2^+ beam trajectory. This beam trajectory makes a single pass through the plasma region. It is confined to the median plane and encircles the magnetic axis. The closest approach of beam to axis (called beam turnaround) lies at a point of tangency to the circular orbit ($R = 8$ cm) for 300-keV protons. This beam trajectory produces the most ordered initial distribution of trapped protons, in the sense that it results in the largest fraction of protons on near-circular orbits. Since dissociative collisions are distributed all along the H_2^+ beam trajectory, the radius of the plasma is fixed by the location of the nearest radial obstruction.

The stability properties of the standard distribution are those of its toroidal core. This toroidal distribution can be isolated by using a mechanical limiter to reduce the plasma radius to a value near the circular orbit radius. The resulting torus has a major diameter equal to the circular orbit diameter and a minor diameter of approximately 1 cm, and contains 2-3% of the population of the standard distribution. For the initial energy spread of the core we take 4 keV, a value from extrapolation of energy-spread data and also half the estimated ripple of the 600-keV molecular-beam power supply.

Experimental determinations of threshold line density for the toroidal distribution produce values[11] of (6-16)$\times 10^5$ energetic protons cm^{-1}. The stability criterion for negative-mass instability[12] indicates a threshold line density of 7×10^5 cm^{-1} for this distribution. The experimental results are therefore in reasonable agreement with this aspect of negative-mass theory.

Measurements of the growth time of the $l=1$ instability made for the standard distribution yield an average value of 40 μsec for a fast-proton population of 1.45×10^9 (total number). We take 7×10^5 cm^{-1} for the threshold line density of the toroidal distribution and find agreement with theory[12] for a toroidal population that is 4.4% of the population of the standard distribution. The 4.4% value is somewhat outside the experimentally determined range for this population ratio, 2-3%, but in view of the uncertainties in assigning and using distribution-function parameters, the agreement of experiment and theory is good.

Other experiments have examined instability threshold as a function of energy spread introduced into the standard plasma configuration by employing noise-driven cyclotron dee structures mounted within the plasma chamber. Thresholds determined during turn-on experiments were essentially identical to those determined in steady state. We therefore use proton-energy spectra determined from measurements of steady-state energy spectra of escaping charge-exchange neutrals. Figure 1 shows the dependence of threshold population on energy spread. Close quantitative agreement with negative-mass theory is obtained if we use the measured full width at half-maximum energy spread for ΔE and 6% of the total population for N_c, the number in the unstable core. Again the agreement of experiment and theory is good.

The usual magnetic field has a field index $[n=(-r/B)dB/dr]$ of 0.09. Changes in coil configuration allowed determinations of threshold variation with ΔE for two other values of n (0.019 and 0.060). The threshold population varied as the product $n(\Delta E)^2$, again in agreement with the stability criterion for negative mass.

Another series of experiments investigated

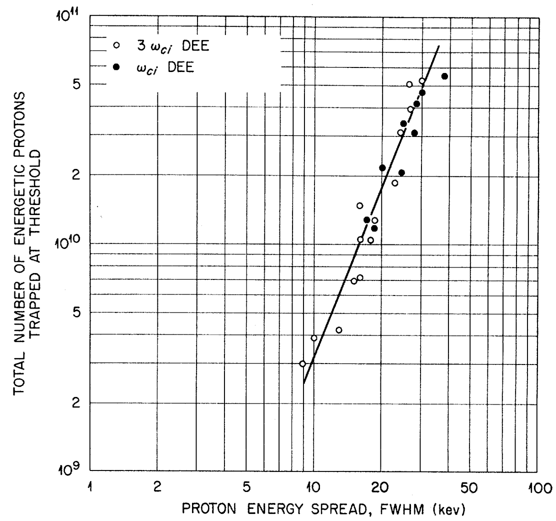

FIG. 1. Population of the standard distribution at instability threshold as a function of proton energy spread (full width at half-maximum) introduced by cyclotron structures mounted within the plasma chamber. Structures suitable for operation at ω_{ci} and at $3\omega_{ci}$ were both employed. In each case the driving potential was the amplified output of a noise generator filtered for a pass band of 1-2 Mc/sec about the harmonic frequency. Energy spreads of less than 15 keV are values from extrapolation of ΔE versus driving potential curves. For zero potential the extrapolation yields $\Delta E\approx 4$ keV.

the sensitivity of threshold to changes of the proton distribution in radial oscillation amplitude. Two controls on this distribution are available. Keeping the molecular-beam trajectory in the median plane but shifting beam turnaround from the circular orbit radius increases the minimum value of radial oscillation amplitude. Reducing the plasma radius decreases the maximum value. In the experiment reported here, both controls were used so as to isolate narrow distributions in radial oscillation amplitude. The experimental data points are shown in Fig. 2, along with thresholds calculated for negative mass. The agreement is good, especially at the smaller values of the abscissa where the approximate orbit calculation is most accurate.

With the standard plasma distribution, the frequency of the $l=1$ mode is 14.60 Mc/sec, the gyrofrequency for the proton circular orbit. The frequency of this mode was examined as a function of minimum oscillation amplitude with equipment capable of frequency definition to 0.05 Mc/sec. With the maximum available displacement of beam trajectory (correspond-

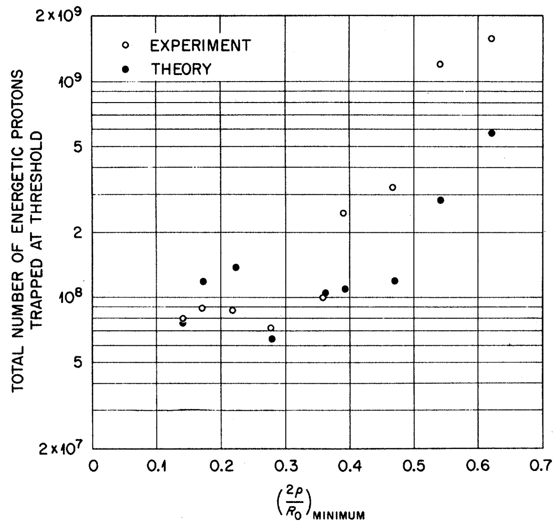

FIG. 2. Populations of isolated toroidal distributions at instability threshold as a function of the minimum radial oscillation amplitude (ρ) of the trapped energetic protons. Experimental points are compared with those calculated from negative-mass theory. The distributions in radial oscillation amplitude were quite narrow, with typical widths of 0.03 on the abscissa scale.

ing to an oscillation amplitude value of 0.6 on the abscissa of Fig. 2), the mode frequency was reduced to 14.55 Mc/sec. The direction and order of magnitude of this frequency shift are in agreement with negative-mass theory.

We now consider the possibility that the observed instability is a mode of drift-cyclotron instability. In order to obtain lowest thresholds in the drift-cyclotron calculations of Mikhailovsky,[7] it is necessary to take considerable $k_{\parallel}$, in which case the drift wave frequency becomes that of Harris[8] for the Burt-Harris plasma model.[9] Lowest threshold density is then given by $\omega_{pe}/\omega_{ci} = 1$ for electrons of zero temperature and somewhat less for warm electrons. Using the results of Harris, we find that threshold can be reduced to the low experimentally observed values ($\omega_{pe} \approx \frac{1}{2}\omega_{ci}$) only in the presence of large shift of unstable frequency away from ω_{ci}. No such shift is observed in this experiment. From this consideration and the demonstrated agreement with negative-mass theory, we conclude that negative mass is the proper mode assignment.

Whether an instability analogous to negative mass may exist in the more complex fields of magnetic wells is an important unresolved question.

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[11]The experimentally determined quantities are believed accurate within factors of about 1.5. The reproducibility of raw data is somewhat more accurate.

[12]The theory of Ref. 5 is modified only slightly when revised to apply to this geometry. The stability criterion becomes

$$N_c < N_0 = \frac{\pi M R^3 n \Omega^2}{8e^2\gamma_l}\left[\frac{\Delta E}{E} + \Delta d_R{}^2 + \Delta d_z{}^2\right]^2,$$

where e and M are the proton charge and mass; R and Ω are the gyroradius and gyrofrequency at the mean energy E of the protons; n is the field-gradient index $-(r/B_z)(dB_z/dr)|_{r=R}$; l is the Fourier harmonic; γ_l (equivalent to g_0 of Ref. 5) is roughly independent of the proton distribution and is determined by the geometry of the unstable core (it is about 6 for $l = 1$, and 5 for $l = 2$); N_c is the population (total number) of protons in the core; and N_0 is the threshold value. The symbols d_R and d_z are the radial and axial betatron oscillation amplitudes scaled by R, and Δd^2 means the spread of d^2 in the distribution. ΔE measures the full width of the proton energy spread. In DCX-1 the oscillation frequency at threshold, ω, is given by $(\omega - l\Omega)/l\Omega \approx -\frac{1}{2}n\Delta d_R{}^2 - \frac{1}{4}n\Delta d_z{}^2$. In the growth-rate experiments reported here, the ΔE term overshadows the betatron-oscillation terms and the growth rate is

$$\mathrm{Im}\omega \approx l\Omega(N_c - N_0)^{1/2}(ne^2\gamma_l/2\pi MR^3\Omega^2)^{1/2}$$

when N_c exceeds N_0. Part of these results are quoted from R. W. Landau and V. K. Neil in University of California Radiation Laboratory Report No. UCRL-14406, 1965 (unpublished). Recent "diochotron" calculations [V. K. Neil and W. Heckrotte, J. Appl. Phys. 36, 2761 (1965)] of corrections for the negative-mass theory yield results which indicate that the radial component, here neglected, of the perturbed electric field is unimportant in these experiments where $\mathrm{Im}\omega/l\Omega \ll n$.

SHAPE OF THE COEXISTENCE CURVE IN THE CRITICAL REGION*

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Recently the form of the coexistence curve of He^4 at the critical point has been discussed by Tisza and Chase[1,2] and by Buckingham and Edwards[3,4] on the basis of variables more "natural" than the traditional ones. Tisza and Chase have analyzed the measurement of the orthobaric densities of He^4, performed by Edwards and Woodbury,[5] translating the Landau[6] expansion in terms of the density $\rho = N/V$ with V fixed instead of the specific volume $v = V/N$ with N fixed. In this way they obtain the following expression for the coexistence curve:

$$\rho_L{}' = -\rho_G{}' = [-(3\alpha/\beta)t]^{1/2}, \qquad (1)$$

where $\rho' = \rho - \rho_c$, α and β are positive constants, and $t = T - T_c$. This formula agrees with experimental results to within about 110 mdeg of T_c, but it yields a coexistence curve with a symmetry about the critical isochore which is in fact only approximately correct.

A different approach has been proposed by Edwards[4] that eliminates this difficulty. He has analyzed the same data by means of the asymptotic form of the coexistence curve proposed by Buckingham[3] as $T \to T_c$,

$$X^2/(1 - \ln X) = -at, \qquad (2)$$

where

$$X = \frac{\rho_L - \rho_G}{\rho_L + \rho_G} = \frac{V_L - V_G}{V_L + V_G} \neq \frac{\rho_L - \rho_G}{2\rho_c} \qquad (3)$$

and a is a positive constant. According to Buckingham this law follows if one takes into account the logarithmic singularity of C_v at the critical point.[7] The expression (2) for the coexistence curve fits the experimental results well to within 230 mdeg of T_c. Edwards thinks that these results confirm the nonanalytic character of the critical point and are evidence of the superiority of the "natural" variable X to the density ρ for displaying the true symmetry of the coexistence curve.

In this Letter we reanalyze the Edwards and Woodbury data, extending Tisza's expansion so as to include second-order terms.[8] We write

$$\frac{1}{V}\left(\frac{\partial^2 F}{\partial \rho^2}\right)_{T,V} = \left(\frac{\partial \mu}{\partial \rho}\right)_T = \alpha t + \beta \rho'^2 + \gamma t \rho' + \delta t^2, \qquad (4)$$

and by development exactly analogous to that of Landau[6] (see also Ref. 5) we obtain, after some algebra,

$$\rho' = -\frac{\gamma}{2\beta}t \pm \left[3t^2\left(\frac{\gamma^2}{4\beta^2} - \frac{\delta}{\beta}\right) - \frac{3\alpha}{\beta}t\right]^{1/2}, \qquad (5)$$

where the negative sign is for the vapor and the positive one for the liquid. According to this law the coexistence curve must be symmetric about a rectilinear diameter whose equation is

$$\tfrac{1}{2}(\rho_L + \rho_G) = \rho_c - (\gamma/2\beta)t. \qquad (6)$$

Figure 1 shows a plot of the mean density against $-t$ for He^4 from the data of Ref. 5. From the slope of the straight line, we obtain

$$\gamma/\beta = (2.75 \pm 0.02) \times 10^{-3}\ \mathrm{g\ cm^{-3}\ {}^\circ K^{-1}}. \qquad (7)$$

Figure 2 shows a plot of the quantity

$$(\rho_L - \rho_G)^2/(-t) \qquad (8)$$

against $-t$. According to Tisza's expansion this quantity should be constant, while with