HEAT CAPACITY OF DILUTE SOLUTIONS OF LIQUID He^3 IN He^4 AT LOW TEMPERATURES*

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The heat capacity of dilute solutions of liquid He^3 in He^4 has been measured for nominal He^3 concentrations of 1.3 and 5.0% down to about 10 mdeg K. The results show that in these concentrations the He^3 is soluble in He^4 down to at least our lowest temperature, in confirmation of the prediction of Edwards et al.[1] Contrary to recent results[2] on the heat capacity of pure He^3 at low temperatures, the ratio of heat capacity to temperature in the present work is constant, within an experimental error of about 2%, from the lowest temperature of 8 mdeg K up to about 35 mdeg K for the 5% mixture. The data for both mixtures are fitted within experimental error by using the same value of the effective mass and the theoretical dependence of heat capacity on temperature and density given by Stoner[3] for an ideal gas of Fermi-Dirac particles. It thus appears that small quantities of He^3 in a background of liquid He^4 may behave as a relatively weakly interacting Fermi system whose density, and hence degeneracy temperature, may be varied at will. The properties of these dilute solutions will be of great interest to study, particularly in relation of those of pure He^3.

The experimental method was similar to that described by Abel et al.[4] The solution of He^3 and He^4 was condensed into an epoxy cell containing powdered cerium magnesium nitrate (CMN), in the form of a right circular cylinder with diameter equal to height. The grain size of the CMN powder was approximately 0.01 cm. Technical problems associated with a possible heat leak due to the He^4 film were solved using the methods described by Vilches and Wheatley.[5] The cell contained 0.50 mole of helium and 13.6 g of CMN. The filling capillary was not pumped. Heat was added electrically. The accuracy of the He^3 concentration was estimated to be about 10% for the 1.3% solution and about 3% for the 5.0% solution. Heat leaks were of the order of one or two erg per minute. The "background" heat capacity of the cell, due primarily to the CMN, was determined with only He^4 in the cell. Above 6 mdeg K it agreed with a determination of CMN heat capacity which was a by-product of our earlier measurements;[2] so it is likely that, at least in the range above 10 mdeg K, the measurements of the background may be trusted.

The results of the measurements are shown in Fig. 1, where the background heat capacity has been subtracted. In spite of excellent conditions, from a cryogenic viewpoint, it was possible to cool the dilute solutions only slightly below 10 mdeg K. Under similar conditions pure He^3 had been cooled to far below 10 mdeg K. The reason is that the thermal time constant in the present work was very long at low temperatures—of the order of half an hour at 10 mdeg K.

For the nominal 5% mixture, the ratio C/T may be used to obtain the ratio of effective mass m^* to atomic mass m by the equation

$$\frac{C}{T}=\left(\frac{\pi}{3}\right)^{2/3}\frac{mk^2}{\hbar^2}V\left(\frac{N_3}{V}\right)^{1/3}\left(\frac{m^*}{m}\right), \qquad (1)$$

where C is the measured heat capacity of a volume V of a mixture containing N_3 He^3 atoms,

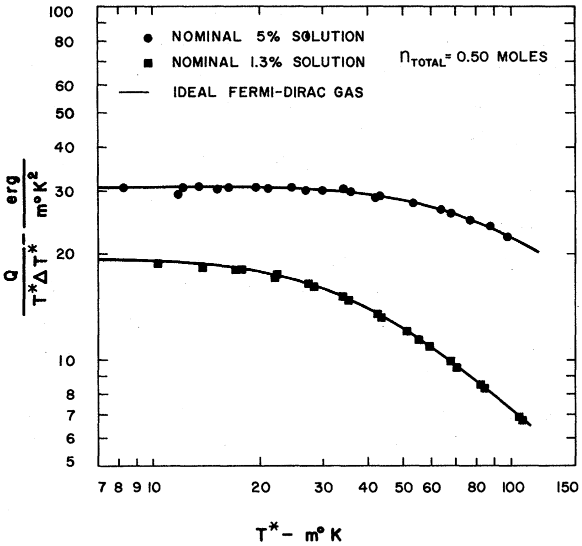

FIG. 1. Ratio of measured heat capacity to magnetic temperature for two dilute solutions of He^3 in He^4. The lines drawn through the data are theoretical and based on the tables of Stoner[3] using values of m^*/m and x given in the text.

k is Boltzmann's constant, and $\hbar$ is $(2\pi)^{-1}$ times Planck's constant. V was determined from the measured number of moles condensed into the cell, using the data of Kerr.[6] For the 5% mixture the ratio C/T is constant within 2-3% from 35 to 8 mdeg K, its average value corresponding to $m^*/m = 2.4_1$ in good agreement with the value 2.5 obtained from second-sound measurements.[7] Using the value 2.4_1 for m^*/m, a value for the He^3 concentration x may be obtained from the higher temperature data, where the dependence on m^*/m is not as great, using the tables for the heat capacity of an ideal Fermi-Dirac gas given by Stoner.[3] In this way we find $x = 0.050_1$ for the nominal 5.0% mixture. Using $x = 0.050_1$ and $m^*/m = 2.4_1$, the smoothed heat-capacity data are fitted to better than 1%, over the full temperature range of measurement, by the theoretical heat capacity given by Stoner. We then make the assumption that the same value of m^*/m would be valid for the nominal 1.3% mixture and determine x for this mixture from the high-temperature data using Stoner's tables, the result being $x = 0.012_9$. The resulting theoretical curve based on this value of x agrees with the experimental data to better than 3% over the full temperature range, there being a systematic deviation, not really outside experimental error, in the direction requiring a smaller value of m^*/m for the 1.3% mixture. Hence, if there is an effect of concentration on m^*/m, it must be less than a few percent in the present concentration range.

No evidence for a phase separation was found in the heat-capacity data. Further evidence, of a qualitative nature, for the absence of a phase separation was obtained from the transient response of a resistance thermometer in the cell. If there had been a phase of nearly pure He^4 below a phase of pure He^3, the resistance thermometer would have responded much more violently during the application of heater power than it actually did, based on observations with only He^4 in the cell. The present experiments also show that the assumptions on which the new He^3-He^4 dilution refrigerator[8] is based are essentially sound. This refrigerator should therefore have a low-temperature refrigerating capacity roughly proportional to T^2 down at least to 10 mdeg K. The present experiments also are in agreement with the original ideas of Landau and Pomeranchuk[9,10] on the nature of dilute solutions of He^3 in He^4.

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