# PARTIALLY CONSERVED AXIAL-VECTOR CURRENT AND THE DECAYS OF VECTOR MESONS*

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The notion of a partially conserved axial-vector current[1] (PCAC) and the current commutation relations that generate the algebra U(2)⊗U(2) have been successfully applied to the calculation of the axial-vector constant renormalization[2] and to the nonleptonic decays of hyperons and $K$ mesons.[3] In this note, we wish to report on further applications of these ideas to the decays of vector mesons.

Let us first consider the following transition process: $A \to B + \pi^i + \pi^j$, in general.[4] The states $A$ and $B$ are specified later. The transition matrix for this process may be expressed as

$$T_{A\to B+\pi^i+\pi^j} = \frac{-1}{(4\omega_i\omega_j)^{1/2}}\int d^4x\int d^4y\,\exp(-ik^ix-ik^jy) \times K_xK_y\langle A_{\rm in}|T[\varphi^i(x),\varphi^j(y)]|B_{\rm out}\rangle, \quad (1)$$

where $\varphi^i(x)$ is the renormalized pion field with isospin $i$, and $K_x$ stands for the Klein-Gordon operator.

With the PCAC relation

$$\partial_\alpha \mathfrak{F}_{i\alpha}^{5}(x) = ic^{-1}\varphi^i(x),\quad c=\frac{g_rK(0)}{M\mu_\pi}\left(\frac{G_V}{-G_A}\right), \quad (2)$$

Eq. (1), after integration by parts,[5] can be transformed into

$$T_{A\to B+\pi^i+\pi^j} = \frac{c^2}{(4\omega_i\omega_j)^{1/2}}\{\int d^4x\int d^4y\,\exp(-ik^ix-ik^jy)K_xK_y \times[\delta(x_0-y_0)\langle A_{\rm in}|[\mathfrak{F}_{i0}^{5}(x),\partial_\beta\mathfrak{F}_{j\beta}^{5}(y)]|B_{\rm out}\rangle + ik_\alpha^i\delta(x_0-y_0)\langle A_{\rm in}|[\mathfrak{F}_{i\alpha}^{5}(x),\mathfrak{F}_{j0}^{5}(y)]|B_{\rm out}\rangle - k_\alpha^ik_\beta^j\langle A_{\rm in}|T(\mathfrak{F}_{i\alpha}^{5}(x),\mathfrak{F}_{j\beta}^{5}(y))|B_{\rm out}\rangle]\}. \quad (3)$$

We now take a limit of both pion four-momenta going to zero, keeping terms up to first order in each pion momentum in Eq. (3), and find

$$\lim_{\substack{k^i\to 0\\ k^j\to 0}}(4\omega_i\omega_j)^{1/2}T_{A\to B+\pi^i+\pi^j} = 2\pi\delta(E_A-E_B)c^2\mu_\pi^4\{\langle A_{\rm in}|[F_i^{5}(0),\dot F_j^{5}(0)]|B_{\rm out}\rangle + ik_\alpha^i\langle A_{\rm in}|[F_{i\alpha}^{5}(0),F_j^{5}(0)]|B_{\rm out}\rangle\}, \quad (4)$$

where $F_{i\alpha}^5(x_0)=\int d^3x\,\mathfrak{F}_{i\alpha}^5(x,x_0)$ with $F_{i0}^5\equiv F_i^5$.

It is convenient to separate our discussions for those processes in which two pions are emitted either in the $S$ state (isospin symmetric) or in the $P$ state (isospin antisymmetric). Equation (4) cannot be applied for states higher than the $P$ state, since we have neglected higher powers of pion momentum.

Noticing that from the commutation relations the first term of Eq. (4) is symmetric with respect to isospin indices, while the second one is antisymmetric, we obtain[6]

$$\lim_{\substack{k^i\to 0\\ k^j\to 0}}(4\omega_i\omega_j)^{1/2}T_{A\to B+\{\pi^i+\pi^j\}} = i(2\pi)\delta(E_A-E_B)c^2\mu_\pi^4\langle A_{\rm in}|u_0(0)|B_{\rm out}\rangle, \quad (5)$$

$$\lim_{\substack{k^i\to 0\\ k^j\to 0}}(4\omega_i\omega_j)^{1/2}T_{A\to B+[\pi^i+\pi^j]} = -(2\pi)\delta(E_A-E_B)c^2\mu_\pi^4[\tfrac{1}{2}(k^i-k^j)_\alpha] \times\langle A_{\rm in}|F_{k\alpha}(0)|B_{\rm out}\rangle, \quad (6)$$

where we have used the commutation relations[7]

$$[F_{i\alpha}^5(0),F_j^5(0)] = i\epsilon_{ijk}F_{k\alpha}(0), \quad (7)$$

and

$$[F_i^5(0),\dot F_j^5(0)] = i\delta_{ij}u_0(0), \quad (8)$$

with

$$F_{i\alpha}(0)\equiv\int d^3x\,\mathfrak{F}_{i\alpha}(x,0).$$

The operator $u_0$ is not specified here. Equation (8) may be regarded as a definition of $u_0$.

It is easily seen from Eq. (6) that the process $A \to B + [\pi^i + \pi^j]$ is now related to that of $A \to B +$ isovector photon in the limit of both $k^i$ and $k^j$ being continued to zero.

Next we state results on the applications of Eq. (6) to the decays of vector mesons.

(i) $A = \rho$ meson, $B =$ vacuum; we get[8]

$$f_{\rho\pi\pi} = c^2 \mu_\pi^{\ 4} m_\rho^{\ 2} / f_\rho, \tag{9}$$

where the constants $f_{\rho\pi\pi}$ and $f_\rho$ are defined by

$$\langle \rho | \pi^i \pi^j \rangle = -(2\pi)^4 \delta^4 (p^\rho - k^i - k^j)(8 m_\rho \omega_i \omega_j)^{-1/2} \times \epsilon_{ijk} \epsilon_\alpha^{\ k} (k^i - k^j)_\alpha f_{\rho\pi\pi}, \tag{10}$$

and[9]

$$\langle \rho | \mathfrak{F}_{3\alpha}(0) | 0 \rangle = (2 m_\rho)^{-1/2} \epsilon_\alpha^{\ 3} m_\rho^{\ 2} / f_\rho. \tag{11}$$

We denote the polarization vector of the $\rho$ meson with isospin $i$ by $\epsilon_\alpha^{\ i}$.

Now, with an additional assumption that $\rho$ mesons universally couple to the isovector current, $f_{\rho\pi\pi} = f_\rho$, we finally obtain

$$f_{\rho\pi\pi}^{\ \ 2} = c^2 \mu_\pi^{\ 4} m_\rho^{\ 2}, \tag{12}$$

or

$$\frac{f_{\rho\pi\pi}^{\ \ 2}}{4\pi} = \left(\frac{g_\gamma^{\ 2}}{4\pi}\right)\left(\frac{-G_A}{G_V}\right)^{-2} \frac{m_\rho^{\ 2}}{2M^2} K^2(0) \tag{13}$$

$$\approx 3.3 K^2(0). \tag{14}$$

If one assumes $K^2(0) \approx 1$, the decay width of the $\rho$ meson comes out to be $\Gamma_\rho \approx 165$ MeV.[10] We believe that the agreement with experimental data is fair.

(ii) $A = \omega$ meson, $B = \pi^0$. We can get a ratio for the decays of $\omega \to \pi^+ + \pi^- + \pi^0$ and $\omega \to \pi^0 + \gamma$. In fact, we have[11]

$$(f_{\omega\text{-}3\omega}) / \mu_\pi^{\ 3} = 2 c^2 \mu_\pi^{\ 3} f_{\omega\pi\gamma}, \tag{15}$$

where $f_{\omega\text{-}3\omega}$ and $f_{\omega\pi\gamma}$ are defined through the relations

$$\langle \omega | \pi^+ \pi^- \pi^0 \rangle = (2\pi)^4 \delta^4 (p^\omega - k^+ - k^- - k^0)(16 m_\omega \omega_+ \omega_- \omega_0)^{-1/2} \epsilon_{\mu\nu\lambda\sigma} \epsilon_\mu^{\ \omega} k_\nu^{\ +} k_\lambda^{\ -} k_\sigma^{\ 0} f_{\omega\text{-}3\pi} / \mu_\pi^{\ 3}, \tag{16}$$

$$\langle \omega | \pi^0 \gamma \rangle = (2\pi)^4 \delta^4 (p^\omega - k^0 - q^\gamma)(8 m_\omega \omega_0 \omega_\gamma)^{-1/2} \epsilon_{\mu\nu\lambda\sigma} \epsilon_\mu^{\ \omega} \epsilon_\nu^{\ \gamma} q_\lambda^{\ \gamma} k_\sigma^{\ 0} e f_{\omega\pi\gamma} / \mu_\pi. \tag{17}$$

It is straightforward, from Eq. (15), to calculate the branching ratio of the two decay modes, which comes out to be

$$\frac{\Gamma(\omega \to \pi^0 + \gamma)}{\Gamma(\omega \to \pi^+ + \pi^- + \pi^0)} \approx 17\%. \tag{18}$$

The observed ratio is about 14%.[12] Again, the agreement is reasonable. It is interesting to note that the numerical value obtained here turns out to be almost the same as that estimated by Gell-Mann, Sharp, and Wagner,[13] although we have not assumed any particular model in order to get the above ratio. This is not surprising, however, since in both methods this ratio only depends upon the $\rho$-meson couplings and we predicted, through Eq. (12), the correct value for the $\rho$-meson width.

(iii) $A = \gamma$, $B = \pi^0$. This is quite analogous to the previous case (ii). We get a relation between $\gamma \to \pi^+ + \pi^- + \pi^0$ and $\pi^0 \to 2\gamma$,

$$f_{\gamma\text{-}3\pi} / \mu_\pi^{\ 3} = 2 c^2 \mu_\pi^{\ 3} f_{\pi^0\text{-}2\gamma}, \tag{19}$$

where the couplings $f_{\gamma\text{-}3\pi}$ and $f_{\pi^0\text{-}2\gamma}$ are defined in Eqs. (16) and (17) with $\omega$ replaced by $\gamma$. We estimate the magnitude of the coupling $f_{\gamma\text{-}3\pi}$ from the observed lifetime of $\pi^0$ and find

$$|f_{\gamma\text{-}3\pi}| \approx 3.7 \times 10^{-2}. \tag{20}$$

The ratio (20) is not far from the estimate given by one of us,[14] although this is smaller than that expected from the SU(3) model.[15]

A few remarks are in order here. If we choose $A = B =$ nucleon in Eq. (6), we would arrive at the Adler-Weisberger formula which connects the axial-vector coupling-constant renormalization to the isospin-antisymmetric part of the forward pion-nucleon scattering amplitude evaluated at threshold. An application of Eq. (5) for the case of $A = B =$ nucleon will be discussed elsewhere.[16]

Finally, we would like to emphasize the fact that in an idealized world where all pion momenta are appropriately continued to zero, various phenomena involving pions are greatly simplified[17]; we have, for instance, the exact Adler-Weisberger relation[2] and $\Delta I = \frac{1}{2}$ rule for the nonleptonic decays of $\Lambda$ and $\Xi$ and $K$ mesons.[3] Moreover, the axial-vector currents are presumably conserved in this limit.[16] It seems that all these features are not far from reality.

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[4]The method we have employed here is somewhat overlapping with those discussed by several authors: S. L. Adler, Phys. Rev. 140, B736 (1965); W. I. Weisberger, to be published; R. F. Dashen, private communication.

[5]The surface terms arising from the partial integration in the time variable are neglected.

[6]We have explicitly antisymmetrized Eq. (5) with respect to $k^i$ and $k^j$, since the two pions are in the isospin antisymmetric state.

[7]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[8]In terms of the decay form factor $F(p_\rho^2, k_i^2, k_j^2)$, the effective decay constant is defined at the value $F(m_\rho^2, \mu_\pi^2, \mu_\pi^2)$, while our constant $f_{\rho\pi\pi}$ is rather related to $F(0,0,0)$. See Ref. 10 for possible corrections.

[9]M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961). The constant $f_\rho$ that appears in Eq. (11) is equivalent to $2\gamma_\rho$ in this reference.

[10]There are several terms that could modify this value. First of all, we must continue the matrix element given here back to the physical mass shell. Corrections coming from the propagators are given by $d_\rho(m_\rho^2)/d_\rho(0)$ and $d_\pi(\mu_\pi^2)/d_\pi(0)$, where $d_\rho(s)$ and $d_\pi(s)$ are the numerators of the exact propagators of the $\rho$ meson and the pion, respectively. With the definition of

$$d_\rho(s^2) = \delta_{ij} d^{(1)}(s^2) + (p_i p_j/m_\rho^2) d^{(2)}(s),$$

we have

$$d^{(1)}(s) = 1 + \frac{m^2 - s}{\pi} \int \frac{\sigma(s')}{s' - s} ds'$$

in the renormalized form. There is no contribution from $d^{(2)}(s)$ for the physical $\rho$ meson at rest. We know that

$$d^{(1)}(m^2)/d^{(1)}(0) = [1 + (m^2/\pi)\int \sigma(s)/s ds]^{-1}$$

is less than unity, since $\sigma(s)$ is positive definite. Presumably, the correction coming from $d_\pi(\mu_\pi^2)/d(0)$ is small. In the case of $d_\rho(m_\rho^2)$, the pole term due to the single $\rho$-meson state is embedded in the continuum. If we take $m_\rho = 765$ MeV and $\Gamma_\rho = 100$ MeV, the correction factor $d_\rho^{(1)}(m_\rho^2)/d_\rho^{(1)}(0)$ turns out to be 0.65. After this correction, we have

$$f_{\rho\pi\pi}^2/4\pi = 2.1,$$

which corresponds to $\Gamma_\rho = 110$ MeV.

[11]Although the matrix element for the decay of $\omega \to \pi^+ + \pi^- + \pi^0$ has the explicit momentum dependence like Eq. (16), there is no contribution from the third term in Eq. (3) that we neglected before. This is clear since the term is symmetric with respect to $k_\alpha^+$ and $k_\alpha^-$ as far as we can keep terms up to linear in each momentum. In order to avoid a possible contribution from the first term, one has to assume the local analog of the commutation relation (8).

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