from the first relation, but the matrix element in this limit seems to be a worse approximation to the physical one as compared with the matrix element given in the text, unless the continuation is made. The limit taken in the text does not contradict with $s$-wave configuration of the di-pion in contrast to the limit given above. In our case, $K_{e4}(\pi^+\pi^-)/K_{e4}(\pi^0\pi^0)=2$ is implicitly predicted. In the case of $K_{e4}(\pi^0\pi^0)$ decay, there does not occur such a problem, and so our estimate of $K_{e4}(\pi^+\pi^-)/K_{e3}$ should be translated into $K_{e4}(\pi^0\pi^0)/K_{e3}$, if one does not adopt explicit symmetrization.

[7]If we use a symmetrized spatial wave function normalized to unity for the di-pion, the factor $c\mu^2/2$ should be replaced by $c\mu^2/\sqrt{2}$, but accordingly the phase-volume integral over final states must be restricted to a hemisphere.

[8]The decay rates with constant form factors are given in Ref. 4 and L. B. Okun', Ann. Rev. Nucl. Sci. 9, 61 (1959).

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# CURRENT ALGEBRAS AND THE SUPPRESSION OF LEPTONIC MESON DECAYS WITH $\Delta S=1$*

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Some time ago, we obtained a set of relations between the semileptonic amplitudes of mesons, which gave an indication that the suppression of $\Delta S=1$ transitions relative to $\Delta S=0$ decays cannot be due to strong-interaction effects.[1] These relations were obtained essentially on the basis of the chiral $U(3)\otimes U(3)$ algebra of hadron currents,[2] partially conserved axial-vector currents (PCAC),[3] and the assumption that the divergence of the axial-vector current acts approximately like a creation or destruction operator for the corresponding pseudoscalar mesons.

In this note we show that the same relations can be derived without the latter assumption, provided we allow one of the meson mass variables to be zero. Irrespective of this mass extrapolation, we find that our formulas give a very direct indication for the absence of specific strangeness-dependent renormalization effects in matrix elements of axial-vector (and vector) currents. Especially, we find that those form factors which are relevant for the determination of the Cabibbo angle[4] are essentially uneffected by the large $K\pi$ mass splitting.

Let us write the matrix elements for semileptonic $K$ decays in the form

$$\langle 0|A_{4\alpha}-iA_{5\alpha}|K^+\rangle=iK_\alpha B_K,$$

$$\langle\pi^0|V_{4\alpha}-iV_{5\alpha}|K^+\rangle$$

$$=(1/\sqrt{2})F_{\pi K}\{(K+\pi)_\alpha+\xi_{\pi K}(K-\pi)_\alpha\},\qquad(1)$$

with similar expressions for the corresponding $\pi$ decays. The invariant functions $F_{\pi K}$ and $\xi_{\pi K}$ have the arguments $(-\pi^2, -K^2; -(K-\pi)^2)$. We can write the relations obtained in Ref. 1 in the form

$$F_{\pi K}(1+\xi_{\pi K})=[F_{\pi K}(1-\xi_{\pi K})]^{-1}=B_K/B_\pi,$$

$$F_{\pi\pi}(0)=F_{KK}(0)=1,\qquad(2)$$

with $-\pi^2=m_\pi^2$, $-K^2=m_K^2$. As far as the momentum-transfer variable $q^2=(K-\pi)^2$ is concerned, we have assumed that $F_{\pi K}$ and $\xi_{\pi K}$ are slowly varying functions for $(m_K-m_\pi)^2\leq -q^2\leq(m_K+m_\pi)^2$.

In order to give a more direct derivation of Eqs. (2), we use the reduction formula and PCAC. Then we have an equation like

$$\langle\bar{K}^0|V_{40}(0)-iV_{50}(0)|\pi^+\rangle$$

$$=\frac{i}{m_K^2B_K}\int d^4x\,e^{-iK\cdot x}(\Box-m_K^2)\langle 0|\theta(-x_0)[\partial_\alpha\{A_{6\alpha}(x)+iA_{7\alpha}(x)\},V_{40}(0)-iV_{50}(0)]|\pi^+\rangle,\qquad(3)$$

and a corresponding one for the matrix element $\langle\pi^0|V_{40}-iV_{50}|K^+\rangle$, where the dependence upon $\pi_\alpha$ is exhibited. Taking the limit $K_\alpha\to 0$ (or $\pi_\alpha\to 0$, respectively), and using partial integration with vanishing surface terms,[5] we find that we can employ the equal-time commutators of U(3)⊗U(3) in order to obtain the relations[6]

$$F_{\pi K}(m_\pi^2, 0; m_\pi^2)[1-\xi_{\pi K}(m_\pi^2, 0; m_\pi^2)] = B_\pi/B_K, \tag{4}$$

$$F_{\pi K}(0, m_K^2; m_K^2)[1+\xi_{\pi K}(0, m_K^2; m_K^2)] = B_K/B_\pi. \tag{5}$$

Furthermore, we find the formulas

$$F_{\pi\pi}(m_\pi^2, 0; m_\pi^2)[1-\xi_{\pi\pi}(m_\pi^2, 0; m_\pi^2)] = 1,$$

$$F_{KK}(0, m_K^2; m_K^2)[1+\xi_{KK}(0, m_K^2; m_K^2)] = 1, \tag{6}$$

and similar relations with the labels $\pi$ and $K$ interchanged. Here $F_{\pi\pi}$ and $F_{KK}$ are the vector form factors related to the charge distribution, and on the mass shell we have, of course, $\xi_{\pi\pi}(-q^2) \equiv \xi_{KK}(-q^2) \equiv 0$.

It is interesting to note that in our earlier derivation of Eqs. (2), we can also take one of the mass variables to zero. We obtain then exactly the expressions (4)-(6). The advantage of the present method is that it gives these equations essentially as a direct consequence of the current algebra and PCAC, granting the specific continuation used in the evaluation of Eq. (3).

As long as the variation of the form factors as functions of $q^2$ is ignored, the most natural solutions of Eqs. (2) are the SU(3) values

$$F_{\pi K} = 1, \quad \xi_{\pi K} = 0, \quad \text{and } B_\pi = B_K,$$

which just correspond to Cabibbo's hypothesis.[4]

On the other hand, the current algebra is assumed to be valid even in the presence of a sizable symmetry breaking. Therefore, Eqs. (4) and (5) are of interest mainly in connection with possible deviations from the SU(3) limit. In this problem the variation of the form factors as functions of their arguments may not be negligible.

A most interesting solution of Eqs. (4) and (5) is obtained with the pole model:

$$F_{\pi K}(-\pi^2, -K^2; -(K-\pi)^2) = g_{\pi K}(-\pi^2, -K^2)m^{*2}/[m^{*2}+(K-\pi)^2],$$

$$\xi_{\pi K}(-\pi^2, -K^2; -(K-\pi)^2) = (K^2-\pi^2)/m^{*2}, \tag{7}$$

where $m^*$ is the mass of the $K^*(890)$ vector meson. The expression for $\xi_{\pi K}$ is determined by the condition that the residue of the $K^*$ pole must vanish in the matrix element for the divergence of the current $V_{4\alpha}-iV_{5\alpha}$ in Eq. (1). The pole model (7) may well be a good approximation for the form factors, especially at the positive values of $-(K-\pi)^2$ considered in the following.

Substitution of Eqs. (7) into the formulas (4) and (5) gives the very simple conditions

$$g_{\pi K}(m_\pi^2, 0) = g_{\pi K}^{-1}(0, m_K^2) = B_\pi/B_K. \tag{8}$$

In contrast to $\xi_{\pi K}$, it is quite plausible that $g_{\pi K}$ is an approximately symmetric and slowly varying function of the mass variables corresponding to the pseudoscalar-meson channels. This conjecture may be reinforced by considering vector-meson pole models analogous to Eqs. (7) for the form factors $F_{\pi\pi}$ and $F_{KK}$ in Eq. (6), in which case we find conditions like $g_{\pi\pi}(m_\pi^2, 0) = g_{KK}(0, m_K^2) = 1$. Hence, it seems reasonable to assume that $g_{\pi K}(m_\pi^2, 0) \approx g_{\pi K}(0, m_K^2)$ in Eq. (8), which then implies

$$B_\pi \approx B_K, \tag{9}$$

as well as $F_{\pi K}(m_\pi^2, m_K^2, 0) \approx 1$, if we are also willing to accept the equality $g_{\pi K}(m_\pi^2, m_K^2) \approx g_{\pi K}(0, m_K^2) \approx 1$. Note that only $F_{\pi K}$ enters into the determination of the Cabibbo angle from $K_{e3}$ decay.[7]

We conclude that the current algebra strongly indicates that the suppression of $\Delta S = 1$ decays of mesons must be due to effects which are related to the structure of the weak interactions. Similar indications for the leptonic decays of baryons have been obtained by several authors,[5,8] although in these cases the sum rules require a very extensive input of empirical information, as well as mass extrapolations which are similar to those encountered in this paper.

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[1]R. Oehme, Ann. Phys. (N.Y.) 33, 108 (1965). [In order to find Eqs. (2), take Eq. (7.12) of this reference with $\rho=0$ and substitute into Eqs. (7.21) and (7.27)]; R. Oehme and G. Segré, Phys. Letters 11, 94 (1964); see also R. Oehme, Phys. Rev. Letters 12, 550, 604(E) (1964).

[2]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[3]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. 703 (1960) [translation: Soviet Phys.—JETP 12, 492 (1961)]; Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[4]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[5]See, for example, W. I. Weisberger, to be published; this paper contains further references.

[6]A relation similar to Eq. (5) has been derived recently by C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966); see also V. S. Mathur, S. Okubo, and L. K. Pandit, to be published.

[7]We must mention here the possibility that the $\kappa(725)$ meson is coupled to the divergence of the strangeness-changing vector current [Y. Nambu and J. Sakurai, Phys. Rev. Letters 11, 42 (1963)]. This would complicate our pole model (7); a possible Ansatz is given by $F_{\pi K}$ as in Eq. (7), but with

$$\xi_{\pi K}(-\pi^2, -K^2; -(K-\pi)^2) = \frac{K^2-\pi^2}{m^{*2}} \frac{\kappa^2-m^{*2}}{\kappa^2+(K-\pi)^2}$$

where $\kappa$ is the mass of the $0^+$ meson. We see that the coupling is strongly dependent upon the mass variables, which presumably makes this model unreliable for our purpose. If we proceed, nevertheless, we find in place of Eq. (8) the relations

$$g_{\pi K}(m_\pi^2, 0)[1+m_\pi^2/(\kappa^2-m_\pi^2)] = B_K/B_\pi,$$

$$g_{\pi K}(0, m_K^2)[1+m_K^2/(\kappa^2-m_K^2)] = B_\pi/B_K.$$

If we can maintain here the assumptions of slow variation and symmetry of $g_{\pi K}$, we obtain roughly $g_{\pi K} \sim 1/\sqrt{2}$ and hence $B_K/B_\pi \sim \sqrt{2}$. Such a value for $B_K/B_\pi$ is in the wrong direction (and too small) for the suppression of $\Delta S=1$ amplitudes on the basis of SU(3) breaking ($B_K/B_\pi \sim \frac{1}{4}$ would be required).

[8]L. K. Pandit and J. Schechter, Phys. Letters 19, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, Phys. Letters 19, 59 (1965); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); S. L. Adler, Phys. Rev. 140, B736 (1965); R. Oehme, Phys. Rev. (to be published).