Table III. Legendre-polynomial expansion coefficients for the $Y_0^*(1520)$ decay distributions in the energy range 1740 to 1780 MeV with respect to the production normal ($I = \sum_K A_K P_K(\cos\Phi)$), where $\cos\Phi = \hat{n}\cdot\hat{\pi}$ and $\hat{n} = K^- \times Y_0^*(1520)/|K^- \times Y_0^*(1520)|$.

Coefficient	Experimental value	Theoretical value $\frac{5}{2}^-$	Theoretical value $\frac{5}{2}^+$
A_0	1.00 ± 0.07	1.0	1.0
A_1	-0.03 ± 0.09	0	0
A_2	-0.91 ± 0.13	-0.7	0.78
A_3	0.06 ± 0.17	0	0
A_4	0.04 ± 0.21	0	0
A_5	-0.07 ± 0.29	0	0

ity of the $Y_1^*(1765)$ is $\frac{5}{2}^-$.

The decay distribution of the $Y_0^*(1520)$ allows a further check on the spin-parity assignment of the $Y_1^*(1765)$. For $J^P = \frac{5}{2}^+$, a distribution of $1 + 0.78P_2(\cos\Phi)$ is expected, while for $J^P = \frac{5}{2}^-$, a distribution of $1 - 0.70P_2(\cos\Phi)$ is predicted. Here we have $\cos\Phi = \hat{n}\cdot\hat{\pi}^+$ in the $Y_0^*(1520)$ c.m. system, and $\hat{n}$ is the production normal $\hat{n} = K^- \times Y_0^*(1520)/|K^- \times Y_0^*(1520)|$. In Fig. 3(b) we present our experimental data; Legendre-polynomial expansion coefficients are shown in Table III. For $E = 1760 \pm 20$ MeV, fits to the theoretical distributions give $\chi^2(\frac{5}{2}^-) = 2.6$ and $\chi^2(\frac{5}{2}^+) = 242.1$ for nine degrees of freedom.

In conclusion, our data indicate the existence of the $Y_1^*(1765)$ hyperon resonance with $M = 1760 \pm 10$ MeV, $\Gamma = 60$ MeV, and the unambiguous spin-parity assignment $\frac{5}{2}^-$.

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QUANTUM NUMBERS AND MASSES OF MESONS AS QUARK-ANTIQUARK SYSTEMS

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The purpose of this note is (a) to make quantum-number assignments and relate the masses of mesons treated as a quark-antiquark ($q\bar{q}$) system, (b) to see if the SU(6) [or U(6)⊗U(6)] type mass splittings of the $J^P = 0^-$, 1^- ($\underline{35} + \underline{1}$) mesons remain for the higher mesons, and (c) to relate mesons to the ($q\bar{q}$) Regge trajectories.

The implications of SU(6) for quark models and in regard to a quark mass $M_q \gtrsim 10$ BeV have been discussed by Nambu,[1] Lipkin,[2] and others. Gell-Mann[3] has derived the orbital angular momentum $\vec{L}$ of quarks[4] from a current algebra supplementing the SU(6) with intrinsic quark parity, U(6)⊗U(6), with $O^L(3)$. [See also Mahanthappa and Sudarshan.[5]]

In Table I we relate the quantum numbers and masses of mesons according to the ($q\bar{q}$) system. The internal dynamics is taken nonrelativistic, though the essential features most likely remain valid[6] in a relativistic discussion. The assignments are quite unambiguous. For $B(1220)$, experimentally $J \geq 1$, $P = ?$, $G = +$. The ($q\bar{q}$) with $L = 1$, $S = 0$ gives $J^{PG} = 1^{++}$ whereas $L = 2$, $S = 0^-$ would contradict G; $L = 2$, $S = 1^-$ would give too many unobserved nearby mesons with $J^P = 3^-$, 2^-, 1^-. The $A_2(1324)$ is consistent with $L = 1$, $S = 1$ ($q\bar{q}$), but also with $S = 0^+$, $L = 2$, e.g., for $qq\bar{q}\bar{q}$. The mass changes in Table I are consistent with changes in L, S, and J of $(q\bar{q})$.

The splitting of $J^P = 0^+,\ 1^+,\ 2^+$ ($^3P_0$, $^3P_1$, $^3P_2$) states is assumed due to spin-orbit and tensor $(q\bar{q})$ potentials $V_{LS}(r)\vec{L}\cdot\vec{S} + V_t(r)S_{12}$. The V_{LS} and V_t may depend on similar regions of r and could contribute equally. Their effect on meson m's are taken to first order. Then

$$\Delta m_{LS} = \tfrac{1}{2}a[J(J+1)-L(L+1)-S(S+1)], \qquad (1)$$

$$\Delta m_t^{(1)} \cong \frac{b[(2\vec{s}_1\cdot\vec{s}_2 + \tfrac{3}{2})\vec{L}^2 - \tfrac{3}{2}(\vec{S}\cdot\vec{L}) - 3(\vec{S}\cdot\vec{L})^2]}{(2L+3)(2L-1)}. \qquad (2)$$

The a and b are spin-orbit and tensor interaction $(q\bar{q})$ parameters. To this order J, J_3, L, S remain good quantum numbers. The second-order effect of V_tS_{12} (e.g., on $^3S_1$) is neglected.

Fitting Eqs. (1) and (2) to $^3P_2[K^{*\prime}(1410)] - {}^3P_0 \times [\kappa(725)]$ and to $^3P_2[A_1(1072)]$, one gets

$$a = 177 \mp 20 \text{ MeV and } b = 171 \mp 20 \text{ MeV}. \qquad (3)$$

These values are very reasonable in view of Table I though the existence of the A_1 resonance is now uncertain.[7] Missing mesons of the $L=1$, $S=1$, 35-plet are shown in square brackets in the table based on Eq. (3). Though not certain, $f(1253)$ is assumed to be ω'', rather than φ''. The φ predictions are less reliable than others. They involve additional assumptions (Ref. 6 and below).

With the above a and b, the unsplit ^3P masses (i.e., before any first-order effect of V_{LS} and V_t) are obtained as

$$\bar{m}(^3P;\ T=1,\ Y=0) = 1164 \mp 40 \text{ MeV}, \qquad (4)$$

$$\bar{m}(^3P;\ T=\tfrac{1}{2},\ Y=\mp 1) = 1250 \mp 40 \text{ MeV}. \qquad (5)$$

In examining SU(6) or U(6)⊗U(6) symmetry breaking within a given L we now use Eqs. (4) and (5) (for $L=1$) along with the $^1P_1$ mesons in the table. For $L=0$, the broken SU(6) m^2 splitting of $S=0$, $S=1$ depends on $S(S+1)$ as well as SU(6) quantum numbers in $\Delta m^2(T, Y, C_2^{(4)}, \mathfrak{M}, \mathcal{S})$.[8] The $S(S+1)$ term is shown by

Table I. Meson assignments. (Predicted mesons shown in [].)

L	SU(6) or U(6)⊗U(6)	S	$SU^U(3)\otimes SU^S(2)$	J^P	Spectroscopic notation	Regge trajectory (see text)	G Parity $=(-1)^{L+S+T}$	Meson[a] (MeV)
0	$\underline{1}$	0	$^1\underline{1}$	0^-	$^1S_0$	A'	+	$X^0(959)$
	$\underline{35}$	0	$^1\underline{8}$	0^-	$^1S_0$	A	−	$\pi(138)$
								$K(496)$
								$\eta(549)$
							+	$\rho(769)$
		1	$^3\underline{8}+{}^3\underline{1}$	1^-	$^3S_1$	D		$K^*(891)$
							−	$\omega(783)$
							−	$\varphi(1020)$
1	$\underline{1}$	0	$^1\underline{1}$	1^+	$^1P_1$	A'	−	$[X'(1640 \mp 170)]$
	$\underline{35}$	0	$^1\underline{8}$	1^+	$^1P_1$	A	+	$B(1220)$
								$K_C(1215 \mp 15)$
							−	$E(1420)$
		1	$^3\underline{8}+{}^3\underline{1}$	0^+	$^3P_0$	C	−	$[\pi'(640 \mp 40)]$
								$\kappa(725)$
							+	$[f'(567 \mp 40);\ \omega'?]$
							+	$[\Phi'(835 \mp 60)]^b?$
				1^+	$^3P_1$	B	−	$A_1(1072 \mp 8)^c$
								$[K^{*\prime\prime}(1158 \mp 40)]$
							+	$[\hat{f}(1000 \mp 20);\ \hat{\omega}'?]$
							+	$[\hat{\varphi}'(1268 \mp 60)]^b?$
				2^+	$^3P_2$	D	−	$A_2(1324 \mp 9)$
								$K^{*\prime}(1410 \mp 10)$
							+	$f(1253 \mp 20)\ [\omega''?]$
							+	$[\varphi''(1520 \mp 60)]^b?$

[a]Data from A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, University of California Radiation Laboratory Report No. UCRL-8030, Pt. I, (unpublished).
[b]See Ref. 10.
[c]See, however, Ref. 7.

Kuo and Radicati[9] to come from $V_S\vec{s}_1\cdot\vec{s}_2$ and Δm^2 from simultaneous spin-unitary spin exchange forces, V_{SU}, on a quark model of baryons. The entire SU(6) pattern for $L=0$, $\underline{35}+\underline{1}$ should repeat for $L=1$ unless V_S and V_{SU} depend on r quite strongly. For $L=0$, $\Delta m[{}^3S(\rho) - {}^1S(\pi)] = 630$ MeV; for $L=1$, $\Delta m[{}^3P(\text{unsplit }\pi) - {}^1P_1(B)] \cong 0$. Similarly, for $L=0$, $\Delta m[{}^3S(K^*) - {}^1S(K)] = 395$ MeV, vs $L=1$, $\Delta m[{}^3P(\text{unsplit }K) - {}^1P(K_c)] \cong 0$. Thus broken SU(6) splittings of the $S(S+1)$ type disappear for $L\neq 0$ indicating a strong r dependence of $V_S(r)$ (especially for $r\to 0$). Additional exchange forces involving L, U, and S could also be involved, however. More detailed calculations[6] show the other (e and f terms of Bég and Singh[8]) SU(6)-type splittings to remain.[10]

The unsplit [Eqs. (4), (5)] $L=0$ to $L=1$ excitation energy of a 35-plet is $\Delta m_L = 614 \mp 150$ MeV (ordinary average of ^1S and ^3S) or $\Delta m_L = 510 \mp 70$ MeV ($\vec{s}_1\cdot\vec{s}_2$ splitting only of ^1S and ^3S). Without regard to $S=0$, $S=1$ splittings, $\Delta m_L(S=0) = 850\mp 115$ MeV and $\Delta m_L(S=1) = 380 \mp 40$ MeV. These values are consistent with an $M_q \gtrsim 10$ BeV and $r_0 \sim 10^{-13}$ cm. Along with Eqs. (1) to (3) they also show that $L=2$, $S=0,1$ $(q\bar{q})$ states would most likely lie above 1600 MeV, only $J^P=1^-$ possibly getting into the region of observed mesons. The missing $L=1$, SU(6) singlet X' is placed at 1640 ∓ 170 MeV using $\Delta m_L = 510$ or 850 MeV.[11]

The above $(q\bar{q})$ system gives the following Regge trajectories over the real L axis (bound states; $L=0, 1, 2, \cdots$) for fixed U(6)⊗U(6) quantum numbers:

$$S=0^-,\ L=J;\ \ J^P=0^-,\ 1^+,\ 2^-,\cdots, \tag{A}$$

$$S=1^-,\ L=J;\ \ J^P=1^+,\ 2^-,\cdots, \tag{B}$$

$$S=1^-,\ L=J+1;\ \ J^P=0^+,\ 1^-,\cdots, \tag{C}$$

$$S=1^-,\ L=J-1;\ J^P=1^-,\ 2^+,\ 3^-,\cdots. \tag{D}$$

In (D), the missing $J^P=0^+$ would correspond to the $L=-1$ "nonsense term."[12] In Table I, $^1S_0(0^-)$ and $^1P_1(1^+)$ lie on (A), the $^3P_1(1^+)$ and $^3P_0(0^+)$ are alone, and $^3S_1(1^-)$ and $^3P_2(2^+)$ lie on (D). The 0^- and 1^+ on (A) would further split with respect to "signature," $(-1)^L$, as would 1^- and 2^+ on (D). However the Δm_L data above indicate $|(-1)^L V_{\text{exch}}|$ to be small (<300 MeV), if any. No two trajectories of (A) and (B) are expected to be parallel because of spin-unitary spin mixing of SU(6).

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[5]K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 163 (1965), assume the SU(6)⊗O(3) invariance of strong interactions without model. This yields nearly degenerate lowest mesons with $J^P=0^-$, 1^-, and 2^-.

[6]Further considerations and results which could not be included in this short note are given in O. Sinanoğlu, Center for Theoretical Studies Report No. CTS-HE-65-2 (unpublished); also in an extended account, O. Sinanoğlu, Phys. Rev. (to be published).

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[10]A general mass formula based on $(q\bar{q})$ dynamics and for generalization of SU(6)-type splittings is given in detail in Ref. 6. Considerations based on these give an unsplit $\bar{m}(^3P;\ \varphi\text{-like}) = 1360\mp 40$ MeV[6] from which by Eqs. (1)-(3), a $\varphi''(1520\mp 60)$, a $\hat{\varphi}'(1268\mp 60)$, and a $\varphi'(835\mp 60)$ follow. For completeness these too are included in Table I.

[11]It has been pointed out that in addition to Mahanthappa and Sudarshan [Ref. 5; see also K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 458 (1965)], E. Borchi and R. Gatto [Phys. Letters 14, 352 (1965)] have also discussed the $\underline{35}$ $L=1$ supermultiplet. The former authors assign $P=(-1)^L$, the latter $P=+1$. Both are phenomenological rather than based on a de-

tailed dynamics (see also Ref. 6) of the $(q\bar{q})$ system. They consider tensors of an abstract group to get symmetry breaking and only $\vec{L}\cdot\vec{S}$ to split $J=0$, 1, and 2. The detailed assignments and mass predictions differ considerably from the present one. The SU(6) singlets and $\Delta m(n',L)$ dependence of mass on dynamical quantum numbers are not considered. While the present papers were at journals, further work on 35 $L=1$ treated as $\tilde{U}(12)$ kinetic supermultiplets with various phenomenological mixing effects [R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. 140, B1579 (1965)] has also appeared.

[12]See, for example, R. G. Newton, The Complex J Plane (W. A. Benjamin, Inc., New York, 1964).

COMPLEX INHOMOGENEOUS LORENTZ GROUP AND COMPLEX ANGULAR MOMENTUM

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The purpose of this Letter is to point out that two of the most important properties of complex angular momentum—the property of generalizing integral or half-integral angular momentum, and the property of characterizing the asymptotic behavior of the S matrix in the crossed channel—can be obtained from group-theoretical arguments.[1]

Our procedure is to construct an S matrix invariant under the complex inhomogeneous Lorentz group (ICLG).[2] That it is natural to do so follows from the fact that if the S matrix satisfies certain analyticity requirements, the Bargmann-Hall-Wightman theorem[3] guarantees that invariance under the Poincaré group implies invariance under ICLG. To the best of our knowledge, there is no reason to believe the S matrix not to be invariant under ICLG.

Assuming the S matrix to be invariant under ICLG, it is then useful to construct the irreducible representations of this group.[1] Because ICLG is not a symmetry group—only the Poincaré group $\mathcal{P}$, a subgroup of ICLG, is a symmetry group—we are free to construct nonunitary representations of ICLG, as long as a state which corresponds to a physical-particle transforms under a unitary representation of $\mathcal{P}$.[4]

A state of an irreducible representation of ICLG is characterized by a complex momentum $\Pi_\mu = P_\mu + iQ_\mu$ with complex "mass" $M^2 = \Pi_\mu \Pi^\mu$. For a given Π_μ, the state transforms under an irreducible representation of the little group for the corresponding mass. For $M \neq 0$, the little group is the homogeneous Lorentz group, whose irreducible representations are labeled by $\chi = (\rho; n)$, a pair consisting of the complex number ρ and the integer n. It is χ which plays the role of complex angular momentum.

A single, spinless "particle" is described by the state with the transformation property

$$T(\alpha_\mu, \Lambda_\nu{}^\mu)\,|\Pi^\mu\rangle_1 = \exp[i\,\mathrm{Re}(\Pi^\mu \alpha_\mu)]\,|\Lambda_\nu{}^\mu \Pi^\nu\rangle_1,$$
$$(\alpha_\mu, \Lambda_\nu{}^\mu) \in \mathrm{ICLG}. \tag{1}$$

Under the Poincaré group, it transforms unitarily by[5]

$$T(a_\mu, L_\nu{}^\mu)\,|P_\mu + iQ_\mu\rangle_1 = \exp[i(P^\mu a_\mu)]\,|L_\nu{}^\mu P^\nu + iL_\nu{}^\mu Q^\nu\rangle,$$
$$(a_\mu, L_\nu{}^\mu) \in P. \tag{2}$$

The two-particle state

$$|\Pi_\mu{}^1\rangle_1 \otimes |\Pi_\mu{}^2\rangle_1 = |\Pi_\mu{}^1, \Pi_\mu{}^2\rangle$$

can be reduced. In the center-of-mass system, it has total momentum

$$\Pi_\mu = \Pi_\mu{}^1 + \Pi_\mu{}^2 = (\Pi_0{}^1, \vec{\Pi}) + (\Pi_0{}^2, -\vec{\Pi}) = (\Pi_0{}^1 + \Pi_0{}^2, \vec{0}),$$

"mass" $S^{1/2} = [\Pi_\mu \Pi^\mu]^{1/2}$, and it contains all "spins" χ. That is, for every χ there exists a projection operator A_χ such that

$$A_\chi\,|\Pi_\mu{}^1, \Pi_\mu{}^2\rangle = |\Pi_\mu; g^{1,2}\rangle_\chi, \tag{3}$$

a state which transforms according to the irreducible representation $[S^{1/2}, \chi]$ of ICLG. $g^{1,2}$ is the complex rotation which takes a vector in the z direction into the direction $\vec{\Pi}$.