terpretation of the $(R+V)$ function. The variables are now the momenta, and

$$R_{kp'} \equiv \int V_{kq} 8\pi M q^2 dq\, T_{qp'}{}^{(2)},$$

with Lovelace's normalization[4] and M as the reduced mass. Then

$$T_{kp} = T_{pk} = \frac{R_{kp}+V_{kp}}{1-\int(R_{kq}+V_{kq})8\pi Mq^2dq/(k^2-q^2+i\epsilon)},$$

and

$$\mathrm{Im}\,T_{pk}{}^{-1} = \frac{V_{kk}+R_{kk}}{V_{kp}+R_{kp}}(4\pi^2Mk).$$

This resembles the usual on-shell unitarity equation with a modification to k. Some other aspects of this work, including the relevance to bootstrap dynamics, will be discussed later.

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SPIN-PARITY DETERMINATION OF THE $Y_1^*(1765)$†

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Measurements of the K^-p total cross section at about 1-BeV/c incident-K^- momenta have shown a broad and asymmetric peak.[1] Further investigations led Barbaro-Galtieri, Hussain, and Tripp to suggest that two hyperon resonances with spin $\frac{5}{2}$ exist in this energy region—one an $I=0$ resonance at an energy about 1815 MeV with positive parity, the other, $I=1$ at about 1765 MeV and negative parity.[2] In this paper, we present data from the reaction $K^-+n\rightarrow\Sigma^-+\pi^++\pi^-$ which confirms that the $Y_1^*(1765)$ exists and that the reported spin-parity assignment, $\frac{5}{2}^-$, is correct.[3]

This study is based on 2100 of our events which fit the hypothesis $K^-+n\rightarrow\Sigma^-+\pi^++\pi^-$. This particular reaction has the advantage of being pure $I=1$ and having all pions visible; thus no effects from the strongly produced $Y_0^*(1815)$ are present. The data were obtained from a separated K^- beam in the Lawrence Radiation Laboratory's new 25-inch bubble chamber filled with deuterium. The incident K^- momenta were 828, 930, 1025, and 1112 MeV/c which, neglecting Fermi momentum, corresponds to a K^--n c.m. energy of 1700 to 1845 MeV.

In Fig. 1 we present the $\Sigma^-\pi^+$ invariant-mass distribution at various K^-n c.m. energies. It is evident that the reaction $K^-+n\rightarrow\Sigma^-+\pi^++\pi^-$

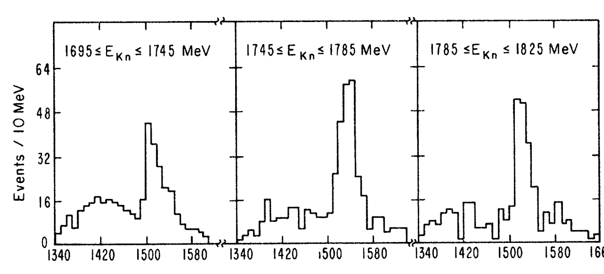

FIG. 1. Invariant mass of the $\Sigma^-\pi^+$ system produced in the reaction $K^-+n\rightarrow\Sigma^-+\pi^++\pi^-$.

is dominated by production of the well-known $J^P = \frac{3}{2}^-$, $Y_0^*(1520)$ hyperon resonance. This leads us to look for the presence of the $Y_1^*(1765)$ in the cross section for the process $K^- + n \rightarrow Y_0^*(1520) + \pi^-$. Because of the deuteron Fermi momentum, a given incident K^- momentum gives rise to a range of K^-n total c.m. energies. Nevertheless, it is interesting to look at the cross section for our reaction at each beam momentum. Figure 2(a) shows the cross section for $K^- + n \rightarrow Y_0^*(1520) + \pi^-$, assuming that the neutron in the deuteron is free. Here, as throughout this paper, we define the $Y_0^*(1520)$ by the condition that the invariant mass of the $\Sigma^-\pi^+$ system be in the range 1520 ± 25 MeV; the results of our analysis are not sensitive to the exact choice for the $Y_0^*(1520)$ width. Despite the considerable overlap in total K^-n c.m. energies between the various beam momenta, an enhancement is clearly indicated in the region of 930 MeV/c, or 1760-MeV K^-n c.m. energy.

One can go further. Knowing the deuterium wave function, the path length for each momentum, and values of the beam momenta, one can predict the expected distribution of K^-n c.m. energies. In Fig. 2(b), we plot the ratio of the number of experimental events to the area under the expected distribution curve for the intervals indicated for the reaction $K^- + n \rightarrow Y_0^*(1520) + \pi^-$; the enhancement around 1760 MeV is apparent. An examination of our data yields the resonance parameters $M = 1760 \pm 10$ MeV and $\Gamma = 60$ MeV, the width being very dependent on the assumed background.

If, as it appears, the $Y_1^*(1765)$ decays into

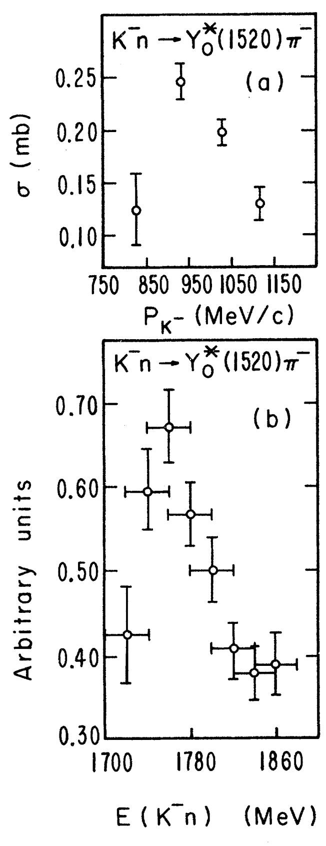

FIG. 2. (a) Cross sections for the reaction $K^- + n \rightarrow Y_0^*(1520)\pi^-$ at various incident momenta. (b) Ratio of the number of experimental events to the area under the theoretical K^-n c.m. energy distribution curve for the reaction $K^- + n \rightarrow Y_0^*(1520) + \pi^-$.

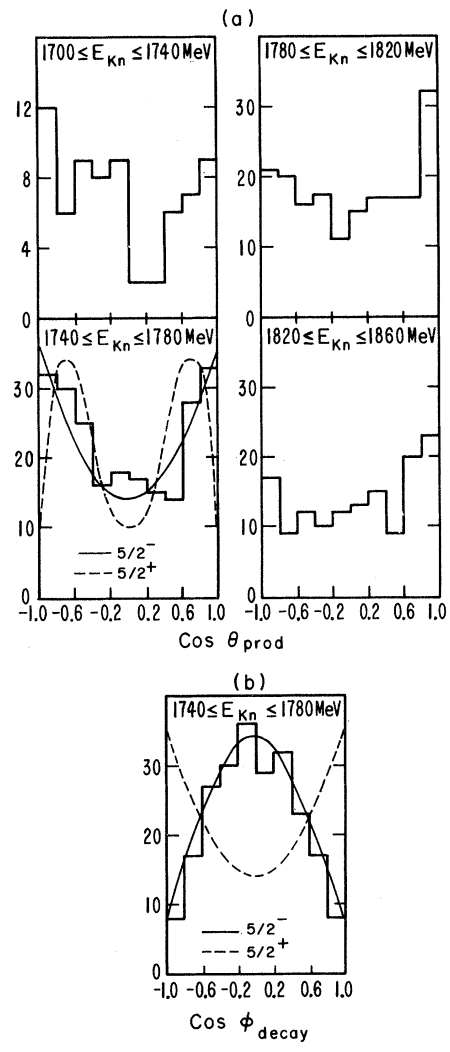

FIG. 3. (a) Production angular distributions for the $Y_0^*(1520)$. (b) Decay angular distribution of the $Y_0^*(1520)$ with respect to the production normal.

$Y_0^*(1520)+\pi^-$, we have an excellent means to determine its spin and parity. At these energies the nonresonating pion travels an average of 10 F during a $Y_0^*(1520)$ mean life; therefore, it is plausible to consider the channel to be dominated by the two-step process $K^-+n\to Y_0^*(1520)+\pi^-$ followed by the decay $Y_0^*(1520)\to\Sigma^-+\pi^+$.

Since the $Y_0^*(1520)$ has $J^P=\frac{3}{2}^-$, the reaction $K^-+n\to Y_0^*(1520)+\pi^-$ does not suffer from the Minami ambiguity associated with $0+\frac{1}{2}\to 0+\frac{1}{2}$ processes. Also, it allows a lower decay orbital angular momentum and thus a simpler decay distribution. Following arguments similar to those of Minami,[4] we observe the following: If the K^-n system forms a $Y_1^*(1765)$ resonance with a spin and parity of $\frac{5}{2}^-$, it can decay into $Y_0^*(1520)+\pi^-$ via a P- or F-wave orbital state. Since the higher orbital angular-momentum state is associated with a higher centrifugal barrier, decay via P wave is greatly favored. For such decay of the $Y_1^*(1765)$, the production angular distribution of the $Y_0^*(1520)\pi^-$ system is expected to be $1+2\cos^2\theta$ or $1+0.8P_2(\cos\theta)$, where $P_2(\cos\theta)$ is the Legendre polynomial of order two, and $\cos\theta=\hat{K}^-\cdot\hat{\pi}^-$.

Figure 3(a) shows the angular distribution of the $Y_0^*(1520)$ for events with total K^-n energies in the indicated intervals. As we have done in considering the production cross sections, the events from various K^- momenta have been summed and redivided according to the total c.m. energy of the constrained $Y_0^*(1520)\pi^-$ system.

We have fitted these angular distributions to the Legendre polynomial expansion $I=\sum_n A_nP_n(\cos\theta)$; the expansion coefficients are presented in Table I for various K^-n c.m. energy intervals. In the range 1760 ± 60 MeV, expansion to $P_2(\cos\theta)$ is both necessary and sufficient to fit the experimental data. For the particular choice $E=1760\pm 20$ MeV, χ^2 for a fit to $1+0.8P_2(\cos\theta)$ is 6.4 for nine degrees of freedom.

To see whether another spin and parity assignment of the $Y_1^*(1765)$ can give rise to a similar angular distribution and whether a reasonable background can explain the small deviation from the $1+0.8P_2(\cos\theta)$ distribution expected for a pure $\frac{5}{2}^-$ resonance decaying via pure P wave, we present in Table II the contributions of various partial-wave amplitudes, up to $J=\frac{5}{2}$. A thorough examination of Table II shows that only a dominant $(\frac{5}{2}^-P)$ partial wave with a small $(\frac{3}{2}^+S)$ background can yield angular distributions in good agreement with the observed data. No other reasonable combination of partial-wave amplitudes can yield a similar distribution. In particular, a pure resonance of spin and parity $\frac{5}{2}^+$ decaying via D wave would yield a distribution $1+10\cos^2\theta-10\cos^4\theta$. Fitting our data to this distribution gives $\chi^2=26.2$ for $E=1760\pm 20$ MeV. In fact, we have also checked the contribution from $J=\frac{7}{2}$ partial-wave amplitudes which is too cumbersome to be included in Table II. Again no other reasonable combination of partial-wave amplitudes can fit our experimental distribution.

We make another observation about the reaction $K^-+n\to\Sigma^-+\pi^++\pi^-$. If the $Y_1^*(1765)$ is $\frac{5}{2}^+$, both the $Y_0^*(1405)\pi^-$ and $Y_0^*(1520)\pi^-$ channels will decay by D wave. The larger Q value in the $Y_0^*(1405)\pi^-$ channel would favor it over the $Y_0^*(1520)\pi^-$ channel. However, if the $Y_1^*(1765)$ is $\frac{5}{2}^-$, it must decay into $Y_0^*(1405)+\pi^-$ by F wave, while it may decay into $Y_0^*(1520)+\pi^-$ by P wave. Centrifugal-barrier arguments would then favor $Y_0^*(1520)$ production, even though that channel has a lower Q value. Figure 1 shows dominant $Y_0^*(1520)$ production and suppressed $Y_0^*(1405)$ production, indicating again that the spin-par-

Table I. Legendre-polynomial expansion coefficients for the $Y_0^*(1520)$ production angular distributions, $I=\sum_n A_nP_n(\cos\theta)$, at various K^-n c.m. energies.

E_{Kn} range (MeV)	Coefficients A_0	A_1	A_2	A_3	A_4	A_5
1700 to 1740	1.00 ± 0.12	-0.22 ± 0.24	0.66 ± 0.32	0.11 ± 0.40	0.10 ± 0.42	-1.26 ± 0.51
1740 to 1780	1.00 ± 0.07	-0.08 ± 0.13	0.69 ± 0.16	0.26 ± 0.21	0.02 ± 0.24	0.09 ± 0.31
1780 to 1820	1.00 ± 0.07	-0.01 ± 0.14	0.63 ± 0.18	0.21 ± 0.23	0.12 ± 0.25	0.41 ± 0.33
1820 to 1860	1.00 ± 0.09	0.26 ± 0.16	0.50 ± 0.22	0.06 ± 0.26	0.47 ± 0.30	-0.16 ± 0.38

Table II. Partial-wave-amplitude contributions to the $Y_0^*(1520)$ production angular distribution $I=\sum A_N P_N(\cos\theta)$. (J^P, L) implies decay from a state of spin and parity J^P via L wave.

Partial amplitude term	$J^P L$	Interference terms	Coefficients A_0	A_1	A_2	A_3	A_4	A_5
1	$(\frac{1}{2}^- P)$		0.56					
2	$(\frac{1}{2}^+ D)$		0.56					
3	$(\frac{1}{2}^+ D)$		1.1					
4	$(\frac{3}{2}^+ S)$		1.1					
5	$(\frac{3}{2}^- P)$		1.1		−0.9			
6	$(\frac{3}{2}^- F)$		1.1		+0.9			
7	$(\frac{5}{2}^- P)$		1.7		1.4			
8	$(\frac{5}{2}^- F)$		1.7		1.1		−0.7	
9	$(\frac{5}{2}^+ D)$		1.7		0.7		−1.7	
10	$(\frac{5}{2}^+ G)$		1.7		1.7		0.93	
		(2, 1)		1.1				
		(3, 1)		−1.6				
		(3, 2)			−1.6			
		(4, 1)		1.6				
		(4, 2)			1.6			
		(4, 3)			−2.3			
		(5, 1)			−0.7			
		(5, 2)		−0.7				
		(5, 3)		1.0				
		(5, 4)		0.8		−1.8		
		(6, 1)			2.1			
		(6, 2)		2.1				
		(6, 3)				−3.0		
		(6, 4)		0.6		2.4		
		(6, 5)			−1.4			
		(7, 1)			−2.6			
		(7, 2)				−2.6		
		(7, 3)		3.7				
		(7, 4)		−0.74		−3.0		
		(7, 5)			1.7			
		(7, 6)			−0.24		−4.7	
		(8, 1)			2.1			
		(8, 2)				2.1		
		(8, 3)				−3.0		
		(8, 4)		3.6		−0.6		
		(8, 5)			1.5		−2.9	
		(8, 6)			1.2		+2.9	
		(8, 7)			−1.4		−3.6	
		(9, 1)				−1.3		
		(9, 1)			−1.3			
		(9, 3)			1.8			
		(9, 4)			1.3		−3.1	
		(9, 5)		3.4		−2.6		
		(9, 6)		−0.5		−1.9		
		(9, 7)		0.6		2.4		
		(9, 8)		0.55		1.8		−4.8
		(10, 1)				3.1		
		(10, 2)			3.1			
		(10, 3)					−4.4	
		(10, 4)			1.3		3.2	
		(10, 5)				−2.0		
		(10, 6)		4.0		2.0		
		(10, 7)				−0.5		−6.7
		(10, 8)		0.4		2.0		3.5
		(10, 9)			−1.0		−2.5	

Table III. Legendre-polynomial expansion coefficients for the $Y_0^*(1520)$ decay distributions in the energy range 1740 to 1780 MeV with respect to the production normal ($I=\sum_K A_K P_K(\cos\Phi)$, where $\cos\Phi=\hat{n}\cdot\hat{\pi}$ and $\hat{n}=K^-\times Y_0^*(1520)/|K^-\times Y_0^*(1520)|$.

Coefficient	Experimental value	Theoretical value $\frac{5}{2}^-$	$\frac{5}{2}^+$
A_0	1.00 ± 0.07	1.0	1.0
A_1	-0.03 ± 0.09	0	0
A_2	-0.91 ± 0.13	−0.7	0.78
A_3	0.06 ± 0.17	0	0
A_4	0.04 ± 0.21	0	0
A_5	-0.07 ± 0.29	0	0

ity of the $Y_1^*(1765)$ is $\frac{5}{2}^-$.

The decay distribution of the $Y_0^*(1520)$ allows a further check on the spin-parity assignment of the $Y_1^*(1765)$. For $J^P=\frac{5}{2}^+$, a distribution of $1+0.78P_2(\cos\Phi)$ is expected, while for $J^P=\frac{5}{2}^-$, a distribution of $1-0.70P_2(\cos\Phi)$ is predicted. Here we have $\cos\Phi=\hat{n}\cdot\hat{\pi}^+$ in the $Y_0^*(1520)$ c.m. system, and $\hat{n}$ is the production normal $\hat{n}=K^-\times Y_0^*(1520)/|K^-\times Y_0^*(1520)|$. In Fig. 3(b) we present our experimental data; Legendre-polynomial expansion coefficients are shown in Table III. For $E=1760\pm20$ MeV, fits to the theoretical distributions give $\chi^2(\frac{5}{2}^-)=2.6$ and $\chi^2(\frac{5}{2}^+)=242.1$ for nine degrees of freedom.

In conclusion, our data indicate the existence of the $Y_1^*(1765)$ hyperon resonance with $M=1760\pm10$ MeV, $\Gamma=60$ MeV, and the unambiguous spin-parity assignment $\frac{5}{2}^-$.

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QUANTUM NUMBERS AND MASSES OF MESONS AS QUARK-ANTIQUARK SYSTEMS

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The purpose of this note is (a) to make quantum-number assignments and relate the masses of mesons treated as a quark-antiquark ($q\bar{q}$) system, (b) to see if the SU(6) [or U(6)⊗U(6)] type mass splittings of the $J^P=0^-$, 1^- ($\underline{35}+\underline{1}$) mesons remain for the higher mesons, and (c) to relate mesons to the ($q\bar{q}$) Regge trajectories.

The implications of SU(6) for quark models and in regard to a quark mass $M_q\gtrsim10$ BeV have been discussed by Nambu,[1] Lipkin,[2] and others. Gell-Mann[3] has derived the orbital angular momentum $\vec{L}$ of quarks[4] from a current algebra supplementing the SU(6) with intrinsic quark parity, U(6)⊗U(6), with $O^L(3)$. [See also Mahanthappa and Sudarshan.[5]]

In Table I we relate the quantum numbers and masses of mesons according to the ($q\bar{q}$) system. The internal dynamics is taken nonrelativistic, though the essential features most likely remain valid[6] in a relativistic discussion. The assignments are quite unambiguous. For $B(1220)$, experimentally $J\geq1$, $P=?$, $G=+$. The ($q\bar{q}$) with $L=1$, $S=0$ gives $J^{PG}=1^{++}$ whereas $L=2$, $S=0^-$ would contradict G; $L=2$, $S=1^-$ would give too many unobserved nearby mesons with $J^P=3^-$, 2^-, 1^-. The $A_2(1324)$ is consistent with $L=1$, $S=1$ ($q\bar{q}$), but also with $S=0^+$, $L=2$, e.g., for $qq\bar{q}\bar{q}$. The mass changes in Table I are consistent with changes in L, S,