impossible to answer this question in the present adiabatic model, but application of the random-phase approximation, taking the vibrations into account dynamically, should shed light on the problem.

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TOTAL CROSS SECTION FOR 14-MeV NEUTRONS USING ALIGNED ^{165}Ho NUCLEI

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We have measured the effect of nuclear deformation on the total cross section of ^{165}Ho using 14-MeV neutrons and an aligned target. This is an extension of our previous work[1] done with neutrons of energy 0.35 MeV, in which the data were explained very nicely in terms of the nonadiabatic coupled-channel calculation (NACC).[2] Since the energy is high in the present case, we should be able to use the adiabatic coupled-channel calculation (ACC).[2] As is shown below, the present experimental data are indeed well explained by this calculation.

The aligned ^{165}Ho target was obtained by cooling a metal single crystal[3] to 0.33°K using the National Bureau of Standards ^{3}He refrigerator.[4] The atomic moments in this temperature region are canted out of the basal plane of the hcp lattice by a small angle (~10°) and form a periodic spiral spin structure.[5] Owing to the large hyperfine interaction a high degree of nuclear alignment[6] ($f_2 = 0.31$ for 0.33°K) is obtained for each group of nuclei whose atomic moments lie along a common axis. There are perhaps 12 of these axes lying essentially in the basal plane. This degeneracy (of having more than one alignment axis) can be removed by lining up the atomic moments with a magnetic field. In the presence of a field we have not only an aligned target, but a polarized target as well. However, in the present experiment we require only nuclear alignment. Since we restrict ourselves to a total cross-section measurement, we do not have to remove the degeneracy of having many alignment axes in the basal plane as long as this plane is perpendicular to the beam direction.[1]

The ^{165}Ho single crystal used in these measurements, although rather large for a rare-earth metal crystal, was nonetheless small when considered as a nuclear target for 14-MeV neutrons. The available area of the crystal (~1 cm^2) and its thickness (1.08 cm) put stringent requirements on the source of 14-MeV neutrons used; namely, a well collimated small beam with inherent high counting stability. The last requirement was needed because the change in transmission due to nuclear alignment was expected to be rather small. A finely collimated beam of 14-MeV neutrons was obtained by careful collimation of the alpha particle produced in the reaction ^{3}H$(d,n)^4$He and by detecting it in fast coincidence with its associated neutron. The coincidence alpha pulse provides the accurate and stable neutron normalization required. The National Bureau of Standards 2-MV Van de Graaff was used to provide a 1-μA, 300-keV deuteron beam. Thin Ti-T targets (175 μg/cm^2) were used as the neutron source.

The total cross section of unoriented ^{165}Ho was measured using two polycrystalline samples, with the same beam and detector conditions as those used for the cryogenic target measurements. This geometry was not optimum for a transmission measurement (the in-scattering corrections were rather large), but was tolerated since we wanted to make these measurements under the same conditions as those needed for the aligned-target ones. The in-scattering correction was made using a the-

Table I. Results of the transmission measurements on unoriented ^{165}Ho.[a]

Measurement No.	Transmission[b] $t = 1.259$ cm	$t = 3.340$ cm
1	0.8289 ± 0.0011	
2		0.6198 ± 0.0009
3	0.8324 ± 0.0009	
4	0.8330 ± 0.0010	
5		0.6196 ± 0.0011
6	0.8310 ± 0.0012	
Weighted mean	0.8313 ± 0.0005	0.6197 ± 0.0007
σ_t (b)[c]	5.26 ± 0.08	5.26 ± 0.09
	$\bar{\sigma}_t = 5.26 \pm 0.06$ b	

[a]These measurements were carried out at room temperature using polycrystalline samples and under the same beam and detector conditions as those used for the cryogenic target.

[b]These data have been corrected for background only. The errors quoted are the standard deviation of the mean.

[c]These results have been corrected for in-scattering.

oretical angular distribution generated by ACC[2] and using the optical-model parameters [Eq. (4) below] that fitted the total cross section and the deformation effect. The results of these transmission measurements are shown in Table I. Each transmission measurement was made up of 41 consecutive 10-minute runs with the sample alternately in and out of the beam. The data showed no systematic trends and exhibited the 0.1% reproducibility from measurement to measurement consistent with the total number of counts recorded for each measurement, which was typically 10^6.

The transmission of the ^{165}Ho single crystal at 0.33°K ($f_2 = 0.31$) and at 4.2°K ($f_2 = 0.005$) was measured in a fashion similar to the polycrystalline room-temperature measurements described above. 10 such transmission measurements were made; four at 4.2°K, four at 0.33°K, and two additional measurements at 0.33°K with reduced ^{3}He level. The results of these measurements are shown in Fig. 1. Calculation made of the level of ^{3}He in the sample tube during normal running of the refrigerator indicated that it was more than a millimeter below the bottom of the beam. Although this is an adequate safety factor (the beam-crystal alignment was ±0.5 mm, it was felt that a further check was desirable. The refrigerator was operated with only $\frac{2}{3}$ of its normal charge of ^{3}He in the system. The results of these two measurements with reduced ^{3}He (see Fig. 1) agreed well with the other four 0.33°K measurements. The fractional change in the total cross section, $\Delta\sigma_t/\sigma_t$, due to nuclear alignment is given by

$$\frac{\Delta\sigma_t}{\sigma_t} = \frac{\sigma_{t,a} - \sigma_t}{\sigma_t} = \frac{\ln(T/T_a)}{\ln(1/T) - (Nt\sigma)_{\mathrm{Cu}}}, \tag{1}$$

where $\sigma_{t,a}$ and T_a are the total cross-section and transmission ratio, respectively, for the aligned target; T is the unaligned transmission ratio; N, t, and σ are the nuclear density, thickness, and 14-MeV total-neutron cross section, respectively, for two thin copper plates used to hold the ^{165}Ho single crystal in its mounting basket in the cryostat. The measured transmission T and T_a were corrected for in-scattering. The angular distribution for the aligned target was calculated using our value of f_2 and the same optical-model parameters as those used for the unaligned angular distribution. This correction increases the effect by a small amount (4%). The resulting value of $\Delta\sigma_t/\sigma_t$ is $+(3.48 \pm 0.75)$%. Using the value for σ_t given in Table I, we obtain (for our value of nuclear alignment)

$$\Delta\sigma_t = +183 \pm 39 \text{ mb}, \tag{2}$$

where the positive sign indicates a larger cross section for nuclei aligned perpendicular to the incident beam than for randomly oriented nuclei. If we ignore the contribution of the higher order nuclear alignment parameters (f_4

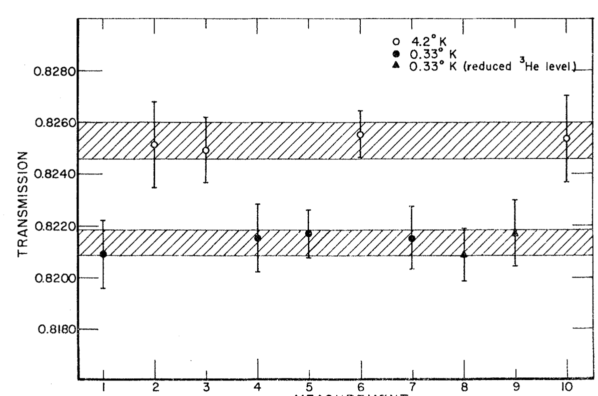

FIG. 1. Transmission data for the ^{165}Ho single crystal. The errors shown here are the standard deviation of the mean. The shaded areas represent the standard deviations of the weighted averages of the measurements at 0.33 and 4.2°K.

and f_0), we would predict a value of 340 ± 70 mb for complete nuclear alignment (the $m = \pm\frac{7}{2}$ is the only magnetic substate populated in this case) perpendicular to the beam. For the case of complete alignment parallel to the beam, we would predict a decrease in the cross section of about twice this amount (~680 mb). Thus our result is consistent with the recent experimental data of Shelley et al.[7]

In order to check that the effect we observe is really due to nuclear alignment and not to some systematic error (e.g., if holmium metal had an abnormally large volume-expansion coefficient in the 4.2 to 0.33°K temperature region, we would observe an apparent change in the cross section when in reality the nuclear density was changing), we made a series of transmission measurements on a "dummy" holmium sample. This sample was cut out of a polycrystalline rod of holmium metal into the same shape and mounted in the cryostat in the same way as the single crystal. There can, of course, be no net nuclear alignment at any temperature in a polycrystalline sample. Transmission measurements were made at 4.2 and 0.33°K in exactly the same way as those made on the single crystal, and the results (see Table II) indicate the absence of any systematic errors in our measured alignment effect.

Table II. Results of the transmission measurements on polycrystalline ^{165}Ho.[a]

Measurement No.	Transmission[b] $T = 0.33$°K ($f_2 = 0.00$)	$T = 4.2$°K ($f_2 = 0.00$)
11	0.8229 ± 0.0010	
12		0.8225 ± 0.0014
13	0.8243 ± 0.0008	
14	0.8211 ± 0.0012	
15		0.8242 ± 0.0011
Weighted mean	0.8231 ± 0.0006	0.8235 ± 0.0009

[a]This sample was nearly identical in shape and size to the single-crystal sample.

[b]The errors quoted are the standard deviation of the mean.

By using the asymptotic form of the scattered wave in ACC [Eq. (69) of Ref. 2], one can easily write down the theoretical expression for the total cross section σ_t for any orientation of the target (and the polarization, if any, of the projectile):

$$\sigma_t = (4\pi/k_1^2)\sum_{ii'}\sum_{m_s m_s' M_1 M_1'}\sum_{ljl'j'\bar{m}_j}(2l'+1)(l\tfrac{1}{2}0m_s|jm_s)(l'\tfrac{1}{2}0m_s'|j'm_s')$$
$$\times\sum(jj'm_s-m_s'|JM_J)(jj'\bar{m}_j-\bar{m}_j|J0)(I_1JM_1M_J|I_1M_1')(I_1JK0|I_1K)$$
$$\times(-)^{m_s'-\bar{m}_j}\,\mathrm{Im}[C_{lj;l'j'}^{\bar{m}_j}a_{m_s}^{(i)}a_{m_s'}^{(i)*}b_{M_1}^{(i')}b_{M_1'}^{(i')*}]. \tag{3}$$

The meaning of all the notations that appear in (3) can be found in Sec. V of Ref. 2. The scattering coefficients $C_{lj;l'j'}^{\bar{m}_j}$ are obtained by solving the coupled equation in ACC [Eq. (68) of Ref. 2]. The optical-model parameters [Eqs. (1) and (3) of Ref. 2] used are as follows:

$$V=46,\quad W=0,\quad W_D=7.5,\quad V_{s0}=7.5$$
$$\text{(all in MeV);}$$
$$r_0=\bar{r}_0=1.25,\quad a=0.65,\quad \bar{a}=0.47 \text{ (all in fm);}$$
$$\beta=0.3. \tag{4}$$

With these parameters we get

$$\sigma_t(\text{unoriented}) = 5.296 \text{ b and } \Delta\sigma_t = +176 \text{ mb}, \tag{5}$$

which are in complete agreement with our experiment (within the experimental error).

Since the optical-model parameters in (4) are consistent with the elastic-scattering analysis[8] of 14-MeV neutrons by various targets and with the analysis[9] of the scattering of 17.5-MeV protons by ^{165}Ho, the good agreement obtained here allows us to conclude that the present results can be explained by the coupled-channel calculation. It should be noted that the value 0.3 for the deformation parameter β can be used here as well as for 0.35-MeV neutrons.[1]

The corresponding value of $\Delta\sigma_t/\sigma_t$ for a black nucleus[10] was calculated to be 2.13%, which is much smaller than that observed (and predicted by ACC); therefore, it is clear that 14 MeV is not yet a sufficiently high energy to allow the use of the black-nucleus model, a conclusion that is not unexpected.

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NEW NUCLIDIC MASS RELATIONSHIP*

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A great deal of work has been done in trying to obtain a formula that will give the masses of all nuclides in the periodic table. These formulas range from the semiempirical type, which are based on a physical picture of the nucleus and involve a small number of parameters, to phenomenological varieties which employ a large number of parameters.[1] Such a formula, based usually on the masses of known nuclides, may be used to predict the particle stability of undiscovered isotopes. A mass formula, however, will in general be too approximate to make such predictions to a sufficient degree of accuracy. Away from the valley of stability, the inaccuracies in the simpler formulas are of orders of a few MeV whereas the usual binding energy against neutron emission for extremely neutron-rich nuclei would be of this magnitude. It is possible to develop certain simple relations between nuclidic masses,[1-3] which will be valid, independently of the actual variation of mass with atomic number and charge. Such relations will certainly not replace a mass formula, but may be very useful because of their simplicity and greater accuracy in making ground-state mass predictions.

It is the purpose of this communication to present such a relation, based on an independent-particle model of the nucleus, and to show the agreement of this relationship between the known relevant nuclidic masses. Using this relationship, certain definite statements may then be made about the stability against nucleon emission for several light nuclei of current interest.[4,5]

The interaction energy of a system of nucleons is composed of an electrostatic term and a nuclear term. Adopting the independent-particle picture, nucleons are described as moving in a self-consistent single-particle field. Each single-particle level is fourfold degenerate, corresponding to the operations of time-reversal and isospin conjugation. A proton and a neutron with spins up and down may therefore occupy each level. The bulk of the two-body nuclear interaction is absorbed into the single-particle Hartree-Fock energies, and the residual interaction will be primarily between nucleons in the same level. No assumptions are made about the quantitative aspects