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[7]This division in Hilbert space takes the place of the configuration-space boundary of the Wigner-Eisenbud R-matrix theory, and has the advantage that it does not disturb the optical potential scattering.

[8]It follows immediately that if there is present an optical potential which produces by itself a phase shift δ_0, then the total phase shift is $\delta_0+\delta$, where δ is given by Eq. (10). The resulting cross section will then have an over-all non-Lorentzian interference shape, as well as fine structure.

[9]The analog states studied by P. Richard, C. F. Moore, D. Robson, and J. D. Fox [Phys. Rev. Letters 13, 343a (1964); see also D. Robson, Phys. Rev. 137, B535 (1964)], seem to fall in this category as the hall-way-state theory would not predict the observed disappearance of fine structure on the high-energy side of the resonance.

$I^2(I+1)^2$ CORRECTION TO NUCLEAR ROTATIONAL SPECTRA*

Eugene R. Marshalek

Department of Physics, Brookhaven National Laboratory, Upton, New York, and Department of Physics, University of Notre Dame, Notre Dame, Indiana†

and

John B. Milazzo

Applied Mathematics Department, Brookhaven National Laboratory, Upton, New York, and Technical Research Group, Melville, New York†

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The energies of members of the ground-state rotational band of even-even deformed nuclei can be represented by the expansion

$$E_I = \tfrac{1}{2}(\hbar^2/\mathcal{I})I(I+1) + \mathcal{B}I^2(I+1)^2 + \cdots \quad (1)$$

for not too large values of the angular-momentum quantum number I. Formulas were derived earlier by one of the authors[1] for the moment of inertia $\mathcal{I}$ and the coefficient $\mathcal{B}$ by considering rotating solutions of the Hartree-Fock-Bogoliubov equations. Under the model assumption that nucleons interact through pairing and quadrupole forces, the moment of inertia $\mathcal{I}$ was shown to agree with the formula of Beliaev,[2] which has previously been numerically evaluated by Nilsson and Prior.[3] In the present paper numerical computations of the $\mathcal{B}$ coefficient are presented for the first time. This coefficient may be decomposed as follows:

$$\mathcal{B} = \mathcal{B}_{\text{Coriolis}} + \mathcal{B}_\Delta + \mathcal{B}_\lambda + \mathcal{B}_\beta + \mathcal{B}_\gamma. \quad (2)$$

The rather complicated expressions for these terms will not be reproduced here, since they are defined by Eqs. (29), (61), (68), and (74) of Ref. 1. The term $\mathcal{B}_{\text{Coriolis}}$ alone can be derived from a fourth-order perturbation treatment of the cranking model applied to independent quasiparticles, with both the self-consistent field and gap parameters Δ_n and Δ_p held fixed. The term $\mathcal{B}_\Delta$ arises from the weakening of the pair correlation, i.e., to a decrease in the neutron and proton gap parameters Δ_n and Δ_p due to rotation, with the self-consistent field held fixed (Mottelson-Valatin effect at low rotational frequencies[4]). The term $\mathcal{B}_\lambda$ is a small contribution which arises from a readjustment of the Lagrange multipliers λ_n and λ_p necessary to keep the numbers of neutrons and protons fixed in the presence of the Coriolis perturbation. The terms $\mathcal{B}_\beta$ and $\mathcal{B}_\gamma$ arise from the centrifugal stretching of the self-consistent field and include changes in Δ_n and Δ_p due to changes in the nuclear quadrupole moments. One may interpret $\mathcal{B}_\beta$ as corresponding to mixing of the ground-state band with the β-vibrational band and $\mathcal{B}_\gamma$ as corresponding to mixing with the γ-vibrational band.

The expressions corresponding to Eq. (2) were evaluated on the IBM 7094 computer using a code developed by one of the authors (J.B.M.). In the calculations, the average field in the absence of rotation was furnished by the Nilsson potential.[5] All the Nilsson single-particle levels from oscillator quantum number $N=0$ through $N=7$ were used, with modifications recommended by Mottelson and Nilsson.[6]

It was found convenient to use the corresponding single-particle eigenfunctions computed by Rassey in cylindrical coordinates.[7,8] The deformation of the Nilsson potential was measured by the parameter $\beta = \kappa(m\omega_0^2)^{-1}\langle\sum r^2 Y_{20}\rangle$, i.e., a parameter proportional to the total mass quadrupole moment of neutrons and protons, which was chosen so as to exactly reproduce the experimental electric quadrupole moments. The strength of the quadrupole-quadrupole force, κ, which was assumed to be the same for neutron-neutron, proton-proton, and neutron-proton interactions, was chosen so as to guarantee Hartree self-consistency at the chosen deformation β. The parameter κ may be written as $\kappa = k4\pi m\omega_0^2(3AR_0^2)^{-1}$. Simple estimates, some based on the pure harmonic-oscillator potential, give $k = 1$ and $\hbar\omega_0 \approx 41A^{-1/3}$ MeV, if the nuclear rms radius R_0 is chosen to have the experimental value.[9-11] The actual values of k which were used ranged from 0.77 to 0.87 in the rare-earth region and from 0.82 to 0.87 in the actinide region, using the above estimate for $\hbar\omega_0$.

The strengths of the neutron-neutron and proton-proton pairing forces, G_n and G_p, respectively, were chosen so as to exactly reproduce the values of Δ_n and Δ_p extracted from experimental data by Nilsson and Prior.[3] As usual, a cutoff on the pairing force was used. It was assumed that the pairing force scatters nucleons in single-particle states lying within a region of width $\hbar\omega_0$ symmetric about the chemical potentials. This gave the values in the rare-earth region $G_nA = 20$-22 MeV and $G_pA = 27$-29 MeV, while in the actinide region $G_nA = 19$-21 MeV and $G_pA = 24$-27 MeV were obtained.

The stability of the self-consistent solutions was checked by computing the force constant for β oscillations, C_β [see Eq. (78a) of Ref. 1]. If $C_\beta < 0$, the stationary solutions are unstable, while if $C_\beta > 0$, the solutions are stable.[12] Unfortunately, in the rare-earth region instability was obtained for Sm^{152}, Sm^{154}, Gd^{154}, and Gd^{156}, while Gd^{158} was near the bound of instability. All other rare earths were stable. In the actinide region $C_\beta < 0$ only for Th^{226}, which may be a transitional nucleus. At the beginning of a region of deformed nuclei, C_β is small and its calculation, which involves the difference of two large numbers, is very sensitive to the choice of parameters and single-particle potential. The instabilities may also reflect the inadequacy of the quadrupole and pairing forces.

The results of the calculation of the coefficient $\mathcal{B}$ are presented in Table I. The corresponding moments of inertia have also been recalculated and generally agree well with those of Nilsson and Prior.[3] The small differences are accounted for by the differences in the details of the calculations, for example, different choices of β.

The total theoretical $\mathcal{B}$ values in the rare-earth region exceed the experimental values by factors of 2-4. However, the situation is not really as bad as it would appear. Since the theoretical values of $\hbar^2/2\mathcal{I}$ generally exceed the experimental ones, the fact that the theoretical $\mathcal{B}$ coefficients are too large in magnitude and negative in sign means that the rotational energies calculated from Eq. (1) will be in better agreement with experiment than if the calculated $\mathcal{B}$ coefficients were smaller. Also, the theoretical expression for $\mathcal{B}$ is proportional to $(\hbar^2/2\mathcal{I})^4$, so any change in the choice of parameters, like the gap parameters, which brings the moment of inertia into line with experiment is likely to reduce the $\mathcal{B}$ coefficient in magnitude. However, for Hf^{176}, the theoretical and experimental moments of inertia nearly coincide, but the theoretical $\mathcal{B}$ value is still about twice the experimental one in magnitude.

In the actinide region, the calculated values of $\hbar^2/2\mathcal{I}$ are only slightly larger than the experimental ones, and the calculated $\mathcal{B}$ coefficients are considerably closer to the experimental ones.

The most interesting result of the calculations is that most of the contribution to $\mathcal{B}$ is due, not to the vibration-rotation interaction terms $\mathcal{B}_\beta$ and $\mathcal{B}_\gamma$, but to $\mathcal{B}_{\text{Coriolis}}$ and $\mathcal{B}_\Delta$. A possible interpretation of the latter terms is that they correspond to an additional term in the nuclear Hamiltonian like $(\mathcal{B}_{\text{Coriolis}} + \mathcal{B}_\Delta) \times (\hat{I}^2 - \hat{I}_3^2)^2$, which would provide an $I^2(I+1)^2$ contribution to the energy without, however, mixing physical bands.[13] In most phenomenological treatments of rotational motion, it has been assumed that vibration-rotation band mixing (centrifugal distortion), which seems to correspond to the terms $\mathcal{B}_\beta$ and $\mathcal{B}_\gamma$ in the present model, account for all of the deviation from the $I(I+1)$ rule in even-even nuclei. Actually, recent analyses of experimental branching ratios, assuming band mixing, indicate that $\mathcal{B}_\gamma$ and $\mathcal{B}_\beta$ account for only a small fraction ($\lesssim 10\%$) of the total $\mathcal{B}$ coefficient,[16,17] in qualitative

Table I. Theoretical moments of inertia and $\mathcal{B}$ coefficients. The total theoretical $\mathcal{B}$ coefficient is the sum $\mathcal{B} = \mathcal{B}_{\text{Coriolis}} + \mathcal{B}_\Delta + \mathcal{B}_\lambda + \mathcal{B}_\beta + \mathcal{B}_\gamma$. The experimental $\mathcal{B}$ coefficients are obtained from the ground-state band using at least the 2^+, 4^+, and 6^+ states. The values of $\mathcal{B}_\beta$ are not given for the cases with unstable self-consistent solutions.

Nucleus	$\hbar^2/2\mathcal{I}$ (keV) Theory	$\hbar^2/2\mathcal{I}$ (keV) Experiment	Contributions to $-\mathcal{B}$, theory (eV) $-\mathcal{B}_{\text{Coriolis}}$	$-\mathcal{B}_\Delta$	$-\mathcal{B}_\lambda$	$-\mathcal{B}_\beta$	$-\mathcal{B}_\gamma$	$-\mathcal{B}$	$-\mathcal{B}$, Experiment (eV)
Sm^{152}	30.8	21.4	77.8	47.8	−0.7	unstable	4.1		193
Sm^{154}	21.6	13.9	33.0	26.8	−0.01	unstable	1.0		37
Gd^{154}	31.9	21.6	86.5	51.1	−1.0	unstable	5.4		193
Gd^{156}	21.5	15.0	35.0	27.8	−0.1	unstable	1.3		28.8
Gd^{158}	17.8	13.3	18.2	16.9	−0.1	barely stable	0.6		14.2
Gd^{160}	16.2	12.7	16.9	15.6	−0.2	6.9	0.5	39.7	19
Dy^{160}	20.2	14.6	28.1	27.4	−0.1	24.9	1.4	81.7	24.7
Dy^{162}	17.6	13.5	19.9	20.8	−0.2	6.8	0.8	48.1	10.8
Dy^{164}	15.8	12.3	16.4	19.9	−0.03	2.1	0.4	38.7	9.82
Er^{164}	17.8	15.4	20.8	23.4	−0.2	9.7	0.7	54.5	
Er^{166}	15.9	13.5	17.1	21.4	−0.003	3.3	0.4	42.1	14.5
Er^{168}	15.3	13.3	11.0	17.7	−0.02	3.6	0.2	32.5	5.6
Er^{170}	14.7	13.4	11.5	18.6	−0.004	6.1	0.15	36.3	
Yb^{170}	15.5	14.0	13.3	19.5	−0.1	5.8	0.2	38.6	
Yb^{172}	14.4	13.2	11.2	17.7	−0.06	6.4	0.1	35.3	8.93
Yb^{174}	14.1	12.8	12.7	17.1	−0.3	2.4	0.1	32.1	
Hf^{176}	15.0	14.8	16.8	20.8	−0.5	4.8	0.2	42.0	18.5
Hf^{178}	16.2	15.6	17.8	24.6	−1.0	3.0	0.3	44.7	12.9
Hf^{180}	18.5	15.6	18.2	27.6	−0.8	6.8	0.7	52.4	10.5
W^{182}	18.7	16.8	18.2	31.5	−0.5	6.9	0.9	57.0	15.3
W^{184}	20.6	18.7	35.2	31.1	−2.2	8.5	1.9	74.6	28.3
W^{186}	24.2	20.7	53.2	30.1	−4.7	14.2	3.6	96.3	
Th^{226}	9.61	12.3	9.20	3.37	−0.92	unstable	0.07		
Th^{228}	8.66	9.70	5.79	2.96	−0.50	8.40	0.04	16.69	20
Th^{230}	7.84	8.87	3.95	2.92	−0.21	1.20	0.03	7.88	
Th^{232}	7.08	8.36	2.64	2.97	−0.03	0.31	0.02	5.91	8.8
Th^{234}	6.41	8.00	1.98	2.89	−0.006	0.18	0.02	5.06	
U^{232}	7.11	7.98	3.00	2.59	−0.08	0.62	0.02	6.15	7.0
U^{234}	6.37	7.29	2.23	3.08	−0.01	0.11	0.01	5.42	6.97
U^{236}	6.29	7.59	1.78	2.53	−0.002	0.03	0.01	4.35	6.9
U^{238}	6.11	7.50	1.98	2.44	−0.01	0.02	0.01	4.43	6.2
Pu^{238}	5.71	7.38	1.49	2.18	−0.003	0.006	0.01	3.68	4.1
Pu^{240}	5.63	7.18	1.68	2.05	−0.015	0.004	0.008	3.73	5.6
Cm^{244}	5.88	7.16	1.22	1.55	−0.03	0.01	0.004	2.75	2.1

agreement with the present calculations. However, our calculated values of $\mathcal{B}_\gamma$ are an order of magnitude smaller than the not-too-accurate experimental values in most cases.

It is interesting to note that the ordinary cranking model, which neglects the changes in the gap parameters and self-consistent field, provides only the contribution $\mathcal{B}_{\text{Coriolis}}$, which agrees fairly well with the experimental values of $\mathcal{B}$ in the rare-earth region. However, the additional term $\mathcal{B}_\Delta$ provided by the more general theory spoils the agreement.

It is of some interest to consider the derivatives $d\mathcal{I}/d\beta$ and $d\mathcal{I}/d\gamma$ defined in Ref. 1, which enter into the calculations of $\mathcal{B}_\beta$ and $\mathcal{B}_\gamma$, respectively. In Table II, there are presented theoretical values of $(\beta/2\mathcal{I})d\mathcal{I}/d\beta$ and $(\sqrt{3}/2\mathcal{I}) \times d\mathcal{I}/d\gamma$. If the moment of inertia were to depend on deformation through the relation $\mathcal{I} \propto \beta^2 \sin^2(\gamma - 2\pi/3)$ as is assumed in many phenomenological discussions of vibration-rotation interaction, then the above ratios (evaluated at the equilibrium deformation) would be unity. However, the calculated microscopic quantities are in most cases only 0.5 to 0.25. Since $\mathcal{B}_\beta$ and $\mathcal{B}_\gamma$ depend on the squares of the

derivatives, it is not surprising that our estimates of the vibration-rotation interaction are significantly smaller than phenomenological estimates.

The present calculations are only tentative. The possibility of other choices of parameters, and the effect of including the contribution of the pairing force to the self-consistent field, which was omitted in the calculations (but were included in the derivations in Ref. 1), are being studied. Finally, a more general application of the time-dependent Hartree-Fock-Bogoliubov theory, including the collective vibrations dynamically, is being applied to the problem by one of the authors.

Table II. Theoretical derivatives of the moment of inertia with respect to deformation. The derivative with respect to β is taken with γ held constant, but with Δ_n and Δ_p and also λ_n and λ_p changing with β. The derivative with respect to γ is taken with all other parameters held constant. The parameters β and γ are defined by $Q_{20} = m\omega_0^2\kappa^{-1}\beta\cos\gamma$ and $Q_{22} = Q_{2-2} = 2^{-1/2}m\omega_0^2\kappa^{-1}\beta\sin\gamma$, where the $Q_{2\mu}$ are the total neutron and proton mass quadrupole moments. For the case $\vartheta \propto \beta^2\sin^2(\gamma-2\pi/3)$, $(\beta/2\vartheta)d\vartheta/d\beta = (\sqrt{3}/2\vartheta)d\vartheta/d\gamma = 1$, if the derivatives are evaluated with $\gamma = 0$ (axial symmetry). The calculated derivatives are independent of the parameter κ.

Nucleus	β	$\frac{\beta}{2\vartheta}\frac{d\vartheta}{d\beta}$	$\frac{\sqrt{3}}{2\vartheta}\frac{d\vartheta}{d\gamma}$
Sm^{152}	0.237	1.145	0.623
Sm^{154}	0.274	0.862	0.548
Gd^{154}	0.228	1.097	0.653
Gd^{156}	0.262	0.800	0.588
Gd^{158}	0.285	0.574	0.558
Gd^{160}	0.294	0.458	0.539
Dy^{160}	0.250	0.674	0.604
Dy^{162}	0.269	0.547	0.570
Dy^{164}	0.282	0.403	0.500
Er^{164}	0.261	0.634	0.538
Er^{166}	0.276	0.489	0.465
Er^{168}	0.276	0.566	0.372
Er^{170}	0.268	0.767	0.322
Yb^{170}	0.268	0.647	0.339
Yb^{172}	0.276	0.740	0.268
Yb^{174}	0.275	0.502	0.290
Hf^{176}	0.260	0.594	0.312
Hf^{178}	0.244	0.465	0.369
Hf^{180}	0.234	0.568	0.448
W^{182}	0.230	0.534	0.470
W^{184}	0.219	0.501	0.561
W^{186}	0.216	0.489	0.608
Th^{226}	0.202	0.680	0.234
Th^{228}	0.205	0.538	0.217
Th^{230}	0.210	0.434	0.218
Th^{232}	0.219	0.316	0.231
Th^{234}	0.211	0.306	0.222
U^{232}	0.226	0.410	0.222
U^{234}	0.221	0.249	0.212
U^{236}	0.232	0.155	0.228
U^{238}	0.234	0.137	0.218
Pu^{238}	0.237	0.083	0.222
Pu^{240}	0.242	0.069	0.207
Cm^{244}	0.280	−0.143[a]	0.173

[a]The fact that $d\vartheta/d\beta$ is negative gives the anomalous result that β decreases with rotation. This result may be a consequence of the gap in the single-particle spectrum at neutron number 152. It also is possible that if oscillator shells beyond $N = 7$ are included, one would obtain a small positive derivative.

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†Present addresses.

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impossible to answer this question in the present adiabatic model, but application of the random-phase approximation, taking the vibrations into account dynamically, should shed light on the problem.

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TOTAL CROSS SECTION FOR 14-MeV NEUTRONS USING ALIGNED ^{165}Ho NUCLEI

H. Marshak and A. C. B. Richardson*

National Bureau of Standards, Washington, D. C.

and

T. Tamura†

Oak Ridge National Laboratory, Oak Ridge, Tennessee

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We have measured the effect of nuclear deformation on the total cross section of ^{165}Ho using 14-MeV neutrons and an aligned target. This is an extension of our previous work[1] done with neutrons of energy 0.35 MeV, in which the data were explained very nicely in terms of the nonadiabatic coupled-channel calculation (NACC).[2] Since the energy is high in the present case, we should be able to use the adiabatic coupled-channel calculation (ACC).[2] As is shown below, the present experimental data are indeed well explained by this calculation.

The aligned ^{165}Ho target was obtained by cooling a metal single crystal[3] to 0.33°K using the National Bureau of Standards ^{3}He refrigerator.[4] The atomic moments in this temperature region are canted out of the basal plane of the hcp lattice by a small angle (~10°) and form a periodic spiral spin structure.[5] Owing to the large hyperfine interaction a high degree of nuclear alignment[6] ($f_2 = 0.31$ for 0.33°K) is obtained for each group of nuclei whose atomic moments lie along a common axis. There are perhaps 12 of these axes lying essentially in the basal plane. This degeneracy (of having more than one alignment axis) can be removed by lining up the atomic moments with a magnetic field. In the presence of a field we have not only an aligned target, but a polarized target as well. However, in the present experiment we require only nuclear alignment. Since we restrict ourselves to a total cross-section measurement, we do not have to remove the degeneracy of having many alignment axes in the basal plane as long as this plane is perpendicular to the beam direction.[1]

The ^{165}Ho single crystal used in these measurements, although rather large for a rare-earth metal crystal, was nonetheless small when considered as a nuclear target for 14-MeV neutrons. The available area of the crystal (~1 cm^2) and its thickness (1.08 cm) put stringent requirements on the source of 14-MeV neutrons used; namely, a well collimated small beam with inherent high counting stability. The last requirement was needed because the change in transmission due to nuclear alignment was expected to be rather small. A finely collimated beam of 14-MeV neutrons was obtained by careful collimation of the alpha particle produced in the reaction ^{3}H$(d,n)^4$He and by detecting it in fast coincidence with its associated neutron. The coincidence alpha pulse provides the accurate and stable neutron normalization required. The National Bureau of Standards 2-MV Van de Graaff was used to provide a 1-μA, 300-keV deuteron beam. Thin Ti-T targets (175 μg/cm^2) were used as the neutron source.

The total cross section of unoriented ^{165}Ho was measured using two polycrystalline samples, with the same beam and detector conditions as those used for the cryogenic target measurements. This geometry was not optimum for a transmission measurement (the in-scattering corrections were rather large), but was tolerated since we wanted to make these measurements under the same conditions as those needed for the aligned-target ones. The in-scattering correction was made using a the-