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[12]If all $|j\rangle$ represent a complete orthogonal set of final states and $\mathfrak{M}$ is the transition matrix, then we have

$$\Gamma_j^S = |\langle j|\mathfrak{M}|K_S\rangle|^2,$$

$$\Gamma_j^L = |\langle j|\mathfrak{M}|K_L\rangle|^2,$$

and

$$(\tau_S^{-1}/2+\tau_L^{-1}/2-i\Delta m)(p_S^*p_L-q_S^*q_L)$$

$$=\zeta=\sum_j\langle j|\mathfrak{M}|K_S\rangle^*\langle j|\mathfrak{M}|K_L\rangle=\sum_j(\Gamma_j^S\Gamma_j^L)^{1/2}\exp(i\varphi_j),$$

where φ_j is the phase difference between $\langle j|\mathfrak{M}|K_L\rangle$ and $\langle j|\mathfrak{M}|K_S\rangle$. Therefore we can write

$$|\zeta|\le\sum_j(\Gamma_j^S\Gamma_j^L)^{1/2}.$$

If j is used to describe several states of decay of the orthogonal set, the above inequality is still valid as a result of the Schwarz inequality.

[13]Inequality (7) is weaker than inequality (5); it has already been reported in Ref. 2.

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[16]When we sum up all known decay rates of $|K_L\rangle$, also taking into account measurements of some of the ratios of the decay rates, and compare the result with the value of τ_L^{-1} determined by exponential decay, the experimental accuracies allow us to write only the inequality

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EQUAL-TIME COMMUTATORS AND K-MESON DECAYS*

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The impressive, quantitative account given by Adler[1] and Weisberger[2] of axial-vector renormalization effects in β decay has stimulated a search for further tests of the underlying hypothesis: notably, the weak-interaction current algebra proposed by Gell-Mann[3] and the notion of a partially conserved axial-vector current (PCAC). In the present note, we report some additional applications of these ideas to leptonic decays of K and π mesons. We also discuss briefly the implications of a certain speculative generalization of the Gell-Mann equal-time commutation relations.

In the first instance, we are concerned with the commutation relations between weak currents and are led, via PCAC, to equations connecting the form factors which arise in the three leptonic decay modes for K mesons, K_{l2}, K_{l3}, K_{l4}. Similar relations connect π_{l2} and π_{l3}. In all of these cases, one or another of the objects which become related is an off-mass-shell matrix element (a pion mass being set equal to zero). In the absence of a reliable procedure for estimating off-mass-shell corrections, we have no choice but to hopefully confront the predictions, uncorrected, with on-mass-shell experimental information. The agreement is in general good, and in two cases, spectacular.

In the second instance, we conjecture a simple expression for the equal-time commutator of axial-vector current and weak, nonleptonic Hamiltonian. Taken together with PCAC, this leads to off-mass-shell relations between $K\to 2\pi$ and $K\to 3\pi$ modes and to other consequences for $K\to 3\pi$ decay. Here the results are far from good.

All of our results proceed from the following identity.[4] Let

$$M_\mu = i\int dx\, e^{-iq\cdot x}\theta(x_0)\langle\beta|[j_\mu^{\gamma}(x), B(0)]|\alpha\rangle. \quad (1)$$

Then, from a four-dimensional integration by parts (with formally dangerous neglect of surface integrals), we find

$$iq_\mu M_\mu = i\int dx\, e^{-iq\cdot x}\theta(x_0)\langle\beta|[\partial_\mu j_\mu^{\gamma}(x), B(0)]|\alpha\rangle$$
$$+ i\int dx\, e^{-iq\cdot x}\delta(x_0)\langle\beta|[j_0^{\gamma}(x), B(0)]|\alpha\rangle. \quad (2)$$

We shall take $j_\mu^\gamma = A_\mu^\gamma$ to be that member of the octet of axial-vector currents which has the quantum numbers of the π^γ meson ($\gamma = +, -, 0$). According to the PCAC hypothesis, we relate the divergence of this current to the pion field φ^γ in the familiar manner,

$$\partial_\mu A_\mu^{\gamma} = (g_A m\mu^2/g_r)\varphi^\gamma, \quad (3)$$

where g_A is the ratio of axial vector to vector β-decay coupling constants, g_r is the strong pion-nucleon coupling constant, m is the nucleon mass, and μ is the pion mass.

In all applications we set $q\to 0$. Then the left-hand side of Eq. (2) must vanish, unless M_μ has a pole at $q_\mu = 0$. In the cases studied no such pole term arises. Thus

$$i\int dx\, \delta(x_0)\langle\beta|[A_0^{\gamma}(x), B(0)]|\alpha\rangle$$
$$= -\frac{ig_A m}{g_r}\int dx(\mu^2 - \Box)\theta(x_0)\langle\beta|[\varphi^\gamma(x), B(0)]|\alpha\rangle. \quad (4)$$

For our first set of examples, let us consider leptonic decays of the K^+ meson, choosing the state $|\alpha\rangle$ to be that of a K^+ meson and choosing $B(0) = J_\mu(0)$ to be the $\Delta S/\Delta Q = 1$ current (vector plus axial vector) responsible for strangeness-changing leptonic decays. On the left-hand side of Eq. (4), we then encounter one of the celebrated current-current commutators. If for A_0^γ we choose the neutral current ($\gamma = 3$, corresponding to the quantum numbers of the π^0 meson), we have

$$[A_0^{3}(x), J_\mu(0)]_{x_0=0} = \tfrac{1}{2}\delta(\vec{x})J_\mu(0). \quad (5)$$

Then the left-hand side of Eq. (4) becomes the matrix element relevant for the decay $K^+\to\beta + l^+ + \nu$. The right-hand side is proportional to the matrix element for the decay $K^+\to\beta+\pi^0 + l^+ + \nu$, evaluated at zero four-momentum q for the π^0 meson. More precisely, we see that

$$\langle\beta|J_\mu|K^+\rangle = -(2ig_A m/g_r)\langle\beta\pi^0|J_\mu|K^+\rangle(2q_0)^{1/2},$$
$$q(\pi^0)\to 0. \quad (6)$$

Choosing $|\beta\rangle = |0\rangle$ to be the vacuum state, we obtain a relation between the amplitudes for K_{l2} and K_{l3} decays (the latter evaluated off mass shell). A similar procedure also relates π_{l2} and π_{l3} decays. With $|\beta\rangle = |\pi^0\rangle$ chosen to be a one-pion state, we relate K_{l3} and K_{l4} decays in the modes involving neutral pions (the latter amplitude, again, being off mass shell).

Returning to Eq. (4), if for A^γ we now choose the negatively charged current (corresponding to the quantum numbers of the π^- meson), we have

$$[A_0^{(-)}(x), J_\mu(0)] = 0, \quad (7)$$

which implies that the amplitude for $K^+\to\beta+\pi^+ + l^+ + \nu$ vanishes in the limit of zero four-momentum for the π^+ meson. In particular, choosing for $|\beta\rangle$ the π^--meson state, we have

$$(2q_0)^{1/2}\langle\pi^+\pi^-|J_\mu(0)|K^+\rangle\to 0, \quad q(\pi^+)\to 0. \quad (8)$$

We now turn to specific results for the K-meson (and π-meson) lepton decays.

K_{l2} vs K_{l3} decays.—The covariant matrix element relevant to $K_{l2}{}^{+}$ decay is

$$(2K_0)^{1/2}\langle 0|J_\mu|K^+\rangle = (G/\sqrt{2})m_K f_K K_\mu, \qquad (9)$$

where m_K is the K-meson mass, K_μ is the K-meson momentum, and f_K is a dimensionless constant. For the decay $K^+ \to \pi^0 + l^+ + \nu$, with q the pion momentum, the relevant matrix element is

$$(2K_0)^{1/2}(2q_0)^{1/2}\langle \pi^0|J_\mu|K^+\rangle = (G/\sqrt{2})[f_+(K+q)_\mu + f_-(K-q)_\mu], \qquad (10)$$

where $f_\pm$ are dimensionless form factors that depend, in general, on the variables $K\cdot q$ and q^2 (the latter is the square of the π^0 mass and is not, alas, experimentally variable): $f_\pm = f_\pm(K\cdot q, q^2)$. Choosing $|\beta\rangle = |0\rangle$ in Eq. (6) we have

$$|f_K| = |2g_A m/g_V m_K|\,|f_+(0,0) + f_-(0,0)| = 0.32\,|f_+(0,0) + f_-(0,0)|. \qquad (11)$$

Experimentally, the form factors $f_\pm$ appear to depend very little on the variable $K\cdot q$, for the pion on its mass shell. Ignoring off-mass-shell corrections then, we suppose that the right-hand side of (11) can be approximated by the experimentally determined form factors. We adopt the experimental values[5] $f_-/f_+ = 0.46 \pm 0.27$ and $|f_+| = 0.16 \pm 0.01$. The right-hand side of Eq. (11) thus has the value 0.074 ± 0.014. The experimental value for the left-hand side is $|f_K| = 0.070 \pm 0.001$. The agreement is surprisingly good.

For the decay process $\pi^+ \to l^+ + \nu$ and $\pi^+ \to \pi^0 + l^+ + \nu$ a similar analysis holds, with structures analogous to Eqs. (9) and (10) and predictions analogous to Eq. (11), with $m_K \to \mu$. Here, however, we have that $f_- = 0$, and the prediction is[6]

$$|f_\pi| = |g_A m/\mu g_V|\,|f_+(\pi)|. \qquad (12)$$

The left-hand side has experimental value 0.94. The right-hand side has experimental magnitude 0.85. The agreement is again very good indeed.

K_{l3} vs K_{l4} decays.—Consider first the decay mode $K^+ \to \pi^+ + \pi^- + l^+ + \nu$. The structure of the relevant matrix element is

$$(2K_0)^{1/2}(2p_0)^{1/2}(2q_0)^{1/2}\langle \pi^+\pi^-|J_\mu|K^+\rangle = \frac{G}{\sqrt{2}}\frac{1}{m_K}\left\{F_1(q+p)_\mu + F_2(q-p)_\mu + F_3(K-p-q)_\mu + \frac{F_4}{m_K{}^2}\epsilon_{\mu\nu\rho\sigma}K_\nu p_\rho q_\sigma\right\}, \qquad (13)$$

where q and p are, respectively, the π^+ and π^- momenta. The form factors F_i depend on the variables $K\cdot p$, $K\cdot q$, $p\cdot q$, and of course on masses, notably on q^2. Setting $q=0$ we have, from Eq. (8),

$$F_1 = F_2, \quad F_3 = 0, \qquad (14)$$

all form factors here being evaluated for zero π^+ four-momentum, hence off the mass shell. There exists experimental information for the mode $K^+ \to \pi^+ + \pi^- + e^+ + \nu$, where the form factor F_3 is essentially undetectable because of the smallness of the positron mass, so nothing can be said about the second of Eqs. (14). However, the on-mass-shell data for the ratio F_1/F_2, averaged over the spectrum, indicates a value $F_1/F_2 = 0.8 \pm 0.3$, in good agreement with the first of Eqs. (14).[7]

On the other hand, if we now set the momentum of the negative π meson equal to zero, we find for this limit the results

$$2|f_+| = |g_A m/g_V m_K|\,|F_1 + F_2|, \quad |f_+ + f_-| = |g_A m/g_V m_K|\,|F_3|, \qquad (14')$$

with the form factors F_i now evaluated for $p(\pi^-) = 0$. Insofar as we suppose that the form factors are independent of momenta, these results are clearly in disagreement with those of Eq. (14). Indeed, if we simply ignore the momentum dependence of the form factors F_i, then we conclude from the data of Birge et al.[7] that the right-hand side of the first of Eqs. (14′) has a value 0.70 ± 0.07, whereas the left-hand side has the value 0.32. Of course the two sets of formulas (14) and (14′) are not formally in disagreement, since they refer to different limits, $q(\pi^+) \to 0$ and $p(\pi^-) \to 0$, respectively. These limits are, however, unachievable physically. When better data are available, one might attempt a better test of the formulas by comparison with the physical limits in which the appropriate three-momenta (in the K-meson rest frame) are set in turn equal to zero.

For the decay mode $K^+\to 2\pi^0+l^++\nu$, the relevant matrix element has a structure identical to that of Eq. (13) and here, setting the four-momentum q of one of the neutral pions equal to zero, we have from Eq. (6) the equations

$$|2f_+| = |2g_A m/g_\gamma m_K|\,|F_1-F_2|,$$
$$|f_+ + f_-| = |2g_A m/g_\gamma m_K|\,|F_3|, \qquad (15)$$

relating the modes $K^+\to\pi^0+l^++\nu$ and $K^+\to 2\pi^0+l^++\nu$. Again, the form factors on the right are evaluated at zero momentum for one of the π^0 mesons. There does not yet exist any experimental information on the $K^+\to 2\pi^0+l^++\nu$ decay mode. However, if we assume rough constancy of the form factors for this mode, as well as for $K^+\to\pi^++\pi^-+l^++\nu$, then from the $\Delta I=\frac{1}{2}$ rule we have $F_2(2\pi^0)=0$ and $F_1(2\pi^0)=(1/\sqrt{2})F_1(\pi^+\pi^-)$. For $|F_1(\pi^+\pi^-)|$, the data of Birge et al.[7] indicate a value 2.5 ± 0.3. With these assumptions, and with neglect of off-mass-shell corrections, we find for the right-hand side of the first of Eqs. (15) the magnitude 0.56 ± 0.06. For the left-hand side the experimental magnitude is 0.32 ± 0.02. The agreement is not spectacular; but it is not totally discouraging, in view of the possible gravity of off-mass-shell corrections in a situation where strong final-state interactions must surely be important.

In the course of the above study, we were led to consider the possibility of applying similar techniques to nonleptonic weak decays. Suppose in Eq. (4) we set $B(0)=H(0)$, where H is the Hamiltonian density responsible for weak, strangeness-changing, nonleptonic processes. With respect to SU(3), we suppose that H transforms like $K^0+\bar{K}^0$, and we distinguish by H^+ and H^- the parity-conserving and parity-nonconserving parts of H. Concerning the equal-time commutator on the left-hand side of Eq. (4), it is tempting to generalize the quarklike symbolism employed by Gell-Mann for the currents to a similar structure for H, namely, $H=H^+ + H^-\sim\bar{\psi}(1+\gamma_5)\psi$ [SU(3) indices on the quark fields are suppressed]. This leads to the commutation relations

$$[A_0^{\,3}(x),H^\pm(0)]_{x_0=0}=\tfrac{1}{2}\delta(\vec{x})H^\pm(0), \qquad (16)$$

$$[A^{(-)}(x),H(\Delta S=1)]_{x_0=0}$$
$$=[A^{(+)}(x),H(\Delta S=-1)]_{x_0=0}=0. \qquad (17)$$

The conjecture of Eqs. (16) and (17) is of course independent of the possible literal existence of quarks. Indeed, it is suggested on a current-current model for nonleptonic interactions if the currents themselves obey the Gell-Mann algebra and if they are effectively coupled in a local manner. We have supposed further that neutral current terms are supplied so that H transforms like a member of an SU(3) octet.

At any rate, on the basis of these conjectures we are led, following the same procedures employed above, to a number of implications for $K\to3\pi$ decays. Namely, we find that the amplitude for the decay $K^+\to2\pi^++\pi^-$ must vanish in the limit of zero four-momentum of the π^- meson; similarly, the amplitude for the decay $K^+\to2\pi^0+\pi^+$ must vanish in the limit of zero four-momentum of one of the π^0 mesons; and similarly, the amplitude for $K^0\to\pi^++\pi^-+\pi^0$ must vanish in the limit of zero four-momentum for either of the charged mesons. These are not very pleasant predictions, but they may reflect the significance of off-mass-shell effects in a state with strong final-state interactions.

On the other hand, for the decay $K_2^{\,0}\to\pi^++\pi^-+\pi^0$, in the limit of zero four-momentum of the π^0 meson, we find

$$A(K_1^{\,0}\to\pi^++\pi^-)$$
$$=(2g_A m/g_\gamma)A[K_2^{\,0}\to\pi^++\pi^-+\pi^0;q(\pi^0)=0], \qquad (18)$$

where A is the invariant amplitude for the decay process in question (small CP-nonconserving effects are ignored); similarly, for zero four-momentum of any of the π^0 mesons in $K_2^{\,0}\to3\pi^0$ decay, we have

$$A(K_1^{\,0}\to2\pi^0)=(2g_A m/g_\gamma)A(K_2^{\,0}\to3\pi^0). \qquad (19)$$

Concerning Eq. (18), we know that the $K_2^{\,0}\to\pi^++\pi^-+\pi^0$ amplitude varies appreciably with π^0 energy. We therefore approximate the right-hand side of Eq. (18) with the physical amplitude, evaluated at zero three-momentum in the $K_2^{\,0}$ rest frame, ignoring off-mass-shell corrections. From experimental information on this amplitude, at $\vec{q}=0$, we infer for the right-hand side of Eq. (18) a magnitude $(0.38\pm0.04)\times10^{-6}$. For the left-hand side we have the magnitude $(0.79\pm0.02)\times10^{-6}$. The agreement is not impressive. Concerning Eq. (11), we have no experimental data on the $K_2^{\,0}\to3\pi^0$ decay spectrum; and for present purposes we suppose the spectrum to be flat. Neglecting off-mass-shell effects, we therefore use the net decay rate to infer for the right-hand side of Eq. (19) the magnitude $(0.40\pm0.04)\times10^{-6}$. For the left-hand side the experimental magnitude is $(0.56\pm0.01)\times10^{-6}$.

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PHOTOPRODUCTION OF THE ETA PARTICLE AT 800-1000 MeV. A COMPARISON BETWEEN THE πN AND THE ηN SYSTEM.

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Photoproduction on protons of the eta particle,

$$\gamma+p\rightarrow\eta+p, \tag{1}$$

has been measured at the Frascati 1100-MeV electron synchrotron, for incident photon energies from 800 to 1000 MeV. Our results allow a comparison with the pion production channel[1,2]

$$\pi^-+p\rightarrow\eta+n \tag{2}$$

as well as a comparison between the cross sections for pion and for eta production in both input channels, $\gamma+N$ and $\pi+N$ (N stands for nucleon). A coherent picture in the $T=\frac{1}{2}$ isospin state is obtained. We find that the η-N system is dominated at low energy by a state different from the known π-N resonances, in good agreement with the suggested S_{11} η-N resonance.[3]

The experimental arrangement is similar in principle to the one used in our previous set of measurements,[4] but the apparatus has a much larger acceptance and a better resolution. The experimental layout is shown in Fig. 1. The γ-ray beam from the electron synchrotron is incident upon a 7.4-cm liquid H_2 target. Protons are detected in the proton telescope (PT), which is a combination of counters and four spark chambers.

On the line of flight of the η there is a total-absorption lead-glass Cherenkov counter C with an anticoincidence counter in front (S_5) to detect γ rays. The energy of the γ ray detected by C is measured by a pulse-height analyzer and recorded on each photograph of the spark chambers. All our spark-chamber events are triggered by a coincidence between a proton in PT and a photon in C. The eta is detected by two different methods, as already described by us.[4]

The first one (step method) is based on the energy distribution of the protons from Reaction (1) in PT. An example of this distribution is given in Fig. 2(a). It shows the energy spectrum of the protons detected in coincidence with a photon in C of energy $E_C\geqslant 420$ MeV. In this case most of the events come from Reaction (1). An alternative method for detecting process (1) consists in plotting the energy distribution of the protons for a "monochromatic" incident γ-ray beam at a given angle in the laboratory. This is simply obtained by taking the difference between the proton energy spectra at two different values of the maximum energy E_0 of the bremsstrahlung beam, normalized for the same number of equivalent quanta. This gives direct evidence