We are indebted to M. Tigner for assistance in setting up the experiment and to D. R. Yennie, S. L. Adler, and C. Mistretta for valuable conversations on the interpretation of the data.

---

*Work supported in part by the National Science Foundation.

[1]C. W. Akerlof, W. W. Ash, K. Berkelman, and M. Tigner, Phys. Rev. Letters 14, 1036 (1965); K. Berkelman, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Hamburg, 1965 (to be published).

[2]R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957); M. Gourdin, Nuovo Cimento 21, 1094 (1961); L. N. Hand, thesis, Stanford University, 1961 (unpublished); S. M. Berman, Phys. Rev. 135, B1249 (1964); see also Ref. 1.

[3]In terms of the σ_U and σ_L defined in Ref. 1, T and L are given by $T=(\sigma_U 2M|\vec{k}|)(W^2-M^2)^{-1}$ and $L=(-\sigma_L 2M|\vec{k}|k^2)(W^2-M^2)^{-1}K_0^{-2}$. Both T and L are positive definite.

[4]R. R. Wilson, Nucl. Instr. & Methods 1, 101 (1957).

[5]K. Berkelman, J. M. Cassels, D. N. Olson, and R. R. Wilson, in Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 757.

[6]In this experiment it was impossible to reach higher W and θ values, because the angle between the incident beam direction and the pion spectrometer could not be made less than 20° because of geometrical limitations.

[7]When the final pion energy is narrowly fixed as in this experiment, the radiative correction to the electroproduction cross section is very similar to the correction one applies to elastic e-p scattering. The normalized cross sections are then very insensitive to radiative effects. We are indebted to D. R. Yennie for helpful suggestions on this point.

[8]S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. 111, 329 (1958).

[9]L. N. Hand, Phys. Rev. 129, 1834 (1963). References to earlier work are given in this paper.

[10]G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[11]S. Fubini et al. do not explicitely include F_π in their formulation; they assume it to be unity, the normalization value at $k^2=0$. To include the F_π dependence, one multiplies F_π into each term in the amplitude which does not already include a nucleon form factor. The resulting expression is still gauge invariant for any value of F_π. In evaluating the cross sections we have omitted the small resonance terms arising from the 33-state projections of the pion-pole amplitude. This has generally been done in recent analyses of photoproduction; see, for example, G. Höhler and W. Schmidt, Ann. Phys. (N.Y.) 28, 34 (1964).

[12]Likewise, the predicted $k^2\to 0$ limit of T is too large compared with the π^0 180° photoproduction data at $W=1210$ MeV.

[13]It is somewhat reassuring, however, to note that the theoretical $k^2\to 0$ limit of T agrees fairly well with the π^+ 0° photoproduction data at both values of W.

[14]The target pions in this experiment are off the mass shell by an amount $\Delta-\mu^2=-2.7\mu^2$ and $-1.9\mu^2$ at $W=1210$ and $W=1313$ MeV, respectively; and one is led to ask whether the F_π measured in electroproduction is the same as the F_π of a free pion. The $(\Delta-\mu^2)$-dependent correction to the pion-pole term (form factor and propagator) may in fact turn out to be quite small, since the generalized Ward identity [F. E. Low, Phys. Rev. 110, 974 (1958)] can be used to show, first, that the correction must vanish in the low-k^2 limit, and second, that the correction vanishes for any k^2 provided that the pion-charge structure can be accounted for by diagrams in which a single particle (a vector meson) couples the photon to the pion (D. R. Yennie, private communication).

[15]R. R. Wilson and J. S. Levinger, Ann. Rev. Nucl. Sci. 14, 135 (1964).

PARAMETERS OF THE $K_0\bar{K}_0$ SYSTEM*

Philippe H. Eberhard

Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 17 December 1965)

When no conservation law is assumed—P, C, CP, T, or CPT—Sachs[1] has shown that the time dependence of a K_0-$\bar{K}_0$ mixed state can still be described with the help of a short-lived state $|K_S\rangle$ and a long-lived state $|K_L\rangle$:

$$|K_S\rangle=p_S|K_0\rangle+q_S|\bar{K}_0\rangle,$$

$$|K_L\rangle=p_L|K_0\rangle-q_L|\bar{K}_0\rangle. \quad (1)$$

Several authors[1-4] have set limits for the parameters p_S, q_S, p_L and q_L under different assumptions concerning the decay processes—either CPT invariance, the $\Delta I=\frac{1}{2}$ rule, or excluding the possibility of accidental cancellation. To our present knowledge, it has not been pointed out yet how well all these parameters are known from already performed experiments, from the properties of unitarity, and from an assumption of negligible C violations in strong interactions only.

Of the eight real parameters that define the four complex parameters p_S, q_S, p_L, and q_L five can be chosen by convention if no conservation law is assumed. We take the following conventions:

$$p_S = \cos\alpha_S e^{i\theta/2}, \quad p_L = \cos\alpha_L e^{-i\theta/2},$$
$$q_S = \sin\alpha_S e^{-i\theta/2}, \quad q_L = \sin\alpha_L e^{i\theta/2}. \qquad (2)$$

The following restrictions can be imposed:

$$0 \leq \alpha_S \leq \pi/2,$$
$$0 \leq \alpha_L \leq \pi/2,$$
$$-\pi/2 \leq \theta \leq \pi/2. \qquad (3)$$

Without assuming any invariance in the decay process, we determine α_S, α_L, and θ from known experimental results.[5-11] Our result is given in Eq. (20).

If $|j\rangle$ is a particular state into which the K mesons can decay, we call Γ_j^S (Γ_j^L) the rate of decay of a pure $|K_S\rangle$ ($|K_L\rangle$) state into $|j\rangle$ and

$$\kappa = \langle K_S|K_L\rangle = p_S^* p_L - q_S^* q_L$$
$$= \cos(\alpha_S + \alpha_L)\cos\theta$$
$$-i\cos(\alpha_S - \alpha_L)\sin\theta. \qquad (4)$$

Unitarity gives the relation[12]

$$|\kappa| \leq \frac{2\sum_j(\Gamma_j^S\Gamma_j^L)^{1/2}}{|\tau_S^{-1} + \tau_L^{-1} - 2i\Delta m|}, \qquad (5)$$

where[5]

$$\tau_S = |K_S\rangle \text{ lifetime} = (0.88 \pm 0.01)10^{-10} \text{ sec},$$
$$\tau_L = |K_L\rangle \text{ lifetime} = (5.8 \pm 0.6)10^{-8} \text{ sec},$$

and

$$\Delta m = \text{mass difference between } |K_S\rangle \text{ and } |K_L\rangle$$
$$= (0.55 \pm 0.1)\tau_S^{-1}. \qquad (6)$$

From the inequality (5) and using a Schwarz inequality and the relations $\sum_j\Gamma_j^S = \tau_S^{-1}$ and $\sum_j\Gamma_j^L = \tau_L^{-1}$, we get the relation[13]

$$|\kappa| \leq \frac{2(\tau_S/\tau_L)^{1/2}}{|1 + (\tau_S/\tau_L) - 2i\Delta m\tau_S|} \cong 5\times10^{-2},$$
$$|\kappa|^2 \leq 3\times10^{-3}. \qquad (7)$$

Three experiments have measured the short-life decay rate of K_0 into $\pi^+\pi^-$.[6] According to our description and convention, those results are a measurement of the quantity

$$\Gamma_{\pi^+\pi^-}^S \sin^2\alpha_L/(1-|\kappa|^2) = (0.36 \pm 0.01)\tau_S^{-1}. \qquad (8)$$

Several experiments have measured the branching ratio of $|K_S\rangle$ into $2\pi^0$ to $|K_S\rangle$ into $\pi^+\pi^-$:[7,8]

$$\Gamma_{2\pi^0}^S/\Gamma_{\pi^+\pi^-}^S = 0.46 \pm 0.02. \qquad (9)$$

Two experiments have measured the ratio R of the reactions $\bar{p}+p \to K_0 +$ (system of particles with negative strangeness) and $\bar{p}+p \to \bar{K}_0 +$ (system of particles with positive strangeness) when the K_0 or the $\bar{K}_0$ decays into two charged pions.[9] We assume that C-invariance violations in strong interactions are small compared to the accuracy of the experiments; therefore, equal amounts of K_0 and $\bar{K}_0$ are produced. Considering that the ratio R is about 1 and that the state $|K_L\rangle$ decays into $\pi^+\pi^-$ no more than 0.2% of the time,[10] while the state $|K_S\rangle$ decays into $\pi^+\pi^-$ with a branching ratio of at least 0.36 [from relations (7) and (8)], we conclude that only the short-lived state $|K_S\rangle$ contributed sensibly to the $\pi^+\pi^-$ decay, whenever a K_0 or a $\bar{K}_0$ was produced initially. Therefore R is a measurement of $\tan^2\alpha_L$. Combining both experiments, we get

$$\tan^2\alpha_L = 1.0 \pm 0.3. \qquad (10)$$

Moreover, combining (7), (8), (9), and (10) we get

$$\Gamma_{\pi^+\pi^-}^S = (0.72 \pm 0.023)\tau_S^{-1}$$

and

$$\Gamma_{2\pi^0}^S = (0.33 \pm 0.018)\tau_S^{-1}. \qquad (11)$$

We also can get the total rate for decay into two pions, $\Gamma_{\pi^+\pi^-}^S + \Gamma_{2\pi^0}^S$, by using Refs. 6 and 7, and combine it with the direct measurement of $\Gamma_{\pi^+\pi^-}^S + \Gamma_{2\pi^0}^S$ given in Ref. 8 [using also relations (7) and (10)]:

$$\Gamma_{\pi^+\pi^-}{}^S + \Gamma_{2\pi^0} = (1.034 \pm 0.03)\tau_S{}^{-1}. \quad (12)$$

Because the sum of all decay rates of the $|K_S\rangle$ state should add up to $\tau_S{}^{-1}$, by using Eq. (12) one can set an upper limit to the decay rates into all final states that are not two pions:

$$\Gamma_{\text{not }2\pi}{}^S < 0.03\tau_S{}^{-1}. \quad (13)$$

We can now turn to the $|K_L\rangle$ decay rates. Christenson et al.[10] have measured

$$\Gamma_{\pi^+\pi^-}{}^L \cong 2\times10^{-3}\tau_L{}^{-1}. \quad (14)$$

An upper limit for the $2\pi^0$ decay mode of the $|K_L\rangle$ state,

$$\Gamma_{2\pi^0}{}^L < 8\times10^{-3}\tau_L{}^{-1}, \quad (15)$$

has been set.[11] Therefore most of the $|K_L\rangle$ state decays into states that are not two pions:

$$\Gamma_{\text{not }2\pi}{}^L \cong \tau_L{}^{-1}. \quad (16)$$

Substituting (11), (13), (14), (15), and (16) in Eq. (5), we get

$$|\kappa| \leq \frac{2[(\Gamma_{\pi^+\pi^-}{}^S\Gamma_{\pi^+\pi^-}{}^L)^{1/2} + (\Gamma_{2\pi^0}{}^S\Gamma_{2\pi^0}{}^L)^{1/2} + (\Gamma_{\text{not }2\pi}{}^S\Gamma_{\text{not }2\pi}{}^L)^{1/2}]}{|\tau_S{}^{-1}+\tau_L{}^{-1}-2i\Delta m|} < (1.3\pm0.45)\times10^{-2}. \quad (17)$$

Relation (4) implies that

$$|\kappa|^2 = \cos^2(\alpha_S+\alpha_L)\cos^2\theta + \cos^2(\alpha_S-\alpha_L)\sin^2\theta. \quad (18)$$

In (18), $\cos(\alpha_S-\alpha_L)$ cannot be close to zero because of relation (10) and the restrictions (3). Therefore (17) and (18) imply that $\alpha_S+\alpha_L \cong \pi/2$, and $\theta\cong 0$. More precisely, we have

$$[\alpha_S+\alpha_L-(\pi/2)]^2+\theta^2 < 1.7\times10^{-4}. \quad (19)$$

The value given in Eq. (19) can be increased by a factor of 2 if we stretch the error by one more standard deviation.

Relation (19) represents a condition for both $\alpha_S+\alpha_L$ and θ. Using (19) and (10), we have

$$\alpha_S = \pi/4 \pm 0.015,$$

$$\alpha_L = \pi/4 \pm 0.008,$$

$$\theta = 0 \pm 0.013. \quad (20)$$

These values show that our parameters are determined without assumption of a conservation law in weak interactions. They are close to their theoretical value if CP were conserved, i.e.,

$$\alpha_S = \alpha_L = \pi/4,$$

$$\theta = 0. \quad (21)$$

Relation (20) shows that the measurement of $|K_L\rangle$ decay rates[14] (made under the assumption that p_S, q_S, p_L, and q_L are close to $1/\sqrt{2}$) are valid. We could now split the $\Gamma_{\text{not }2\pi}$ into several categories, but this would not improve our result (20) sensibly because of the uncertainties about the $|K_S\rangle$ decay rates in the already performed experiments[15] and about possible unknown modes of decay of $|K_L\rangle$.[16]

The errors quoted in (20) result from quadratic combination of experimental errors and the condition of unitarity (5). Because the latter is an absolute limit and should not be interpreted as a Gaussian error, the probabilities of situations corresponding to several standard deviations in the errors in α_S and θ are somewhat less than those expected for a Gaussian distribution.

Since this work was performed, we have seen a preprint by Bell and Steinberger[17] in which our unitarity condition (5) is derived. They obtain a somewhat better constraint than our relation (17) for $|\kappa|$ under reasonable assumptions about a few decay rates.

I am indebted to Professor L. W. Alvarez for his support, Professor F. S. Crawford, Jr., for enlightening discussions, and Dr. A. Barbaro-Galtieri for making available some experimental data used in this paper.

---

*This work was done under the auspices of the U. S. Atomic Energy Commission.

[1]R. Sachs, Ann. Phys. (N.Y.) 22, 239 (1963).

[2]T. D. Lee and L. W. Wolfenstein, Phys. Rev. 138, B1490 (1965).

[3]S. Weinberg, Phys. Rev. 110, 782 (1958).

[4]T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[5]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Lawrence Radiation Laboratory Report No. UCRL-8030, 1965 (unpublished). The values of lifetimes have been determined only by experiments measuring exponential decay of the particle (A. Barbaro-Galtieri, private communication). The value of Δm used is an average given by M. L. Good at the Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, October 1965 (to be published).

[6]F. S. Crawford, M. Cresti, R. Douglass, M. Good, G. Kalbfleisch, M. L. Stevenson, and H. Ticho, Phys. Rev. Letters 2, 266 (1959); M. Schwartz, in Proceedings of the Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 727; J. A. Anderson, F. S. Crawford, B. B. Crawford, R. L. Golden, L. J. Lloyd, G. W. Meisner, and L. Price, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 836.

[7]C. Baglin et al., Nuovo Cimento 18, 1043 (1960); M. Chretien et al., Phys. Rev. 131, 2208 (1963).

[8]J. L. Brown et al., Nuovo Cimento 19, 1155 (1961); J. L. Brown, J. A. Kadyk, G. H. Trilling, B. P. Roe, D. S. Sinclair, and J. C. Vander Velde, Phys. Rev. 130, 769 (1963).

[9]R. Armenteros et al., Phys. Letters 17, 170 (1965); C. Baltay et al., Phys. Rev. Letters 15, 591 (1965).

[10]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[11]L. Criegee, J. D. Fox, H. Frauenfelder, A. O. Hanson, G. Moscati, C. F. Perdrisat, and J. Todoroff, in Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, October 1965 (to be published).

[12]If all $|j\rangle$ represent a complete orthogonal set of final states and $\mathfrak{M}$ is the transition matrix, then we have

$$\Gamma_j^S = |\langle j|\mathfrak{M}|K_S\rangle|^2,$$

$$\Gamma_j^L = |\langle j|\mathfrak{M}|K_L\rangle|^2,$$

and

$$(\tau_S^{-1}/2+\tau_L^{-1}/2-i\Delta m)(p_S^*p_L-q_S^*q_L)$$

$$=\zeta=\sum_j\langle j|\mathfrak{M}|K_S\rangle^*\langle j|\mathfrak{M}|K_L\rangle=\sum_j(\Gamma_j^S\Gamma_j^L)^{1/2}\exp(i\varphi_j),$$

where φ_j is the phase difference between $\langle j|\mathfrak{M}|K_L\rangle$ and $\langle j|\mathfrak{M}|K_S\rangle$. Therefore we can write

$$|\zeta|\le\sum_j(\Gamma_j^S\Gamma_j^L)^{1/2}.$$

If j is used to describe several states of decay of the orthogonal set, the above inequality is still valid as a result of the Schwarz inequality.

[13]Inequality (7) is weaker than inequality (5); it has already been reported in Ref. 2.

[14]The data considered here for branching ratios are the world compilation quoted in Ref. 5.

[15]J. A. Anderson, F. S. Crawford, R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters 14, 475 (1965); P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and R. J. Plano, Phys. Rev. 140, B127 (1965). An upper limit of $10\Gamma_{3\pi^0}{}^L$ for $\Gamma_{3\pi^0}{}^S$ has been given us by the Paris-Milan-Padua Collaboration (B. Aubert, private communication).

[16]When we sum up all known decay rates of $|K_L\rangle$, also taking into account measurements of some of the ratios of the decay rates, and compare the result with the value of τ_L^{-1} determined by exponential decay, the experimental accuracies allow us to write only the inequality

$$\Gamma_{\text{unknown}}^L<0.08\tau_L^{-1}$$

(A. Barbaro-Galtieri, private communication).

[17]J. S. Bell and J. Steinberger, CERN Report No. 65/1524/5-TH. 605, 1965 (unpublished); lectures given at the International Conference on Elementary Particles, Oxford, September 1965 (unpublished).

EQUAL-TIME COMMUTATORS AND K-MESON DECAYS*

C. G. Callan and S. B. Treiman

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 8 December 1965)

The impressive, quantitative account given by Adler[1] and Weisberger[2] of axial-vector renormalization effects in β decay has stimulated a search for further tests of the underlying hypothesis: notably, the weak-interaction current algebra proposed by Gell-Mann[3] and the notion of a partially conserved axial-vector current (PCAC). In the present note, we report some additional applications of these ideas to leptonic decays of K and π mesons. We also discuss briefly the implications of a certain speculative generalization of the Gell-Mann equal-time commutation relations.

In the first instance, we are concerned with the commutation relations between weak currents and are led, via PCAC, to equations con-