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RECENTLY PROPOSED RELATIONSHIP FOR SUPERCONDUCTORS

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(Received 27 October 1965)

Utilizing an approximate expression for the specific heat of superconductors, Lewis[1] obtained a general thermodynamic relation between the energy gap of a superconductor and the slope of the critical field at the critical temperature (T_c). It was emphasized by Schafroth[2] that Lewis's expression is insensitive to the exact form chosen for the specific heat. Noting this, and the fact that Lewis's implied energy gap can be approximately[3] identified with the energy gap at zero temperature [$\Delta(0)$], it is shown below that numerically Lewis's expression approximates a relationship which was recently proposed by Toxen[4] [see Eq. (3) below]. This indicates, we believe, that Toxen's relation is a numerical coincidence rather than a fundamental relationship for superconductors.

We now outline briefly Lewis's method. The specific heat (C_s) for superconductors in our adopted approximation is given by

$$C_s/\gamma T_c = A\exp(-\alpha T_c/T). \quad (1)$$

Here $\alpha \approx \Delta(0)/kT_c$,[3] k is the Boltzmann constant, and A is a parameter. (We remark that a weak temperature dependence of A will not affect the argument.) γ is given by the electronic specific heat of the normal metal (C_n) at low temperatures, i.e.,

$$C_n \simeq \gamma T.$$

Equating the entropy of the normal and superconducting metal at T_c, we get

$$\int_0^{T_c} C_s \frac{dT}{T} = \gamma T_c,$$

or

$$AE(\alpha) = 1,$$

where

$$E(\alpha) = \int_\alpha^\infty \frac{e^{-x}}{x} dx.$$

This determines A. A somewhat more complicated, but straightforward, thermodynamic argument gives[1,2]

$$-\left(\frac{dH_c}{dT}\right)_{T_c} \frac{T_c}{H_0}\Bigg|_{\text{theor}} = \left[\frac{e^{-\alpha} - E(\alpha)}{2e^{-\alpha} - (1+2\alpha)E(\alpha)}\right]^{1/2} \equiv F(\alpha). \quad (2)$$

Here H_0 is the critical field at zero temperature and $(dH_c/dT)_{T_c}$ is the slope of the critical field at T_c; $F(\alpha)$ is defined by the above equa-

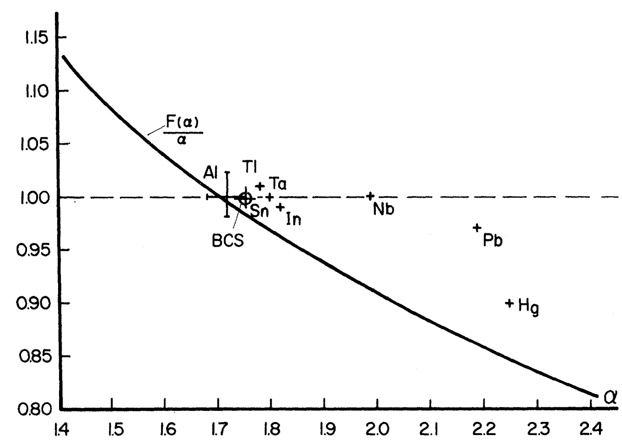

FIG. 1. Solid line depicts $F(\alpha)/\alpha$ as a function of α [see Eq. (2)]. Dashed line shows Toxen's relationship [see Eq. (3)]. The experimental points plotted are $-(dH_c/dT)_{T_c}[kT_c^2/\Delta(0)H_0]$ as a function of $\alpha \simeq \Delta(0)/kT_c$. The BCS value in the weak-coupling limit is shown by a circle.

tion.

The following empirical relation was proposed by Toxen[4]:

$$-\left(\frac{dH_c}{dT}\right)_{T_c}\frac{T_c}{H_0}\bigg|_{\text{expt}}=\frac{\Delta(0)}{kT_c}\simeq\alpha. \tag{3}$$

In Fig. 1, $\alpha^{-1}F(\alpha)$ is plotted as a function of α for the range of interest. Also plotted are the experimental points of $-(dH_c/dT)_{T_c}(T_c/H_0)[\Delta(0)/kT_c]^{-1}$ vs $\Delta(0)/kT_c$. The experimental data are those quoted by Toxen except for more recent data on lead[5] and niobium.[6] Toxen's relation is shown in the figure by a dashed line.

It can be seen that there is a reasonable agreement between the experimental points and our calculated curve, which is not surprising in view of the known[2] insensitivity of Lewis's method to the precise form of C_s chosen. From the figure we note that for α between 1.55 and 1.85, $F(\alpha)=\alpha\pm5\,\%$. This is the range within which most experimental values of α lie. It is our contention that this numerical property of $F(\alpha)$ accounts for the correctness of Toxen's relationship [Eq. (3)]. For $\alpha<1.55$ or $\alpha>1.85$ we expect that $(dH/dT)_{T_c}(T_c/H_0)[\Delta(0)/kT_c]^{-1}$ would follow our curve, and that they would depart from the dashed line describing Toxen's relation. The experimental points available do show this trend, indicating, we believe, that Toxen's relation is a numerical coincidence (though a strikingly good one).

We would like to thank Professor A. Ron and Mr. A. Lonke for helpful discussions.

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EXTENSIONS OF THE MOMENTUM-TRANSFER THEOREM*

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(Received 22 November 1965)

Recently Lippmann,[1] in this Journal, has discussed the extension of the momentum-transfer theorem[2,3] to systems more complicated than the (elastic or inelastic) collisions of electrons with atomic hydrogen. Lippmann[1] also discussed extensions of the theorem to other observables, so as to derive, e.g., an energy-transfer theorem. In his discussion, Lippmann took exception to some remarks concerning the validity of the symbolic methods customarily employed in scattering theory. These remarks, from a preprint version of the paper which proved the momentum-transfer theorem for e-H collisions, were accurately quoted by Lippmann, but do not appear in the actually published paper,[3] because I already had decided the remarks were not wholly defensible.[4] Nevertheless there remain some differences between Lippmann's and my views of the status of the momentum-transfer theorem and its extensions. Making these differences explicit is the primary objective of this Letter.

In Lippmann's derivation of the momentum-transfer theorem, the starting point is

$$(\Psi^{(+)},\ \ [p_{1z}H-Hp_{1z}]\Psi^{(+)}), \tag{1a}$$

the "expectation value" of the commutator between the Hamiltonian H and p_{1z}, the momentum operator (along its incident direction) of the incident particle. Lippmann relates (1a) to the momentum-transfer cross section via symbolic methods. My starting point has been much the same as Lippmann's, namely, the