also. For half-lives in the 10^{-9}- to 10^{-3}-sec range, the same ferromagnetic environment can be used for resonance destruction of the angular distribution.[6] For this range of lifetime one needs to take advantage of the rf-amplitude enhancement in a ferromagnetic lattice while for longer lifetimes this may not be necessary.[7] For conventional spin-rotation measurements in an external magnetic field, the recoil atoms should go into nonmagnetic cubic metals, to avoid as far as possible attenuating interactions. Experiments along these lines are clearly feasible, and preliminary work is in progress.

In summary, our results have shown that a large and reasonably uniform degree of alignment is present throughout the neutron and gamma-ray cascades following heavy-ion nuclear reactions. This has been demonstrated to be very useful for spin and multipolarity assignments in spectroscopic studies of the de-exciting product nuclei. It promises to be useful also for studies of the nuclear moments and hyperfine interactions of levels in the product nuclei. Taken together with (1) the wide variety of nuclei that can be produced in compound-nucleus reactions, (2) the broad population of levels in a given product nucleus, and (3) the large uniform recoil velocities of compound nuclei, it makes the use of these reactions a general and powerful tool in nuclear spectroscopy.

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DYNAMICS OF THE GEOMAGNETIC TAIL

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In a paper recently published,[1] Ness has reported the experimental evidence of a magnetically neutral sheet behind the earth at a distance of (20-30)R_e (earth radii). This sheet separates regions of oppositely directed magnetic fields in the magnetic tail (Fig. 1). It has been proposed by Ness[1] and subsequent authors that the dynamics of this sheet may have an essential role in geomagnetic phenomena. This suggests as a model the analysis of the stability of a pinch containing a neutral sheet.[2] In fact, theory and laboratory experiments[3-5] clearly show that this configuration is violently unstable as it breaks up into separate pinches (Fig. 2), lying on the neutral sheet, which tend to repel each other. Considering the order of magnitude of the sheet thickness (600 km), and the energy of the electrons there contained, we see immediately that collisional effects (such as resistivity) cannot play a role in the dynamics of the neutral sheet. For this, we can take up the stability analysis of a collisionless pinch,[2,6] with the intention

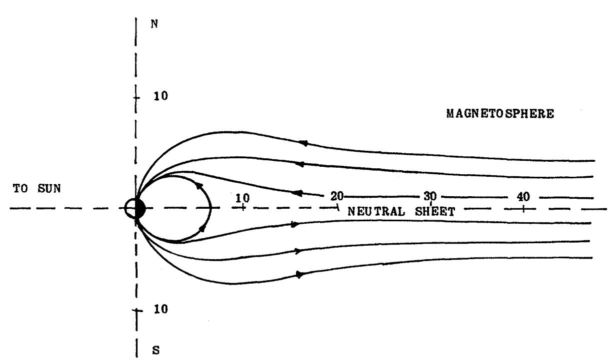

FIG. 1. Projection of magnetic field topology on noon-midnight meridian plane in the vicinity of the neutral sheet (unperturbed configuration). Distance in earth radii.

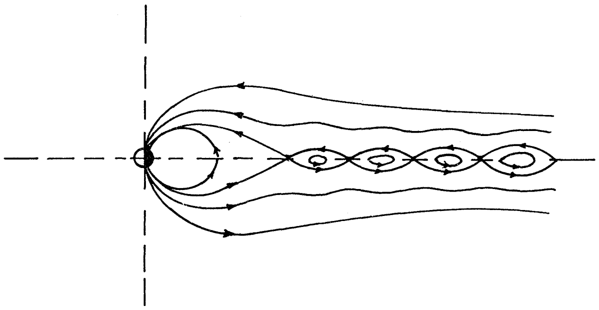

FIG. 2. Effects of the instability on magnetic field topology around the neutral sheet.

to show that the relevant instability has macroscopic effects,[7] as it transforms magnetic energy into kinetic energy, and that it can be suitable to explain the characteristic times of evolution of phenomena observed during auroral events, in the auroral regions, and in the magnetic tail.

We consider for this a one-dimensional model as in Fig. 1 where we set the X axis along the S-N direction, the Z axis on the ecliptic plane pointing from the earth to the sun, and the Y axis perpendicularly to them. The relevant perturbations from this (equilibrium) configuration are constant along the equilibrium current lines, in the Y direction, and have wavelengths parallel to the magnetic field lines, in the Z direction, of the order of the sheet thickness or larger. The result of the instability is a reconnection of initially antiparallel magnetic field lines. This occurs through the transfer of macroscopic energy of the plasma to a relatively small number of electrons through microscopic particle-wave resonant processes. The heated electrons lie on a thin sheet around the plane of zero magnetic field.

All these detailed features have not been observed as yet, but we must remember that the relatively small transverse scale (in the X direction) and the short duration of these phenomena make their direct detection by satellite difficult. Now, we show, in order to prove our point, that by computing the rate at which magnetic energy is transferred to the particles we obtain a growth time in agreement with the one which can be formally derived by the full theory.[6]

Take as equilibrium distribution functions for each species of particle

$$f_j^0 \approx n_0 \exp\{-\Theta_j^{-1}[\tfrac{1}{2}m_j v^2 - V_j(m_j v_y + q_j A_y^0)]\},$$

where v is the total particle velocity, m_j and q_j the mass and the charge, $\Theta_j = kT_j$ represents the temperature, V_j the flow velocity, n_0 the maximum density, and A_y^0 is the only component of the magnetic vector potential. The electric potential is zero as we choose the coordinate frame where $V_i/\Theta_i = -V_e/\Theta_e$. Then using Maxwell's equations, we find for the equilibrium magnetic field $B_y^0 = B_0 \tanh(x/\lambda)$, and for the density $n = n_0/\cosh^2(x/\lambda)$, where $B_0^2/8\pi = n_0(\Theta_i + \Theta_e)$, $\lambda^{-1} = qB_0(V_i - V_e)/2c(\Theta_i + \Theta_e)$, c being the velocity of light and λ the sheet thickness. For the mode we consider, the perturbed magnetic potential A^1 has the form $A^1 \approx A_y^1(x)\exp(i\omega t + ikz)$. Then, if W is the kinetic energy, we have

$$\frac{dW}{dt} = \sum_j q_j \int dx\, E_y^1 v_y f_j^1 d^3v,$$

where E_y^1 is the perturbed electric field $(-1/c) \times (\partial A_y^1/\partial t)$. In the neutral sheet the particle motion can be considered as free and determined by two reflecting walls with distance $d \simeq (r_L\lambda)^{1/2}$, r_L being the mean-particle Larmor radius. Using the linearized collisionless Boltzmann equation, we can write the approximate form of the perturbed distribution function in this region as

$$f_j^1 = (q_j f_j^0/\Theta_j)\{V_j A_y^1 - iE_y^1 v_y(\omega + kv_z)^{-1}\},$$

where the first term on the right-hand side represents the influence of the perturbed magnetic field and the second one represents the interaction between wave and particles. The latter contribution is negligible outside the neutral sheet $(x > d)$ as the particle motion becomes adiabatic and there is no electric field along the magnetic lines of force. Then, since it is physically interesting to consider a low-frequency mode, we assume $\omega \ll kv_{\mathrm{th}i}$ $(v_{\mathrm{th}i}^2 \equiv \Theta_i 2/m)$ and take $(\omega/k + v_z)^{-1} \approx i\pi\delta(v_z) + \mathrm{P}(1/v_z)$. Noticing that the integration of the principal part gives no contribution we find

$$\frac{dW}{dt} = \frac{\pi}{k}\sum_j \frac{q_j}{\Theta_j}\int_{d_j}^{d_j} dx \int d^3v f_j^0 \delta(v_z)(E_y^1 v_y)^2$$
$$-\frac{1}{4\pi}\frac{d}{dt}\int dx \frac{n(x)}{n(0)}\left(\frac{A_y^1}{\lambda}\right)^2$$
$$\equiv \sum_j \frac{d}{dt}W_j^R - \frac{d}{dt}W^M. \qquad (1)$$

Here $W_j{}^R$ represents the increment of kinetic energy for the resonant particles of species j, contained in the region $|x|<d$, and W^M is the decrement of the macroscopic kinetic energy of the remaining particles.

One can see that the kinetic-ion resonant term is smaller than the electron one by the factor $(r_{\mathrm{L}e}/r_{\mathrm{L}i})^{1/2}$. On the other hand, by conservation of the total energy,

$$\frac{dW}{dt}=-\frac{1}{8\pi}\frac{d}{dt}\int dx\,[B^1]^2. \qquad (2)$$

Then comparing the two expressions (1) and (2) we have

$$W_e{}^R=\frac{1}{\tau}\int_{-d_e}^{d_e}dx\int d^3v f_e{}^0\delta(v_z)\,|(A_y{}^1)^2 v_y|^2$$

$$=-\frac{k\Theta_e}{8\pi e^2}\int dx\left\{\left|\frac{\partial A_y{}^1}{\partial x}\right|^2+|A_y{}^1|^2\left[k^2-\frac{2}{\lambda^2\cosh^2(x/\lambda)}\right]\right\}$$

$$=W^M-\frac{1}{8\pi}\int dx(B^1)^2, \qquad (3)$$

where $\tau=-i/\omega$ represents the growth time.

We see from Eq. (3) that the instability is purely growing and that it exists for long wavelengths, such that $k^2\lambda^2<1$. Also, these features are typical of an instability having macroscopic effects. In particular, the growth time is of the order of $(\lambda/r_{\mathrm{L}e})^2 d_e/v_{\mathrm{th}e}$ in agreement with the formal result of Ref. 6. Therefore, Eq. (3) proves clearly that free macroscopic energy is transferred to resonant electrons contained in a slab of thickness $d_e\approx(r_{\mathrm{L}e}\lambda)^{1/2}$ on the ecliptic plane. Using the exact form $\tau=\pi^{-1/2}(2\lambda/r_{\mathrm{L}e})^{3/2}(\lambda/v_{\mathrm{th}e})\Theta_e/(\Theta_e+\Theta_i)$ and the results of measurements by the IMP-1 satellite,[1] we take $2\lambda=600$ km, $B_0\simeq1.6\times10^{-4}$ G, $\Theta_i\simeq1$ keV, and obtain

$$\tau\approx15 \text{ sec if } \Theta_e\simeq10 \text{ eV:}$$

$$\tau\approx5 \text{ sec if } \Theta_i\simeq1 \text{ keV,}$$

Θ_e being the electron temperature previous to the perturbation. In particular we see that the growth time is not very sensitive to the electron temperature. The energy to which particles are accelerated by the instability can be estimated by evaluating the electric field as resulting from the variation of magnetic flux across a contour of height λ. In a time of a few seconds, as consistent with the growth time, the electrons can attain an energy of the order of 10 keV, as observed during auroral events.[8]

This acceleration is maximum at the nodes of Fig. 2 where the magnetic field lines are reconnected. The relevant region, in which the magnetic field is violently perturbed, has a width of the order of 20 earth radii (1.2×10^5 km along the Y axis) and a height of the order of $d_e\approx50$ km corresponding to $\Theta_e\simeq1$ keV. The corresponding area, at the earth's surface, crossed by the same tubes of force is $\approx1.8\times10^3$ km^2, for a magnetic field strength ratio of 3×10^{-4}. If the longitudinal extent of this region is $\approx6\times10^3$ km then the latitude width becomes 0.3 km. Thus the above-mentioned region maps into one, near the earth, which is extended in longitude and very narrow in latitude, reminiscent of auroral arcs,[9,10] that have the same geometrical features.

To support this interpretation we can quote the fact that[10] the oval belt of the aurora coincides roughly with the intersection curve between the ionosphere and the outer magnetosphere. The lines of force of the magnetic tail terminate inside the oval belt. After a violent solar flare, the inner magnetosphere shrinks and correspondingly one observes a displacement of the oval belt toward the equator. During the subsequent auroral substorm, the arcs increase in intensity and move polewards, indicating a reconnection of the magnetic field lines which are the closest to the ecliptic plane, as predicted by the theory.

Our simple analysis cannot explain the presence of energetic electron spikes (electron islands) detected far from the neutral sheet in the magnetic tail. If one attributes their presence to large-amplitude magnetic perturbations[11] (solitons) propagating at an angle with the magnetic field, one can ask the question whether these nonlinear perturbations come from the boundary of the outer magnetosphere or from the instability of the neutral sheet which gives rise to macroscopic fluctuations of the magnetic field.

Finally, if the auroral substorm phenomenon[9,10] can be associated with the instability of the neutral sheet, the recovery time of the substorm may be related to the time to re-form the sheet. For this we can suppose that the particles on the night side have sufficient pressure to blow the outer reconnected magnetic field lines farther up to the stage where a sheet with opposite lines of force is formed.

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ONE-DIMENSIONALITY OF RELATIVISTIC PARTICLE FORCES FOR UNIFORM CENTER-OF-MASS MOTION

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It may be possible to construct instantaneous equations of motion for a Lorentz-invariant classical mechanical description of two interacting particles. Conditions for Lorentz invariance have been formulated as differential equations for the force functions.[1] The problem of finding physically interesting solutions appears to be rather complicated.[2] For one space dimension, solutions with interaction are easy to find. There are even examples which admit representations of the Poincaré group by canonical transformations[3]; for three space dimensions this is impossible.[4,3] In a recent Letter[5] it was suggested that forces which are also Galilean invariant might be particularly interesting; this was illustrated with a one-dimensional example. In three-dimensional space we find that there is no interaction if the acceleration of the center-of-mass coordinate is zero[6]; we show that the acceleration of the center-of-mass coordinate must be zero if the Lorentz-invariant equations of motion are also Galilean invariant.[7]

Consider two particles described by center-of-mass and relative coordinates $\vec{X}=(m_1+m_2)^{-1}\times(m_1\vec{x}^1+m_2\vec{x}^2)$ and $\vec{x}=\vec{x}^1-\vec{x}^2$ and velocities $\vec{V}=\dot{\vec{X}}$ and $\vec{v}=\dot{\vec{x}}$ with translation-invariant equations of motion $\ddot{\vec{X}}=\vec{F}(\vec{x},\vec{v},\vec{V})$ and $\ddot{\vec{x}}=\vec{f}(\vec{x},\vec{v},\vec{V})$. The conditions for Lorentz invariance are[6,8]

$$x_i\dot{F}_j+2V_if_j+2v_iF_j+F_iv_j+f_iV_j-\frac{\partial f_j}{\partial x_k}V_kx_i-\frac{\partial f_j}{\partial v_k}(F_kx_i+v_kV_i+V_kv_i)$$
$$-\frac{\partial f_j}{\partial V_k}(V_kV_i-\delta_{ki})-(m_1-m_2)(m_1+m_2)^{-1}\left\{x_i\dot{f}_j+2v_if_j+f_iv_j-\frac{\partial f_j}{\partial x_k}v_kx_i-\frac{\partial f_j}{\partial v_k}(f_kx_i+v_kv_i)\right\}$$
$$-\frac{m_1m_2}{(m_1+m_2)^2}\frac{\partial f_j}{\partial V_k}(f_kx_i+v_kv_i)=0, \quad (1)$$