PHYSICAL REVIEW LETTERS

VOLUME 16 | 24 JANUARY 1966 | NUMBER 4

ZERO-SOUND EXCITATIONS IN He^4

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(Received 22 November 1965)

It has recently been suggested by Pines[1] that the low-momentum (phonon) excitations observed[2,3] in the energy spectrum for liquid He^4 correspond to zero-sound oscillations, which are a complete analog, for a neutral system, of the plasmons in an electron liquid. Thus, the excitations closely resemble the zero-sound oscillation proposed by Landau for Fermi liquids, rather than ordinary first sound which has been the previously accepted identification.

This interpretation for the phonon mode of excitation was prompted by the inelastic-neutron-scattering experiment in He I and He II by Woods,[3] who has shown that the velocity of phonons with momentum 0.38 Å^{-1} is essentially independent of temperature (a small increase in velocity with temperature is actually observed) for all temperatures less than 2.57°K. Furthermore, neither the energy nor the lifetime of these excitations change appreciably on going from He II to He I.

It seems, therefore, that the low-momentum excitation energy in liquid helium is independent of ρ_S, the superfluid density. Pines has noted that a very similar behavior is displayed by an electron liquid where, for low momenta, the plasmon excitation energy is unaffected by the transition from the normal to the superconducting state and is independent of the superconducting density. For this case,

$$\omega(k)\tau \gg 1, \tag{1}$$

where $\omega(k)$ is the plasmon excitation energy and τ is its lifetime. The experiment by Woods was carried out in this same "collisionless" regime in which the restoring forces responsible for the oscillating sound waves have a very different character than do those responsible for ordinary first sound.

To investigate the validity of this proposed explanation for Woods's experimental results, a detailed calculation of the energy spectrum for densitylike fluctuations in a boson fluid is carried out. The method of procedure is completely analogous to the Pines-Bohm[4] treatment for the electron gas, and the results are found to be in complete agreement with Pines's[1] proposal.

The equation of motion for the density fluctuation

$$\rho_k = \sum_p a_{p+k}^{+} a_p$$

is, for either boson or fermion systems,

$$\ddot{\rho}_k = -\sum_p (k^2/2m + \vec{p}\cdot\vec{k}/m)^2 a_{p+k}^{+} a_p - k^2[NV(k)/m]\rho_k - m^{-1} \sum_{k \neq k} (\vec{k}\cdot\vec{k}')V(k')\rho_{k'}\rho_{k-k'}, \tag{2}$$

where a_k and a_k^+ are the plane-wave annihilation and creation operators, $V(k)$ is the Fourier transform of the two-body interaction potential, and N is the total number of particles per unit volume. The kinetic energy is

$$T_k = k^2/2m,$$

where $\hbar$ has been set equal to unity for convenience. Within the random-phase approximation (RPA), the term which is nonlinear in the density fluctuations may be neglected in Eq. (2). This approximation corresponds to the neglect of coupling between density fluctuations of different momenta, and it is therefore in concert with the experiments[2,3] which, for temperatures less than 2.57°K, were performed in this "collisionless" regime, where the mean time between collisions for the collective modes is very large compared to their period of oscillation, i.e., for $\omega\tau \gg 1$. The first term on the right of Eq. (2) is associated with the random kinetic motion of the individual particles in the system, and for low-momentum excitations it is expected to be very small. To the extent that it may be neglected compared to the remaining potential-energy term in Eq. (2), the equation of motion (with the RPA) shows that the Fourier components of the particle density behave as independent excitations of the system, oscillating with circular frequency $\omega(k)$. That is,

$$\ddot{\rho}_k + \omega^2(k)\rho_k = 0, \tag{3}$$

where

$$\omega(k) = ck = k[NV(k)/m]^{1/2}. \tag{4}$$

At low momenta, where $V(k)$ is assumed to be nearly constant, the quantity c is usually associated with the first-sound velocity but here it will be referred to only as the phonon velocity for obvious reasons.

Just as in electron fluids, the RPA seems to be a valid approximation for boson fluids at low momenta. This is apparent from the work of Feynman[5] who has shown that the excited states $|k\rangle = \rho_k|0\rangle$ are approximate eigenstates for liquid helium at low momenta, where $|0\rangle$ is the ground-state wave function. This result was established by showing that the derived expression for the excitation spectrum agrees with experiment for all momenta $\vec{k} \leq 0.6$ Å^{-1}. For such eigenstates, it is an easy matter to show that the equation of motion for ρ_k must be of the form given by Eq. (3). Therefore, the terms nonlinear in the density fluctuation (RPA terms) must be of negligible importance to the result. From Eqs. (2) and (4) it is evident that the criterion for neglecting the random kinetic motion of the individual particles in the equation of motion is that

$$g^2 = \frac{(k^2/2m + \vec{p}\cdot\vec{k}/m)^2}{\omega^2(k)} \ll 1 \tag{5}$$

for all relevant momenta. Clearly then, only to the extent that Eq. (5) is satisfied can ρ_k possibly be a satisfactory collective coordinate.

It is apparent that Eq. (4) gives, for the phonon velocity c, an expression that is independent of the temperature and the superfluid density, which is substantially what has been observed experimentally. However, it remains to investigate the temperature range of validity for the approximations leading to this result, and to deduce the temperature-dependent corrections to c, if any exist.

It can be shown, just as Pines and Bohm[4] have done for the electron fluid, that the effect of the random kinetic motion of individual particles is simply accounted for in a boson fluid. The resulting expression for the excitation spectrum is[6]

$$1 = 2V(k)\sum_p N_p \frac{(T_{p+k} - T_p)}{\omega^2(k) - (T_{p+k} - T_p)^2}, \tag{6}$$

where N_p is the momentum distribution function. This equation can then be expanded in powers of g^2 provided $g^2 < 1$, and, to second order, the result for $\omega(k)$ at low momenta is

$$\omega^2(k) = c^2k^2 = 2NV(k)T_k + 3N^{-1}\sum_p (\vec{p}\cdot\vec{k}/m)^2 N_p. \tag{7}$$

By averaging over an isotropic momentum distribution, c becomes

$$c(\theta) = [NV(k)/m + 2\overline{T}_i(\theta)/m]^{1/2}, \tag{8}$$

where $\overline{T}_i(\theta)$ is the average kinetic energy of the individual particles and θ is the temperature. This result is very interesting because it shows that the depletion of particles in the single-particle zero-momentum state leads to increased (not decreased) excitation energies, which is contrary to the predictions of other theories[7,8] but is in accord with the experimental work of Woods.

For finite temperatures, the kinetic energy of the individual particles is derived not only from the interparticle interactions but also from thermal excitations. Actually, $\overline{T}_i(\theta)$ is the only temperature-dependent quantity in the expression for the phonon velocity c. The constant quantity $NV(k)/m$ in Eq. (8) is not directly calculable because the interaction potential for He is not precisely known for small interatomic separations, where it is strongly repulsive. As a result, its Fourier transform cannot be accurately evaluated. However, this difficulty is avoided in calculating the change, with respect to temperature, of the phonon velocity. That is, the quantity $\Delta c(\theta) = c(\theta) - c(0) = \Delta\overline{T}_i(\theta)/mc(0)$ is directly determined from $\Delta\overline{T}_i(\theta) = \overline{T}_i(\theta) - \overline{T}_i(0)$, where $c(0)$ is taken to be the accepted experimental value of the phonon velocity extrapolated to 0°K.[9] An approximate expression for $\Delta\overline{T}_i(\theta)$ is obtained from the ideal boson-gas model, which may be expected to yield a reasonable estimate for the change of individual particle kinetic energies with temperature, especially for values of θ on the order of or greater than θ_λ, the lambda temperature. These expressions are[10]

$$\Delta\overline{T}_i(\theta) = \tfrac{3}{2}k\theta(\theta/\theta_\lambda)^{3/2}[\zeta(\tfrac{5}{2})/\zeta(\tfrac{3}{2})], \quad \theta < \theta_\lambda; \tag{9}$$

$$\Delta\overline{T}_i(\theta) = \tfrac{3}{2}k\theta[1 - 0.4618(\theta_\lambda/\theta)^{3/2} - 0.0226(\theta_\lambda/\theta)^3 - \cdots], \quad \theta > \theta_\lambda. \tag{10}$$

The ζ's are the Riemann zeta functions, k is Boltzmann's constant, and $\theta_\lambda = 2.19$°K for He^4. For $c(0) = 239$ m/sec, and with Eqs. (9) and (10), the values of $\Delta c(\theta)$ are calculated and presented in Fig. 1. The points represent the known experimental results[2,3] and the solid line represents our theoretical prediction. It is apparent that this theory correctly predicts the temperature dependence for the phonon velocity over the entire range of temperatures up to and including 2.57°K.

The strength of the parameter g^2 may be shown to represent a rough measure of the coupling between the individual particles and the collective excitations. As a result, the collective modes can exist as independent excitations only

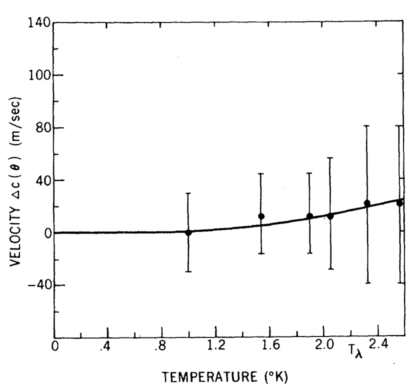

FIG. 1. Change of the phonon velocity with temperature, for liquid helium at normal vapor pressure. The dots represent the experimental results and the solid line represents our theoretical prediction.

if g^2 is small compared with unity. Therefore, in order to establish the validity of our results for all temperatures of interest, it is necessary to examine the change with temperature of g^2. For small excitation momenta, g^2 reduces to

$$g^2 = (\vec{p}\cdot\vec{k}/m)^2/\omega^2(k),$$

and it is apparent that Eq. (5) is then satisfied provided

$$c(\theta) \gg \overline{v}_i, \tag{11}$$

where $\overline{v}_i = \overline{p}/m$ is the average velocity of the individual particles. That is, the system possesses a phonon mode of excitation only if the zero-sound velocity c is large compared to the velocities of the individual particles. Even though this condition may be met at 0°K, for sufficiently large temperatures $\overline{v}_i$ becomes comparable to c. In fact, for $\theta = 2.57$°K, Eq. (10) shows that $\overline{v}_i \simeq 0.22c$, and for temperatures approaching 4.2°K, Eq. (11) is no longer even approximately satisfied. As a result, the phononlike collective modes no longer exist as well-defined independent excitations for temperatures approaching 4.2°K. This prediction agrees well with the experiment of Woods where it is found that, at $\theta = 4.2$°K, the phonon-excitation line is so broad that it is no longer discernable as an observed excitation.

It appears, therefore, that if Eq. (11) holds at $\theta = 0$°K, it also holds over a considerable range of temperatures; that is, approximately for $0 \leq \theta \leq 4.2$°K, but not at higher temperatures. Also note that g^2 is less strongly dependent on temperature for large rather than for small momentum transfers. Thus, thermal broadening begins to play, for small momentum transfers, a more important role at lower temperatures, as pointed out by Pines.

In conclusion, we find that the properties associated with the excitation spectrum for densitylike fluctuations in liquid helium are completely consistent with Woods's experimental results and that these results are also in agreement with the proposal by Pines—that these excitations should more appropriately be associated with zero-sound oscillations in the "collisionless" regime for which $\omega\tau \gg 1$, rather than with ordinary sound for which $\omega\tau \ll 1$.

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[6]R. D. Etters, Bull. Am. Phys. Soc. 10, 721, 113 (1965).

[7]P. Hohenberg and P. Martin, Phys. Rev. Letters 12, 69 (1964).

[8]K. Huang, The Many Body Problem (John Wiley & Sons, Inc., New York, 1958), p. 636.

[9]On the basis of this theory, one would expect that

$$\lim_{k\to 0}\,[NV(k)/m + 2\overline{T}_i(0)/m]^{1/2} \simeq \lim_{k\to 0}\,[NV(k)/m]^{1/2} \simeq c(0) = 239\ \text{m/sec}$$

for He II, which is the value for $c(0)$ to which we have normalized our data. The commonly used phenomenological potentials for helium cannot be expected to yield this value for the phonon velocity because they do not adequately represent the interaction for values of $\vec{r}$ significantly smaller than the equilibrium interatomic separation, nor are they intended to. For example, the familiar 6-12 potential does not even possess a Fourier transform because of its highly singular character at small r, a very unphysical characteristic. However, based on work in progress, we have shown that it is possible to impose a constraint on the phenomenological potential so that the correct experimental value is realized for $c(0)$, and the resulting potential $V(r)$ still agrees with experimental data to within 5% in the vicinity of the equilibrium interatomic separation and at large interatomic separations, which are the only regions where the potential is well known anyway. Thus, it is not unlikely that our results are compatible with known experimental data for $V(r)$ and that all that is required for a complete treatment is a more realistic form for the potential at small $\vec{r}$.

There is, however, another possible interpretation for the quantitative success of this theory. That is, we have in effect actually chosen an effective potential $V_{\text{eff}} = V(k)$, where V_{eff} is such as to yield the observed phonon velocity $c(0)$. This is equivalent to arguing that the short-range correlations which are neglected in the RPA are such as to change $V(k)$ from being simply the Fourier transform of $V(r)$ to that of an effective potential V_{eff}. As to which interpretation is the more correct, it remains to be seen.

[10]F. London, Superfluids (John Wiley & Sons, Inc., New York, 1950), p. 47.

COLLECTIVE SCATTERING OF LASER LIGHT BY A PLASMA

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(Received 23 December 1965)

Several authors[1-5] have reported experiments on the scattering of laser light from electrons in a laboratory plasma. The profiles of the scattered light show a nearly Gaussian distribution, indicating little or no collective effect between the ions and the electrons. It is of interest to observe this collective effect, on the one hand to verify the theory[6,7] which predicts satellite peaks approximately at the plasma frequency, and on the other hand to develop a useful technique for the diagnostics of plasmas. This paper reports an observation of the scattering of light from a pulsed ruby laser by a plasma jet. The profile of the scattered light shows unambiguously the distinct satellite peaks on both sides of the central frequency, indicating strong collective effects between the ions and the electrons.

A plasma jet is used because it is fairly simple to obtain a reproducible plasma with an electron density of 10^{16} to 10^{17} cm^{-3} and an electron temperature of 1 or 2 eV.[8] These conditions make it possible to observe the satellites at a scattering angle of 45°. It is much easier to reduce the stray light when making observations at this large angle as compared to small forward-scattering angles. In addition, since the jet is operated at atmospheric pressure, we need no windows or walls in the neighborhood of the plasma.

The jet is mounted vertically. It draws 280 A at 15 V from a battery and rheostat power supply. The diameter of the jet nozzle is 5 mm. The flow of the argon gas is controlled at a steady velocity of 15 m sec^{-1}.

A TRG giant-pulse ruby laser, with peak power of 10 MW and a pulse duration of 50 nsec, is used. The light from the laser is focused onto a pinhole, and this is then focused with a second lens at the center of the plasma. The light from the laser is monitored with a photodiode. Suitable light baffling and light traps are provided so that the stray light being reflected into the detector system is less than