

conductivity measurements is 10%, the relative error is less than 3%. The error in the absolute value is due to an uncertainty in the "in-series" thermal-conductivity background.

<sup>20</sup>T. Seidel and H. Meissner, Phys. Letters **17**, 100 (1965), also investigated the thermal conductivity of Pb films near  $H_{C3}$ , but did not report on the size dependence of  $(dK_S/dH)_{H_{C3}}$ .

# SIMPLE EXPLANATION OF THE "SHOENBERG ANOMALIES" IN THE de HAAS-van ALPHEN EFFECT\*

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Since the observation by Shoenberg<sup>1</sup> and Joseph and Thorsen<sup>2,3</sup> of a rather large harmonic content in the de Haas-van Alphen (dHvA) oscillations in the noble metals, there has been a concerted effort<sup>4-8</sup> to account for this in terms of the notion that an electron sees  $\vec{B}$  rather than  $\vec{H}$  and that, consequently, related orbits may be magnetically coupled. The simple ad hoc theory adduced by Shoenberg has been rigorously justified by Pippard to the extent that the magnetization may be considered as a function of  $\vec{B}$ , but the further implication that  $\vec{B}$  replaces  $\vec{H}$  in the independent-electron formalism is still highly suspect. In fact, if the electrons are to be allowed to interact magnetically, their Coulomb interactions must also be included and one has the additional complication of dielectric screening which is also of an oscillatory nature. It is tempting to believe, as is suggested by Luttinger's work,<sup>9</sup> that the independent-particle theory survives these complications intact, except for a modification in the over-all amplitude or phase. Instead, in this note it is argued that the anomalous harmonic content owes its origin to a simple, generally overlooked, feature of the independent particle theory, namely, that it is due simply to the FM coupling of the dHvA oscillations brought about by the field dependence of the chemical potential. It is possible to base an argument on the Lifshitz-Kosevich<sup>10</sup> formula quoted by Shoenberg, but the following calculation has some presumptions of exactness and presents, in a simple way, all the features we wish to emphasize.

In a forthcoming paper,<sup>11</sup> the author has calculated the exact free energy (per unit volume)  $F$ , in a uniform magnetic field  $H\hat{z}$ , of a metal having the zero-field band structure

$$\xi(k) = k_x^2/2m_1 + k_y^2/2m_2 + E(k_z),$$

where  $E(k_z)$  is monotonic, but otherwise arbitrary. The result for  $T=0^\circ\text{K}$  is, writing  $k_z = k = k(E)$  and taking for simplicity  $m_1 = m_2 = m$ ,

$$F = n\zeta - \frac{m\mu_0 H}{\pi^3 \hbar^2} \sum_{l=1}^{\infty} l^{-1} \int_0^{\zeta} k(E) \sin\left[\frac{\pi l(\zeta - E)}{\mu_0 H}\right] dE \\ - \frac{m}{\pi^2 \hbar^2} \int_0^{\zeta} k(E)(\zeta - E) dE, \quad (1)$$

where  $n$  and  $\zeta$  are the electron density and chemical potential. This expression is exact, except for the assumption of independent electrons and the neglect of surface effects. However, the chemical potential has still to be determined from the relation  $\partial F/\partial \zeta = 0$ , which becomes

$$\frac{\pi^2 \hbar^2 n}{m} = \sum_{l=0}^{\infty} \int_0^{\zeta} k(E) \cos\left[\frac{\pi l(\zeta - E)}{\mu_0 H}\right] dE. \quad (2)$$

For purposes of explicit calculation we adopt the band structure, shown in Fig. 1,

$$k(E) = a^{-1} \cos^{-1}(1 - E/\gamma), \quad 0 \leq E \leq 2\gamma, \\ = a^{-1}\pi, \quad 2\gamma \leq E \leq 2\gamma + \Delta, \quad (3)$$

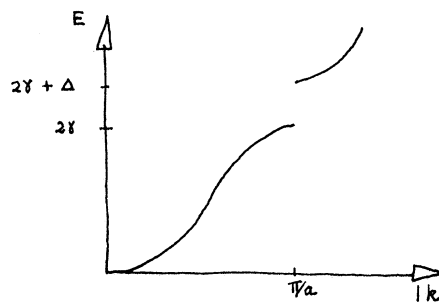


FIG. 1. Model band structure in the  $k_z$  direction.  $\Delta$  denotes the band gap. The band structure throughout the remainder of  $\vec{k}$  space is parabolic.

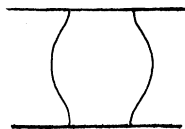


FIG. 2. Fermi surface for the model. The heavy black lines denote the Brillouin-zone boundaries and the surface has cylindrical symmetry about the field direction.

where  $a$  is the lattice spacing (in the  $z$  direction),  $2\gamma$  is the band width, and  $\Delta$  is the energy gap across the first zone face. It is assumed that at zero field the chemical potential  $\xi_0$  lies in the region  $2\gamma < \xi_0 \leq 2\gamma + \Delta$  so the Fermi surface has the shape indicated in Fig. 2. On the basis of the Onsager argument, the dHvA effect should show a "neck" and a "belly" oscillation, and indeed from (1) we find (retaining only the  $l=1$  term)

$$F^{\text{osc}} = \frac{m}{\pi^4 \hbar^2 a} (\mu_0 H)^2 \left( \frac{\mu_0 H}{\gamma} \right)^{1/2} \times \left\{ \cos \left( \frac{\pi \xi}{\mu_0 H} - \frac{\pi}{4} \right) + \cos \left[ \frac{\pi(\xi - 2\gamma)}{\mu_0 H} + \frac{\pi}{4} \right] \right\}, \quad (4)$$

where both the amplitude and phases are in accord with the semiclassical theory. Equation (2) for  $\xi$  becomes

$$\xi - \xi_0 = (\mu_0 H / \pi) \{ \sin(\pi \xi / \mu_0 H) - \sin[\pi(\xi - 2\gamma) / \mu_0 H] - \sin[\pi(\xi - \gamma) / \mu_0 H] J_0(\pi \gamma / \mu_0 H) \}, \quad (5)$$

and to a good approximation (assuming  $\mu_0 H \ll \pi \gamma$ ),

$$\xi - \xi_0 = (\mu_0 H / \pi) [\sin \alpha - \sin \alpha'], \quad (6)$$

where  $\alpha = \pi \xi_0 / \mu_0 H$ ,  $\alpha' = \pi(\xi_0 - 2\gamma) / \mu_0 H$ . For example, the term corresponding to the belly oscillation now becomes

$$\cos[\pi \xi / \mu_0 H - \pi/4] = \cos\{(\alpha - \pi/4) + \sin \alpha - \sin \alpha'\}. \quad (7)$$

Where  $\alpha \sim \alpha'$ , as is the case for the cigar-orbit oscillations<sup>5</sup> in Be, (7) becomes very nearly

$$\cos(\alpha - \pi/4) \cos(\sin \alpha - \sin \alpha'),$$

and we obtain the expected belly oscillation slowly modulated by a complex wave form, as is observed. If  $\alpha'$  disappears or becomes much smaller than  $\alpha$  then (7) becomes, over a very long range of field strengths,  $\cos(\alpha - \pi/4 + \sin \alpha)$ , which shows no modulation. Similar considerations apply also to the neck oscillation. It thus appears possible to account for the wide variety of observed dHvA wave forms directly in terms of the independent electron theory of the effect, without the necessity for rather complicated extra considerations.

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