We would like to acknowledge many discussions with L. R. Walker, R. E. Walstedt, and Y. Yafet.

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# INCOHERENT SCATTERING OF MICROWAVES BY UNSTABLE ELECTRON PLASMA OSCILLATIONS*

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The possibility of using incoherent scattering of electromagnetic waves by plasma oscillations as a diagnostic tool has received considerable attention recently.[1] The importance of this method for the investigation of unstable plasmas has been emphasized by Ichimaru, Pines, and Rostoker[2] and by Drummond.[3] Arunasalam and Brown[4] have observed incoherent scattering from the unstable ion-acoustic mode in a dc discharge, and scattering by driven plasma oscillations has been demonstrated in two recent publications.[5]

In this Letter we report the observation of incoherent scattering of microwaves by high-frequency oscillations resulting from a beam-plasma instability. The experimental arrangement is sketched in Fig. 1. An electron beam of 0.5 A, 15-21 keV, and 2 $\mu$sec duration is injected into the afterglow of a pulsed discharge in neon at a pressure of $3\times10^{-2}$ Torr. Discharge and electron beam are triggered at a repetition frequency of 15 cps. By varying the delay between the initiation of the discharge and the injection of the beam, the beam can be exposed to plasma densities in the range between $10^{13}$ and $10^{11}$ electrons/cm$^3$. The decay time for the plasma is 300 $\mu$sec so the plasma density can be taken as constant during the time the beam is on. In the region of interest the plas-

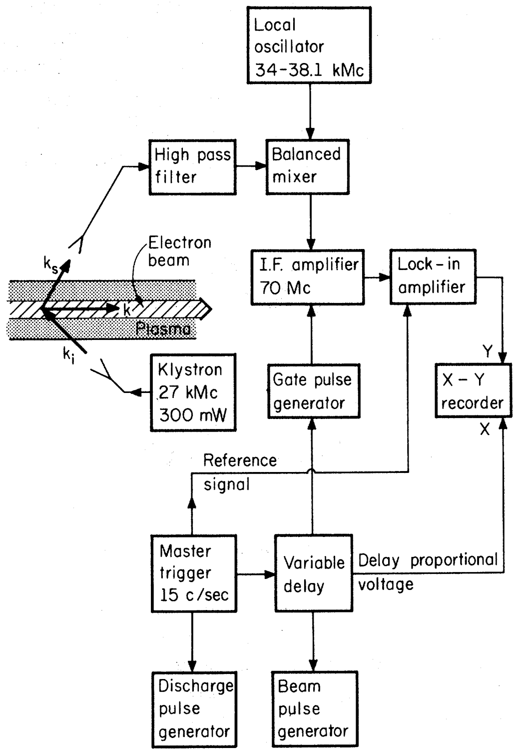

FIG. 1. Schematic of the experiment. The plasma diameter is 4 cm, the beam diameter 1 cm.

ma is at room temperature as inferred from the ambipolar diffusion coefficient. The beam instability, however, increases the electron temperature by a factor of 6 for beam currents in excess of 300 mA. The plasma is illuminated under a variable angle by a 300-mW, 27-kMc/sec cw signal from a klystron. A receiving horn, mounted at an angle with respect to the axis of the discharge, is coupled to a receiver via a high-pass filter of 29-kMc/sec cutoff frequency. The received signal is mixed in a balanced mixer with a local oscillator, variable between 38 and 34 kMc/sec. The difference frequency of 70 Mc/sec is amplified by an i.f. amplifier of 8-Mc/sec bandwidth. The i.f. amplifier is gated on during the beam pulse, and the rectified output is detected in a synchronous fashion by a lock-in amplifier.

The planes of polarization of the transmitting and receiving horns are such that the $E$ vector is perpendicular to the axis of the electron beam. Under these conditions the scattered intensity per solid angle is given by

$$dP_s/d\Omega = 2Nr_0^2(P_0/F)S(k,\omega)\Delta\omega,$$

where $N$ is the total number of electrons in the scattering volume, $r_0$ the classical electron radius, $P_0/F$ the incident power density, $\Delta\omega$ the receiver bandwidth, and $S(k,\omega)$ the dynamical form factor of the density fluctuations. $\vec{k}$ and $\omega$ are determined by $\vec{k}=\vec{k}_s-\vec{k}_i$, $\omega=\omega_s-\omega_i$, where $\vec{k}_i$ and $\omega_i$ are wave vector and frequency of the incident radiation, $\vec{k}_s$ and $\omega_s$ represent the scattered radiation. In the present experiment the most unstable $\vec{k}$ is expected to be parallel to the direction of the electron beam. By varying the angles between $\vec{k}_i$ and $\vec{k}$ as well as between $\vec{k}_s$ and $\vec{k}$, values of $k$ from 4 to 10 $\text{cm}^{-1}$ could be selected.

Figure 2 shows a recorder trace of the received power as a function of electron density in the vicinity of the scattered signal. The two neighboring peaks are due to radiation at higher harmonics of the plasma frequency. Phenomena associated with the radiation will be the subject of a subsequent paper and will not be discussed further at this point.

Figure 3 shows a plot of scattered power versus $k$ as inferred from the angular distri-

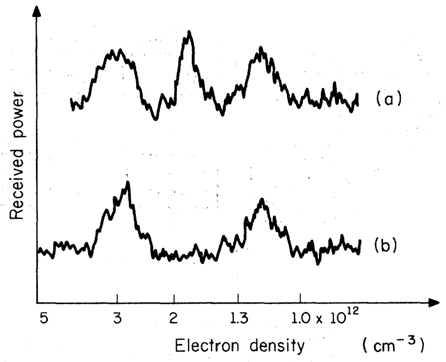

FIG. 2. Received power versus electron density. Frequency and power of incident microwave, 27.0 kMc/sec, 300 mW; local oscillator frequency, 38.1 kMc/sec; angle $(\vec{k}_i,\vec{k})=60°$, angle $(\vec{k}_s,\vec{k})=40°$; electron beam: 500 mA, 21 kV. (a) Incident microwave power on. The center peak is due to scattering, the adjacent peaks are due to radiation at harmonics of the plasma frequency. (b) Incident microwave power off.

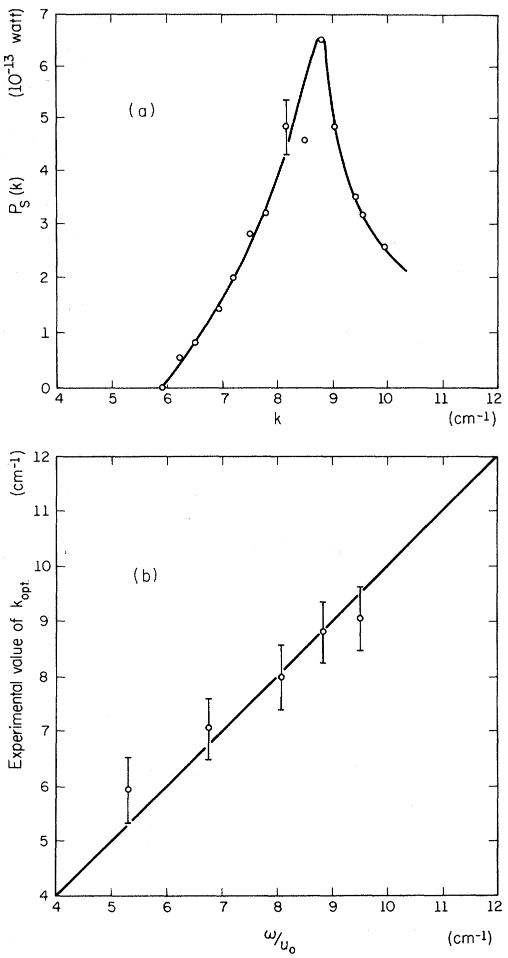

FIG. 3. (a) Example of scattered power as a function of the wave number of unstable waves in the direction of the electron beam. (b) Experimental value of $k_{\text{opt}}$ as a function of $\omega/u_0$.

bution. The maximum of the scattering is expected to occur for the most unstable $k$, which is given to a good approximation by $k_{\text{opt}} = \omega/u_0$, where $u_0$ is the beam velocity and $\omega$ the frequency of the most unstable mode. This is true for a cold as well as a warm beam, the corrections due to finite beam density and thermal velocity spread being within the accuracy of the measurement. $k_{\text{opt}}$ can be changed by varying either $u_0$ or $\omega$. Both parameters were varied and the result is shown in Fig. 3(b) where the experimental values of $k_{\text{opt}}$ are plotted versus $\omega/u_0$ together with the theoretical curve. The width of the $k$ spectrum is determined primarily by the angular resolution of the microwave horns, so that a comparison with theoretical predictions based on the quasilinear theory[6] is not possible at the moment. The appearance of strong radiation at harmonics of the plasma frequency would indicate, however, that mode-mode coupling terms give rise to strong density fluctuation at these frequencies and that these terms may no longer be considered to be small. We seem to be faced, therefore, with a case of "strong turbulence," for which no satisfactory theories seem to exist at the present time. The magnitude of the scattered power indicates that the beam-plasma interaction has enhanced the fluctuation spectrum by a factor $10^9$-$10^{10}$ over the thermal equilibrium value.

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# MINIMUM $B$ AS A PLASMA STABILITY CRITERION*

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We describe a class of toroidal plasma equilibria with concentric flux surfaces in which a magnetic-field minimum can be continuously varied from the interior to the exterior of the plasma with no apparent alteration of its stability. The spatial disposition of $\int dl/B \sim V'(\psi)$, some form of which is frequently interpreted as a stability criterion, is unchanged as the absolute minumum of $B$ moves from inside to outside the plasma. This casts doubt on a widely held intuitive belief concerning the relation of minimum $B$ to plasma stability and shows that "minimum $B$" and "minimum average $B$" are not necessarily related concepts.

It has been widely advocated that, in order to promote stability, plasma configurations should be constructed as magnetic wells (minimum $B$)[1]; and when this is not possible, one should at least try to obtain an average magnetic well.[2] Many examples of open-ended (mirror) magnetic well configurations have been found.[3,4] But it has been claimed that no true minimum-$B$ toroidal configurations exist,[2] and an important recent discovery was the existence of (presumably second-best) toroidal average-magnetic-well configurations.[2,5] Careful ex-