MINIMAL CP NONCONSERVATION IN K^0 DECAY*

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The discovery of the decay $K_L{}^0 \to \pi^+ + \pi^-$ by Christenson, Cronin, Fitch, and Turlay[1] has raised serious doubts concerning the conservation of CP and T in weak interactions. This question can be precisely restated in terms of the treatment by Lee, Oehme, and Yang[2] of the decay properties of the K^0, which are obtained from the eigenvalues and eigenstates of the equation

$$(\Gamma + iM)\psi_{S,L} = \lambda_{S,L}\psi_{S,L}, \tag{1}$$

where Γ and M are 2×2 matrices, and the two components of $\psi_{S,L}$ are the amplitudes of the K^0 and $\bar{K}^0$ components in ψ_S or ψ_L. With CP and T conservation, all the elements of Γ and M must be real and

$$\Gamma_{11} = \Gamma_{22}, \quad M_{11} = M_{22}, \tag{2}$$

$$\Gamma_{12} = \Gamma_{21}, \quad M_{12} = M_{21}. \tag{3}$$

The question is, "Does the $K_L{}^0 \to \pi^+ + \pi^-$ decay require the introduction of imaginary components to Eq. (3) to describe correctly the time development of K^0 amplitudes given by Eq. (1), thus forcing the abandonment of CP and T conservation in the description of particle amplitudes?"

Attention is drawn to one rather unique possibility of minimal CP noninvariance in K^0 decay which would allow the retention of CP and T conservation conditions, Eq. (3), for the K^0 states, yet would be consistent with the known experimental results for $K_L{}^0 \to \pi^+ + \pi^-$. To appreciate its uniqueness, the following analysis is presented.

Applying the conditions of Eqs. (2) and (3) to Eq. (1), one finds that the eigenfunctions are the $CP = \pm 1$ eigenstates

$$\psi_S = (1/\sqrt{2})(K + \bar{K}) \equiv K_1, \quad \psi_L = (1/\sqrt{2})(K - \bar{K}) \equiv K_2. \tag{4}$$

It is now convenient to study the function

$$Z = \sum_n a(K_L \to n)a^*(K_S \to n), \tag{5}$$

which Coleman and Glashow[3] have discussed, where the sum is over all possible decay modes, n, and the a's are the amplitudes of the respective decay modes, including density-of-states factors. From Eqs. (1), (2), (3), (4), and (5) it can be shown[4] that

$$Z = 0, \tag{6}$$

and, conversely, that Eqs. (1), (2), (5), and (6) imply Eq. (3) is valid. Indicating explicitly the known decay modes of the K^0, one obtains

$$\begin{aligned} Z = {} & a(K_2 \to \pi^+ + \pi^-)a^*(K_1 \to \pi^+ + \pi^-) \\ & + a(K_2 \to \pi^0 + \pi^0)a^*(K_1 \to \pi^0 + \pi^0) \\ & + a(K_2 \to 3\pi)a^*(K_1 \to 3\pi) \\ & + a(K_2 \to \pi^+ + l^- + \bar{\nu})a^*(K_1 \to \pi^+ + l^- + \bar{\nu}) \\ & + a(K_2 \to \pi^- + l^+ + \nu)a^*(K_1 \to \pi^- + l^+ + \nu). \end{aligned} \tag{7}$$

This function is useful because the magnitude and approximate phase of the first term, $Z(\pi^+\pi^-)$, is known from experiment.[1,5] The result is indicated in Fig. 1 as a vector from the origin to any point in the shaded region.[6] In order to satisfy the requirements of CP and T invariance of Eq. (1), yet account for the experimen-

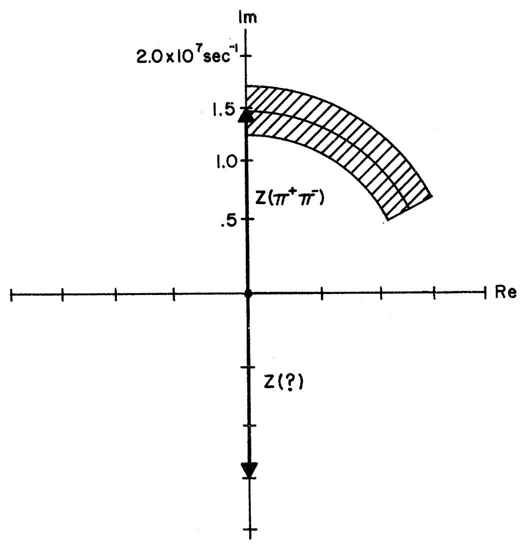

FIG. 1. $Z(\pi^+\pi^-) = a(K_L \to \pi^+ + \pi^-)a^*(K_S \to \pi^+ + \pi^-)$ must lie in or near the shaded region [arbitrarily placed in the first quadrant, corresponding to $M(K_2) > M(K_1)$] according to experiment. $Z(?)$ represents the term in Z which is desired to cancel $Z(\pi^+\pi^-)$. Effects of unequal $\pi\pi$ phase shifts in the $I = 0$ and $I = 2$ final states have been neglected.

tally observed CP nonconserving amplitude represented by $Z(\pi^+\pi^-)$ in Fig. 1, $Z(\pi^+\pi^-)$ must be cancelled by other terms in Eq. (7), whose sum is represented by $Z(?)$ in Fig. 1.

The contribution of the last two terms in Eq. (7), $Z(\pi l\nu)$, can be rewritten:

$$Z(\pi l\nu) = 2i\,\mathrm{Im}[a(K\to\pi^- + l^+ + \nu)a^*(\bar{K}\to\pi^- + l^+ + \nu)]. \quad (8)$$

The assumption that $K_S = K_1$ and $K_L = K_2$ and the conditions imposed on the decay amplitudes by TCP invariance require that all of the terms, such as $Z(\pi l\nu)$, in Z be pure imaginary in the absence of final-state interactions. If $Z(\pi l\nu)$ occurs with the direction and approximate magnitude shown for $Z(?)$ in Fig. 1, one asks if it could cancel a purely imaginary $Z(\pi^+\pi^-)$ compatible with present experimental limits as indicated in the figure. If the $\Delta S = \Delta Q$ rule is exact, $Z(\pi l\nu) = 0$, failing to cancel $Z(\pi^+\pi^-)$. Even with maximal violation of the $\Delta S = \Delta Q$ rule and with the most favorable relative phases, as suggested by Sachs,[7] it is found that[8]

$$|Z(\pi l\nu)| \leq |a(K\to\pi^- + l^+ + \nu)|^2 + |a(\bar{K}\to\pi^- + l^+ + \nu)|^2 \leq (1.09\pm 0.15)(10^7)\ \mathrm{sec}^{-1}, \quad (9)$$

whereas the magnitude of $Z(\pi^+\pi^-)$ is

$$|Z(\pi^+\pi^-)| = (1.45\pm 0.2)(10^7)\ \mathrm{sec}^{-1}. \quad (10)$$

Since recent experiments do not find evidence of a large $\Delta S = -\Delta Q$ amplitude,[9] $Z(\pi l\nu)$ is an unlikely candidate to balance $Z(\pi^+\pi^-)$.

The 3π term in Z,

$$Z(3\pi) = a(K_2\to 3\pi)a^*(K_1\to 3\pi), \quad (11)$$

would be expected to have a much smaller magnitude than $Z(\pi^+\pi^-)$ since it is known that[10]

$$|a(K_1\to\pi^+ + \pi^- + \pi^0)| \lesssim 1.6\,|a(K_2\to\pi^+ + \pi^- + \pi^0)|. \quad (12)$$

Even with the most favorable choice of isotopic spin, $I = 1$, for $a(K_1\to 3\pi)$ and of the phase between amplitudes,

$$|Z(3\pi)| \lesssim 1.6\,|a(K_2\to 3\pi)|^2 = (1.1)(10^7)\ \mathrm{sec}^{-1}, \quad (13)$$

so the $Z(3\pi)$ term is also an improbable source of a cancellation of the observed $Z(\pi^+\pi^-)$ term.

In the term due to the decay into two π^0's,

$$Z(\pi^0\pi^0) = a(K_2\to\pi^0 + \pi^0)a^*(K_1\to\pi^0 + \pi^0), \quad (14)$$

it is known from the dominance of the $I = 0$ $\pi\pi$ state in K_1^0 decay that

$$a^*(K_1\to\pi^0 + \pi^0) = (1/\sqrt{2})a^*(K_1\to\pi^+ + \pi^-). \quad (15)$$

If one assumes that the K_2^0 also decays predominantly into the $I = 0$ $\pi\pi$ state, then Eq. (18) holds for the decay $K_2^0\to\pi^0 + \pi^0$, as well, so

$$Z(\pi^0\pi^0) = \tfrac{1}{2}Z(\pi^+\pi^-). \quad (16)$$

Thus, if K_2^0 decays to an $I = 0$ $\pi\pi$ state, $Z(\pi^0\pi^0)$ can only increase the magnitude of Z.

The last remaining possibility is to assume that the K_2^0 decays into an $I = 2$ $\pi\pi$ state, in which case

$$a(K_2\to\pi^0 + \pi^0) = -\sqrt{2}\,a(K_2\to\pi^+ + \pi^-). \quad (17)$$

In this event, one finds

$$Z(\pi^0\pi^0) = -Z(\pi^+\pi^-), \quad (18)$$

exactly achieving the desired cancellation. If $Z(3\pi)$ and $Z(\pi l\nu)$ are zero, then Eq. (6) is satisfied. Truong[11] has already discussed under more general conditions the possible origin of the decay $K_L^0\to\pi^+ + \pi^-$ in $\Delta I = \frac{3}{2}$ transitions which give rise to an $I = 2$ $\pi\pi$ amplitude. However, the $I = 2$ $\pi\pi$ amplitude required from K_L^0 decay is about 1/26 of the $I = 2$ $\pi\pi$ amplitude present in K^+ decay. This ratio might be more easily accounted for if both $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{5}{2}$ transitions occur in K^+ decay.

It is seen that only the hypothesis of K_L^0 decay into an $I = 2$ $\pi\pi$ state would allow a remarkable special case compatible with experiment where CP noninvariance of a decay mode does not destroy the CP invariance of the K_S^0 and K_L^0 eigenstates. Under these special conditions, one obtains

$$\frac{a(K_L\to\pi^+ + \pi^-)}{a(K_S\to\pi^+ + \pi^-)} = i\exp[i(\delta_2 - \delta_0)]\epsilon, \quad (19)$$

$$\frac{a(K_L\to\pi^0 + \pi^0)}{a(K_S\to\pi^0 + \pi^0)} = -2i\exp[i(\delta_2 - \delta_0)]\epsilon, \quad (20)$$

where ϵ is a real number and δ_0 and δ_2 are the $\pi\pi$ phase shifts in the $I = 0$ and $I = 2$ states, respectively, due to final-state interactions. A unique branching ratio,

$$\frac{|a(K_L\to\pi^0 + \pi^0)|^2}{|a(K_L\to\pi^+ + \pi^-)|^2} = 2, \quad (21)$$

would be expected, since the K_L^0 would be forbidden to decay into an $I = 0$ $\pi\pi$ state. Analogously, the K_1^0 and K_2^0 eigenstates would not be disturbed by a CP-nonconserving, $I = 3$ 3π

amplitude of appropriate phase in K_1^0 decay. Relations similar to Eqs. (19), (20), and (21) for 3π final states are

$$\frac{a(K_S\to\pi^+ +\pi^- +\pi^0)}{a(K_L\to\pi^+ +\pi^- +\pi^0)} = i\exp[i(\delta_3-\delta_1)]\epsilon', \tag{22}$$

$$\frac{a(K_S\to\pi^0 +\pi^0 +\pi^0)}{a(K_L\to\pi^0 +\pi^0 +\pi^0)} = \tfrac{2}{3}i\exp[i(\delta_3-\delta_1)]\epsilon', \tag{23}$$

$$\frac{|a(K_S\to\pi^0 +\pi^0 +\pi^0)|^2}{|a(K_S\to\pi^+ +\pi^- +\pi^0)|^2} = \frac{2}{3}, \tag{24}$$

where ϵ' is a real number, and δ_1 and δ_3 are the corresponding phase shifts due to final-state interactions.

Of the class of theories which might require $K_L^0\to\pi^+ +\pi^-$ to be dominated by an $I=2$ final state and which would maintain $K_S^0=K_1^0$ and $K_L^0=K_2^0$, a simple example is the following:[12] Let the strangeness-changing weak-interaction Hamiltonian be written as the sum of two parts,

$$H = H_{\Delta I=\frac{1}{2}, CP=+1} + H_{\Delta I=\frac{5}{2}, CP=-1}, \tag{25}$$

where the first term, assumed to be dominant, gives the decay $K_1^0\to 2\pi$ with the familiar $\Delta I=\frac{1}{2}$ selection rule. The second, CP-nonconserving term gives rise only to $I=2$ and $I=3$ states in K decay and since CP is required to change, $K_2^0\to(I=2)$ and $K_1^0\to(I=3)$ are the only CP-nonconserving K^0 decay modes.

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[1]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2]T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957).

[3]S. Coleman and S. L. Glashow, to be published.

[4]Using the notation of Ref. 2, $p^2 \equiv \Gamma_{12}+iM_{12}=q^2$ using Eq. (3). Expressing Z in terms of K and $\bar{K}$ decay amplitudes, $Z=(|p|^2-|q|^2)\Gamma_{11}+pq(\Gamma_{12}^*-\Gamma_{12})=0$.

[5]V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).

[6]As the authors of Refs. 1 and 5 present their results differently, the error assignments in Fig. 1 are order-of-magnitude estimates by the author. $\delta=0.8\pm0.2$ is assumed for the K_1^0–K_2^0 mass difference.

[7]R. G. Sachs, Phys. Rev. Letters 13, 286 (1964).

[8]Based upon the decay times and branching ratios given in University of California Radiation Laboratory Report No. UCRL 8030 (unpublished), Pt. I, the following decay rates are employed: $|a(K_1\to 2\pi)|^2=(1.10\pm0.02)(10^{10})\ \text{sec}^{-1}$, $|a(K_2\to 3\pi)|^2=(0.67\pm0.09)(10^7)\ \text{sec}^{-1}$, $|a(K_1\to\pi+l+\nu)|^2 \cong |a(K_2\to\pi+l+\nu)|^2=|a(K\to\pi^-+l^++\nu)|^2+|a(\bar{K}\to\pi^-+l^++\nu)|^2=(1.09\pm0.15)(10^7)\ \text{sec}^{-1}$, $|a(K^+\to\pi^++\pi^0)|^2=(1.75\pm0.04)(10^7)\ \text{sec}^{-1}$.

[9]L. Kirsch et al., Phys. Rev. Letters 13, 35 (1964); B. Aubert et al., Phys. Letters 17, 59 (1965).

[10]J. A. Anderson et al., Phys. Rev. Letters 14, 475 (1965); 15, 645 (1965).

[11]T. N. Truong, Phys. Rev. Letters 13, 358a (1964).

[12]The author thanks Professor S. L. Glashow for suggesting this example.

SU(6) RESULTS DERIVED FROM CHIRAL SU(3)⊗SU(3) ALGEBRA*

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A suggestion by Gell-Mann[1] that the algebra generated by current components may be useful in understanding the systematics of the hadrons even though the Hamiltonian may not be invariant has been fruitfully pursued recently. Lee[2] and Dashen and Gell-Mann[3] used the SU(6) algebra generated by the time component of the vector current and the space components of the axial-vector currents and, by assuming that the matrix elements of the commutator algebra between zero-momentum octet baryons are saturated by intermediate octet and decuplet states, derived many of the results previously obtained from SU(6) invariance. Adler[4] and Weisberger,[5] on the other hand, used the chiral SU(3)⊗SU(3) algebra generated by the time components of the vector and axial-vector currents to calculate the renormalization of the axial-