consider a case where n clusters are $m=1$ and the other (n_c-n) clusters of $m \geqslant 2$ have (n_s-n) spins as a whole. Then the number of independent states $w(n_c, n_s, n)$ is represented as $w={}_{n_c}C_n \times {}_{n_c-n}H_{n_s-n_c}$. Accordingly, the mean value of n, i.e., n_e, is given by

$$n_e=\sum_n nw/W=\sum_{n=1}^{nc-1} n\times {}_{n_c}C_n \times {}_{n_c-n}H_{n_s-n_c}.$$

Using a formula of combination products,

$$\sum_{r=0}^{q} {}_{\alpha}C_{q-r} \times {}_{\beta}C_r = {}_{\alpha+\beta}C_q \quad (\alpha \geqslant q, \beta \geqslant q),$$

n_e reduces to $n_e=n_c(n_c-1)/(n_s-1) \simeq n_c^2/n_s$ when $n_c \gg 1$, $n_s \gg 1$. Similarly, the number of clusters having $m=\lambda$ is obtained as $n_c^2/n_s(1-n_c/n_s)^{\lambda-1}$.

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SYSTEMATICS OF LOW-LYING 0^+ STATES IN LIGHT NUCLEI*

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Recently it has been suggested[1] that the 20.3-MeV 0^+ state of He^4 might be described by a "breathing mode." The present note calls attention to certain systematics[2] of 0^+ excited states in light even-even nuclei, particularly closed-shell nuclei, which show that a breathing-mode[3] description of these states is probably not successful. A more fruitful description in terms of multipair excitations, a pair consisting of a particle and a hole, is indicated.

Table I lists the 0^+ states considered here. Column 2 gives[4] excitation energies E. Column 3 presents a first systematic trend: The energies of these states follow roughly a $1/R^2$ dependence, where R is the nuclear radius. For a breathing mode one expects a $1/R$ energy dependence.[5] The argument can of course be made[1] that the He^4 state represents a breathing mode and that in the other nuclei the corresponding state, lying roughly at 20.3 $(4/A)^{1/3}$ MeV, has not yet been discovered.

Table I. 0^+-0^+ transitions in light nuclei. The energy of the transition is denoted by E, the pair width by Γ_π. The quantities ρ and f are the normalized monopole transition element and fraction of a monopole sum rule exhausted by the transition. Exact definitions are given in the text.

	E (MeV)	$EA^{2/3}$	Γ_π (eV)	ρ^e	f (%)
He^4	20.3	51	$(3.4\pm0.9)\times10^{-4}$ a	0.23	11
C^{12}	7.66	40	$(6.2\pm0.6)\times10^{-5}$ b	0.52	16
O^{16}	6.05	38	$(9.2\pm0.9)\times10^{-6}$ c	0.31	4.0
Ca^{40}	3.35	39	$(1.9\pm0.1)\times10^{-7}$ d	0.12	0.25
Zr^{90}	1.75	35	$(2.2\pm0.2)\times10^{-9}$ d	0.048	···

[a]Ref. 6.
[b]Ref. 7.
[c]Ref. 8.
[d]Ref. 4.
[e]The percentage uncertainty in ρ is approximately one-half of that in Γ_π.

Column 4 in Table I lists the internal pair widths Γ_π of the states, taken from various references.[4,6-8] A second systematic trend can be noted by extracting from Γ_π the normalized transition matrix element $\rho=\langle r^2\rangle_{if}/R^2$, where $\langle r^2\rangle_{if}$ is the monopole matrix element between the initial state i and the final state f and R is the nuclear radius. As has been customary[9] we have put $R=(\frac{1}{2}e^2/mc^2)A^{1/3}=1.41A^{1/3}$ F. Using interpolations of calculations by Zirianova and Krutov,[10] the values for ρ in column 5 of Table I are found. One notes that the normalized transition matrix element for He^4 is comparable to that for C^{12}, O^{16}, and Ca^{40}. A similar conclusion can be drawn by substituting the experimental monopole matrix element into Ferrell's sum rule[11] for $T=0$ to $T=0$ monopole transitions:

$$N=\sum_f 2mE\,|\langle r^2\rangle_{if}|^2/(\hbar^2\langle r^2\rangle), \tag{1}$$

where N = number of nucleons, m = mass of nucleon, $\langle r^2\rangle$ = mean square radius of ground state. One then finds that each of the 0^+-0^+ transitions considered here exhausts only a fraction f of the sum rule, shown in column 6 of Table I. [The fraction f is defined as the ratio of the right-hand side of Eq. (1) to the left-hand side.] It can be seen that the He^4 transition does not exhaust a greater portion of the sum rule than the C^{12} transition, and neither exhausts the full sum rule. Since the model discussed in Ref. 1 does exhaust[1] the full sum rule (1), it does not appear that the He^4 state is well de-

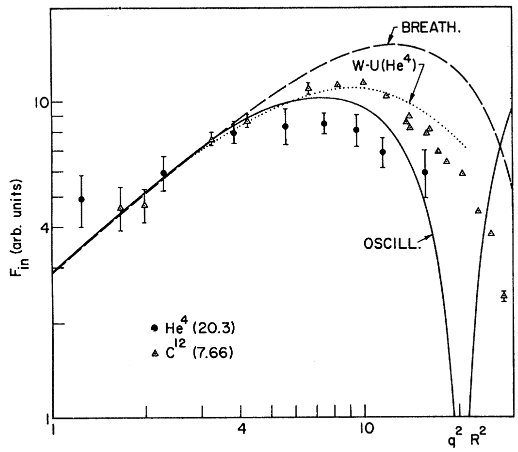

FIG. 1. Shapes of form factors for inelastic electron scattering to 0^+ excited states of He^4 and C^{12}, vs q^2R^2 where q is the momentum transfer and R is the nuclear radius. The value of R has been taken as the equivalent uniform radius of Ref. 12. Solid points, for the 20.3-MeV state of He^4, are taken from Ref. 6. Triangle points, for the 7.66-MeV state of C^{12}, are taken from Ref. 7. The solid ("OSCILL.") and dashed ("BREATH.") lines represent results of the oscillating and the breathing liquid-drop models of Ref. 5. The dotted line ["W-U(He^4)"] gives the prediction for He^4 of the breathing mode model of Ref. 1. Experimental and theoretical form factors have been arbitrarily normalized at low q^2 values.

scribed by this model.

One can also investigate the nature of the 0^+-0^+ transitions by studying the shape of the inelastic electron scattering form factor[7] $F_{\rm in}(q^2)$ as a function of the square of the momentum transfer q^2. Figure 1 compares the shapes of $F_{\rm in}$ for[6] the He^4 and[7] the C^{12} transitions with those calculated by Walecka[5] for an oscillating or breathing quantized liquid drop of radius R.[12] Also shown is the prediction of Ref. 1 for the He^4 transition.[13] All the form factors have been normalized arbitrarily at low q^2 values, where, independently of the model, they are expected[14] to be proportional to q^2. One sees that neither the He^4 nor the C^{12} form factors follow the shapes predicted by the models of Refs. 1 and 5.

Szydlik[15] has recently computed the energies of the lowest states of He^4 on a one- and two-pair model. He finds a 0^+ state close to 20 MeV. Similar success for O^{16} has been obtained[16] by including four-pair excitations. The systematics presented here should encourage further computations along these lines, in particular for $F_{\rm in}$. In this connection one must take into account the fact that the 0^+ state of He^4 is unbound.[17]

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