SUPERCONDUCTORS AND GRAVITATIONAL DRAG*

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Schiff and Barnhill[1] have shown that the electric field inside of a conductor does not vanish when a gravitational field is present. This phenomenon, which is of importance in connection with proposals to determine whether electrically shielded positrons will fall up or fall down, does not depend on the presence of a nonvanishing Riemann tensor but holds already for uniform gravitational fields, i.e., acceleration fields: If the acceleration of the conductor is $-\vec{g}$ then the quantity which vanishes in its interior, and indeed inside any comoving Faraday cage, is not the electric vector $\vec{E}$ but rather the sum $\vec{E}+(m/e)\vec{g}$, where m and e are the mass and charge of the charge carriers.[2] Since the electron is the universal carrier for almost any conceivable electrical cage, this means that a shielded electron will simply float and, if the equivalence principle is valid, a shielded positron will fall twice as fast as a neutral particle.

It is the purpose of this note to point out that a similar phenomenon holds for magnetic fields inside of superconductors. Whenever matter is in motion near a superconductor, so that a Lense-Thirring field[3] is present, the Meissner effect requires the vanishing not of the magnetic vector $\vec{H}$ but of a linear combination of $\vec{H}$ and the Lense-Thirring field. Moreover, it is the flux of the latter combination, through any superconducting ring, which gets quantized in units of $h/2e$ rather than the magnetic field alone.

To discuss the quantum properties of a superconductor in a gravitational field we first compute the Hamiltonian H of a single electron. This can be obtained from the Lagrangian

$$L=-m(-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu})^{1/2}+eA_{\mu}\dot{x}^{\mu}, \tag{1}$$

where e and m are the charge and mass of the electron, $g_{\mu\nu}$ is the metric tensor, A_μ is the electromagnetic vector potential, and dots denote differentiation with respect to the time x^0.[4] One finds

$$H=(g^{ij}g_{0i}g_{0j}-g_{00})^{1/2}[m^2+g^{kl}(p_k-eA_k)(p_l-eA_l)]^{1/2}-g^{ij}(p_i-eA_i)g_{0j}-eA_0, \tag{2}$$

where $g^{\mu\nu}$ is the inverse metric and $\vec{p}$ is the canonical momentum. In the limit of small velocities and weak fields this reduces, after removal of the rest mass, to

$$H=(1/2m)(\vec{p}-\vec{B})^2+V, \tag{3}$$

where

$$V=-eA_0-\tfrac{1}{2}mh_{00}, \tag{4}$$

$$\vec{B}=e\vec{A}+m\vec{h}_0,\ \ \vec{h}_0=(h_{01},h_{02},h_{03}), \tag{5}$$

$$h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu},\ \ \eta_{\mu\nu}=\eta^{\mu\nu}=\mathrm{diag}(-1,1,1,1). \tag{6}$$

For the Hamiltonian of the ensemble of free electrons inside a superconductor, (3) is replaced by

$$H=\sum\{(1/2m)[\vec{p}_n-\vec{B}(\vec{x}_n)]^2+V(\vec{x}_n)\}+V_{\mathrm{int}}, \tag{7}$$

where $\vec{x}_n$ and $\vec{p}_n$ are the canonical variables of the nth electron and V_{int} includes the electron-phonon interaction and the phonon energy. All of the apparatus of the BCS theory[5] may be applied to this Hamiltonian, with the result that the Meissner effect implies the vanishing of the vector

$$\vec{G}=\vec{\nabla}\times\vec{B}=e\vec{H}+m\vec{\nabla}\times\vec{h}_0 \tag{8}$$

inside the superconductor. Moreover, taking into account the Cooper pairs as in the conventional derivation, one easily finds that the total flux of $\vec{G}$ linking a superconducting circuit must be quantized in units of $\frac{1}{2}h$. Since a superconductor is a conductor one also sees that the Schiff-Barnhill field,

$$\vec{F}=-\vec{\nabla}V=e\vec{E}+\tfrac{1}{2}m\vec{\nabla}h_{00}, \tag{9}$$

must vanish inside.

Now consider an experiment in which the superconductor is a uniform circular ring surrounding a concentric, axially symmetric, quasirigid mass. Suppose the mass, initially at rest, is set in motion until a constant final angular velocity is reached. This produces a Lense-Thirring field which, in a coordinate system for which the metric is time independent, takes the form

$$\vec{\nabla}\times\vec{h}_0 = 16\pi\kappa\nabla^{-2}\vec{\nabla}\times(\rho\vec{V}), \qquad (10)$$

where κ is the gravitation constant and ρ and $\vec{V}$ are, respectively, the mass density and velocity field of the rotating mass. If $\vec{H}$ is initially zero then so is $\vec{G}$. Because of the flux quantization condition the flux of $\vec{G}$ through the superconducting ring must remain zero. But since $\vec{\nabla}\times\vec{h}_0$ is nonvanishing in the final state, a magnetic field must be induced. Suppose the rotating mass is kept electromagnetically neutral (which means compensating for any Schiff-Barnhill polarization which may be induced in it). Then the magnetic field must arise from a current induced in the ring. The magnitude of this current will be

$$I = -\frac{m}{eL}\int_S \vec{\nabla}\times\vec{h}_0\cdot d\vec{S} = -\frac{16\pi\kappa m}{eL}\oint(\nabla^{-2}\rho\vec{V})\cdot d\vec{r}, \qquad (11)$$

where S is the area spanned by the ring, L is its self-inductance, and the final integral is taken around the ring. Assuming the dimensions of the ring to be little larger than those of the rotating mass, we obtain the order-of-magnitude estimate

$$I \sim \kappa m M V/ed, \qquad (12)$$

where V is the rim velocity of the rotating mass, M is its mass, and d is its diameter.

The current I arises from an induced motion of electrons on the surface of the superconductor. This motion is in the same direction as the motion of the rotating mass, and represents another instance of the well-known drag effect produced by the Lense-Thirring field. It could, in principle, be detected by leading the superconducting current into a region where the Lense-Thirring field is negligible and measuring its magnetic effect.[6]

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[1]L. I. Schiff and M. V. Barnhill, Bull. Am. Phys. Soc. 11, 96 (1966).

[2]The mass m is here the static rest mass and not the effective mass in a conduction band.

[3]J. Lense and H. Thirring, Physik. Z. 29, 156 (1918).

[4]The following notational conventions are adopted in this paper: Lower case Greek and Latin indices have the ranges 0, 1, 2, 3 and 1, 2, 3, respectively. The signature of the metric is $-+++$. Boldface symbols denote three-vectors. Units are chosen in which $c=1$.

[5]See, for example, C. Kittel, Quantum Theory of Solids (John Wiley & Sons, Inc., New York, 1963), Chap. 8.

[6]R. Forward, Bull. Am. Phys. Soc. 10, 531 (1965).

NON-S-STATE PARAMAGNETIC RESONANCE IN A DILUTE ALLOY

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Various experimental techniques have been applied to the problem of localized magnetic moments in dilute alloys, but paramagnetic resonance measurements have previously been restricted to solid-solution alloys or intermetallic compounds containing Mn,[1-3] Gd,[4-7] or divalent Eu,[5] where a g value close to 2.0 seems to indicate S character for the presumably half-filled $3d$ or $4f$ shell.[8] We have now observed the paramagnetic resonance due to erbium in dilute solution in silver, the effective g value being very close to that observed[9,10] for trivalent Er in cubic environments in nonmetallic matrices, where the $^4I_{15/2}$ state is split by the crystal field to give a ground-state doublet.

The resonance measurements were made at X band (8950 Mc/sec) with a high-frequency-modulation reflection spectrometer, in which the large thermal capacity of the epoxy casing around the cavity and an efficient automatic frequency control made possible measurements at intermediate temperatures while the cavity slowly warmed up from 4.2°K. The specimens were filed powders prepared from an arc-melted silver-0.3 at.% erbium ingot which had been given a homogenizing anneal.