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$SU(6)_W$ AND MESON RESONANCES OF EVEN PARITY*

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It was shown recently that if the three-particle vertices involving the pseudoscalar and vector octet and singlet mesons are invariant to the group $SU(6)_W$, the one-M (-meson) exchange force may bootstrap the M as P-wave, MM bound states.[1] In this model, there are forces in states of even orbital angular momentum that are of the same order as the P-wave forces. These even-l forces will lead to predicted meson resonances of even parity. The purpose of this note is to list the quantum numbers and relative coupling strengths of different decay modes of the predicted even-parity mesons, and to compare some of the results with experiment.

The basic odd-parity mesons M are taken to be degenerate. The $SU(6)_W$-symmetric MMM vertices are given in Ref. 1; alternatively, they may be determined from the work of Sakita and Wali.[2] The method of calculation involves constructing the vertices so that the $SU(6)_W$ symmetry is preserved if one particle is off the mass shell, and then examining the various one-M-exchange amplitudes in Born approximation at the physical threshold energy. The many-channel amplitudes corresponding to particular initial and final orbital angular momenta are then diagonalized. The threshold value of an eigenamplitude is called the eigenvalue. It is assumed that resonances or bound states may be associated with the largest positive eigenvalues, and that the relative constants of interaction of the different two-particle configurations of a composite particle are equal to the relative components of the configurations in the corresponding eigenamplitude.

It is shown in Ref. 1 that the one-M-exchange amplitude at threshold for any specific pair of MM states may be written in the form $A_{ij} \times(\vec{k}-\vec{k}')_i(\vec{k}-\vec{k}')_j$, where i and j label the spatial axes, $\vec{k}$ and $\vec{k}'$ are the initial and final three-momenta in the center-of-mass system, and A_{ij} is an operator in the space of the intrinsic spins of the initial and final mesons. Those amplitudes that are linear in both $\vec{k}$ and $\vec{k}'$ are P-wave amplitudes. However, amplitudes quadratic in either $\vec{k}$ or $\vec{k}'$ are of the same order; these amplitudes represent both S-wave scattering and S-D transitions.

At present there is no reliable way to estimate the relative importance of S-S and S-D amplitudes, since these amplitudes are associated with different phase-space factors.[3] In this note we will neglect the S-S amplitudes, because this assumption leads to simple results, and because D-wave decays of even-parity meson resonances are observed to be important experimentally.

All S-D transitions corresponding to the SU(3) representations $\underline{10}$, $\underline{10}^*$, and $\underline{27}$ vanish, for reasons discussed at the end of this note. Since only S-D transitions are considered, the total S-wave component and total D-wave component of each eigenamplitude are of equal magnitude,

Table I. Quantum numbers of the meson states of positive eigenvalue.

Eigenvalue	States
$(5)^{1/2}$	$(\underline{1},1)^+$
$(11/4)^{1/2}$	$(\underline{1},5)^+$
$(5/4)^{1/2}$	$(\underline{1},3)^-$, $(\underline{8},1)^+$, $(\underline{8},5)^+$, $(\underline{8},3)^+$, $(\underline{8},3)^-$

and the eigenvalues occur in pairs of equal positive and negative values. All eigenamplitudes corresponding to positive eigenvalues are listed in Table I. The eigenvalues are normalized for convenience. The symbol $(a,b)^i$ denotes a multiplet of SU(3) and spin multiplicities a and b, whose $I_z = Y = 0$ members are of C parity i. The octet part of this spectrum is identical to that resulting from the assumption that these resonances are quark-antiquark states, with all P-wave states resonating. In the present model, the relative couplings of the different S-wave MM states, and of the D-wave states, are predicted. The results are

$$S(1,1)^+ = (\tfrac{3}{4})^{1/2}(PP) = (\tfrac{1}{4})^{1/2}(VV)_0,$$
$$D(1,1)^+ = (VV)_2,$$
$$S(1,5)^+ = (VV)_2,$$
$$D(1,5)^+ = (3/11)^{1/2}(PP) + (1/11)^{1/2}(VV)_0 + (7/11)^{1/2}(VV)_2,$$
$$S(1,3)^- = (PV),$$
$$D(1,3)^- = (PV),$$
$$S(8,1)^+ = (\tfrac{3}{4})^{1/2}(PP)^d + (\tfrac{1}{4})^{1/2}(VV)_0{}^d,$$
$$D(8,1)^+ = (VV)_2{}^d,$$
$$S(8,5)^+ = (VV)_2{}^d,$$
$$D(8,5)^+ = (3/20)^{1/2}(PP)^d + (1/20)^{1/2}(VV)_0{}^d + (7/20)^{1/2}(VV)_2{}^d + (9/20)^{1/2}(PV)^f,$$
$$S(8,3)^+ = (PV)^f,$$
$$D(8,3)^+ = (\tfrac{1}{4})^{1/2}(PV)^f + (\tfrac{3}{4})^{1/2}(VV)_2{}^d,$$
$$S(8,3)^- = (\tfrac{1}{2})^{1/2}(VV)_1{}^f + (\tfrac{1}{2})^{1/2}(PV)^d,$$
$$D(8,3)^- = (\tfrac{1}{2})^{1/2}(VV)_1{}^f + (\tfrac{1}{2})^{1/2}(PV)^d. \quad (1)$$

The symbols $S(a)$ and $D(a)$ denote the S- and D-wave functions for the bound state a, and the subscript on the symbol (VV) denotes the total intrinsic spin. The relative components of the singlet-singlet and octet-octet parts of the SU(3) singlet states are given by the formula

$$\psi = (\tfrac{1}{9})^{1/2}\psi_{11} + (8/9)^{1/2}\psi_{88}.$$

The superscripts f and d denote F-type octet-octet-octet coupling, and D-type nonet-nonet-nonet coupling.[4] The relative components of the octet-octet and octet-singlet parts of the D-type octet wave functions are given by the formula

$$\psi^d = (5/9)^{1/2}\psi_{88} + (4/9)^{1/2}\psi_{(81+18)}.$$

In order to indicate the possible effect of ω-φ mixing on the decays of the $(PV)^d$ states, we list the terms of these wave functions that involve the SU(3) singlet and octet isoscalar V mesons ω_1 and ω_8:

$$\psi(I=1) = (2/9)^{1/2}(\pi\omega_1) - (\tfrac{1}{9})^{1/2}(\pi\omega_8) + \cdots,$$
$$\psi(I=0) = (2/9)^{1/2}(X\omega_8) + (2/9)^{1/2}(\eta\omega_1) + (\tfrac{1}{9})^{1/2}(\eta\omega_8) + \cdots,$$
$$\psi(I=\tfrac{1}{2}) = (2/9)^{1/2}(K\omega_1) + (1/36)^{1/2}(K\omega_8) + \cdots. \quad (2)$$

The ω-φ mixing that corresponds to "quark-model type" symmetry breaking is[5]

$$\omega = (\tfrac{2}{3})^{1/2}\omega_1 - (\tfrac{1}{3})^{1/2}\omega_8, \quad \varphi = (\tfrac{1}{3})^{1/2}\omega_1 + (\tfrac{2}{3})^{1/2}\omega_8, \quad (3)$$

where ω and φ are the observed particles.

The experimental data concerning even-parity meson resonances are rather skimpy and have been summarized by several authors.[6] We discuss here only a few salient points related to the data. Since the momentum dependences at threshold of P-P and S-D amplitudes are the same, we have normalized the S-D eigenvalues to be consistent with the P-P eigenvalues of Ref. 1, i.e., the ratio of any two eigenvalues is the ratio of the corresponding S-matrix elements in Born approximation. The eigenvalue describing the P-wave states associated with the P nonet and V octet is 2.[1] If the even-parity (S-D) state $(\underline{1},1)^+$ is identified with the Pomeranchuk trajectory, or with some low-energy phenomenon in $\pi\pi$ scattering, its mass is smaller than the average M mass. Therefore, it is reasonable that the eigenvalue of this state is larger than 2. The eigenvalue $(11/4)^{1/2}$ of the spin-2 singlet of Table I is a little disturbing, since the experimental data favor a spin-2 singlet and octet of comparable masses.[6] However, the eigenvalues are only rough indications of the expected masses, since the large mass splitting of the M multiplet has not been taken into account. It is reasonable that the observed spin-0, even-parity octet be lighter than the spin-2 octet, because of the strong coupling of the former to S-wave PP states.

If the 1220-MeV B meson corresponds to the isovector member of the multiplet $(\underline{8},3)^-$, and if the ω-φ mixing is that of Eq. (3), then the predicted $\pi\varphi/\pi\omega$ branching ratio is zero, as may be seen from Eq. (2). This is consistent with experimental indications.[6] The 1320-MeV A_2 meson, assumed here to be the isovector member of the spin-2 octet, provides a good test of predicted relative PP/PV decay rates, as these modes are both D-wave modes, and the thresholds for the $\pi\rho$, $K\bar{K}$, and $\pi\eta$ channels are comparable. The predicted branching ratios, uncorrected for phase-space differences, are $\pi\rho{:}K\bar{K}{:}\pi\eta=18{:}3{:}2$.[7] This agrees with the observed dominance of the $\pi\rho$ mode.[8] More data will make many other experimental comparisons possible. It is interesting to note that if the ~1500-MeV f' meson is a linear combination of the two isoscalar spin-2 mesons, the tail of the ρ-ρ decay mode should be observable.

We now discuss the reason that certain S-D transitions vanish. Such transitions must vanish for all states of an SU(3) representation R that satisfies the condition

$$\langle\psi_R^{\ m}|[T,\vec{S}]|\psi_R^{\ m}\rangle=-\langle\psi_R^{\ m}|[T,\vec{L}]|\psi_R^{\ m}\rangle=0, \quad (4)$$

where T is the scattering amplitude, $\vec{S}$ and $\vec{L}$ are the total intrinsic spin and the orbital angular momentum, and the $\psi_R^{\ m}$ represent arbitrary spin states of any specific internal state m of the representation R. Any S-D transition must contribute in the forward and backward directions, so we may limit attention to the collinear amplitudes T_c. In our model T_c are $SU(6)_W$ symmetric, and hence commute with all $SU(6)_W$ generators. Since $S^z=W^z$,[9] T_c commutes with S^z, so that it is sufficient to consider the spin-lowering operator S^- in Eq. (4). The transformation properties of the mesons are described conveniently if they are considered as quark-antiquark S states, formed from the three (proton, neutron, and Λ) quarks. (The quarks are not postulated to be physical particles.) The quark-spin- and W-spin-lowering operators S_i^- and W_i^- may be defined so that their matrix elements with respect to the quark i are equal, in which case their matrix elements for the antiquark i are of opposite sign.[9] In general, $S^-=S_p^-+S_n^-+S_\Lambda^-$ is not a linear combination of $SU(6)_W$ generators. However, a simplification occurs if, for each i, the quark i and antiquark i do not both occur in the wave function ψ^m. (An example is an MM wave function of $I_z=2$, which must include two proton quarks and two neutron antiquarks.) For such a wave function, S^- may be replaced by the expression $\sum_i a_i W_i^-$, where a_i is chosen to be 1 if the quark i occurs, or −1 if the antiquark i occurs. This expression commutes with T_c, so the condition of Eq. (4) is valid. The representations $\underline{10}$, $\underline{10}$*, and $\underline{27}$ contain states of internal quantum numbers that cannot be formed from a single quark-antiquark pair, and thus are of the type discussed above. Hence, no S-D transitions are possible for these representations in our $SU(6)_W$-symmetric model.

A similar theorem applies to other, related calculations. For example, $SU(6)_W$-symmetric one-particle exchange contributions to the scattering of a meson M from a member of the 56-fold baryon supermultiplet can lead to S-D transitions only for states of the SU(3) representations $\underline{1}$, $\underline{8}$, and $\underline{10}$.

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RECENT RESULTS ON STRANGENESS-ONE RESONANCES*

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In the last two years a number of strangeness-one resonances have been reported in the literature. There have been resonances involving one K meson and two π mesons reported at energies of 1170,[1,2] 1215,[3,4] 1275,[5] 1320,[6-8] and 1400 MeV.[9-12] Except for the case of the $K^*(1400)$, relatively little firm information exists so far as spin and parity quantum numbers are concerned. If one could make definite quantum number assignments to these rather numerous resonances, then it might be possible to establish the existence of families of such resonances and perhaps understand what sort of symmetry scheme is valid for these relatively energetic systems.

In the process of studying K^+-proton interactions at an energy of 3.54 BeV/c in the 80-inch Brookhaven National Laboratory bubble chamber, we have found evidence for several of these resonant states. We have measured all events consisting of two and four prongs with associated V^0's in a series of 50 000 pictures. These topologies are particularly useful ones in that they present less ambiguous cases than the other topological classes. The reactions that we have studied are listed below.

$$K^+ + p \to K^0 + p + \pi^+, \quad (1)$$

$$\to K^0 + p + \pi^+ + \pi^0, \quad (2)$$

$$\to K^0 + n + \pi^+ + \pi^+, \quad (3)$$

$$\to K^0 + p + \pi^+ + \pi^+ + \pi^-, \quad (4)$$

$$\to K^0 + p + \pi^+ + \pi^+ + \pi^- + \pi^0. \quad (5)$$

$K\pi\pi$-1170.—A $K\pi\pi$ interaction at 1170 MeV was first reported by Wangler, Erwin, and Walker[1] in the reaction $\pi^- + p \to K + Y + 2\pi$. This particular peak was later verified by Miller et al.[2] at Purdue. Higher energy πp interactions did not seem to show the presence of such an enhancement in the $K\pi\pi$ mass spectra. We have observed a peak in the $K^0\pi^+\pi^+$ mass spectrum from Reaction (4) which agrees in mass and width with the $K^*(1170)$ previously reported. The interaction is predominantly associated with $K^0\pi^+\pi^+$ combinations in which one $K^0\pi^+$ pair forms a $K^*(892)$. The data are shown in Fig. 1(a). This establishes the isotopic spin of our peak as $\frac{3}{2}$, but its existence as a resonant particle is more in doubt.

An analysis of a Dalitz plot of the $K^0\pi^+\pi^+$ combinations in the 1170-MeV region suggests that this enhancement may be caused by the final-state interaction shown in the inset in Fig. 1(a). The triangle diagram causes a $K\pi\pi$ enhancement for the range of 1030 to ~1180 MeV and also forces the $\pi^+\pi^+$ system into a mass range less than $\sim 2.5 m_\pi$.[13] We find a 3.8-standard-deviation excess of $\pi^+\pi^+$ pairs in this mass range for $K^0\pi^+\pi^+$ combinations in the 1170-MeV peak. This enhancement occurs at the low-mass end of the $K^*(892)$ band in a Dalitz plot of $M^2(K^0\pi^+)$ versus $M^2(\pi^+\pi^+)$. No such effect is observed for $K^0\pi^+\pi^+$ combinations in a control mass region above 1170 MeV.

$K^*(1400)$.—The $K^*(1400)$, which was initially reported by Haque et al.[9] and later seen in a number of experiments,[10-12] has been assigned to a 2^+ nonet by Glashow and Socolow.[14] We see the effect of the $K^*(1400)$ resonance in several channels; in particular, the mass spectrum of the $K^0\pi^+$ system from Reaction (1) shown in Fig. 1(b).[15] We also see an indication of the 1400-MeV resonance in Reaction (2) in which there is a peak above phase space in the region of 1400 MeV for the $K^0\pi^0\pi^+$ mass spectrum as shown in Fig. 1(c). Although we have also looked for the presence of the decay of the $K^*(1400)$ into $K^0 + \omega^0$ and $K^0 + \eta^0$, we have found no definite evidence for such decay modes, but can only set an upper limit for the branching ratio into these channels. We have also sought evidence for the $K^*(1400)$ in Reaction (3). A plot of the mass distribution for the $K^0\pi^+\pi^+$ in Fig. 1(d) yields no indication of the $K^*(1400)$ in this channel which has an I_z of $\frac{3}{2}$. This is strong