Assuming an average value of $\nu = 85$ MeV/c, then the best fit of the experimental rates is obtained with $\delta = 3.13$ and $\gamma(\nu/m_\mu)^2 = 183\ \mathrm{sec}^{-1}$. With the coupling constants (i) we get $\gamma(\nu/m_\mu)^2 = 184\ \mathrm{sec}^{-1}$. With the constants (ii) we fit the experimental result with $\lambda = 54$ or $\lambda = -8$. For $\lambda = -24$ we get the value 298 sec^{-1} which is too high.

(6) Angular distribution of emitted neutrons.—The angular distribution of the neutrons emitted after the capture of polarized μ^- is described by a parameter α introduced by Primakoff[4] which is defined by

$$\alpha = \frac{G_V^2 - G_A^2 + G_P^2 - 2G_A G_P}{G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P}.$$

From measurement in Ca^{40} and S^{32}, it is known that $\alpha = -1 \pm 0.15$.[14] The calculated value using (i) is $\alpha = -0.36$ in bad agreement with the experiment. More experiments of this kind are needed to clear up this discrepancy completely. With the coupling (ii) we get always $\alpha > 0$, in particular for $\lambda = -24$ we get $\alpha = +0.60$ in worse agreement with the experiment.

From all this it is clear that there is strong experimental evidence in support of the constants (i) and the same evidence rules out completely the second set of constants.

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EQUIVALENT REPRESENTATIONS IN SYMMETRIZED TENSORS

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It has been noted that for many detailed properties of group representations, such as generator matrix elements and certain Clebsch-Gordan coefficients, tensor methods have not been used.[1] The reason is because the basis of decomposed tensors has usually been only implicit.[2] The cause of this trouble is in a portion of a theorem given by Weyl[3] which states that if C is a Young symmetrizer, then the tensors CF form an irreducibly invariant subspace. Except for one-dimensional representations, this is incorrect whether it is interpreted to mean a single tableau and some set of tensors F or whether it is interpreted to mean some set of tableaux (of a given Young pattern) and a single tensor F. This has caused considerable confusion in the physics literature. Most authors use the first interpretation,[4–6] but the second one has also appeared.[7]

We now show how to construct the irreducibly invariant subspaces. Denoting tableaux column and row permutations by q and p, respectively, it is well known that for any given tableau a minimal left ideal is generated by the Young symmetrizer

$$PQ = \sum_{pq} pq\delta_q, \tag{1}$$

where the sums are over all column and row permutations. The left ideals obtained from the standard tableaux are linearly independent and span the whole ring so that a Peirce resolution of the unit element (e) of the ring can be written

$$e = \sum_{i,\mu} (N^\mu/G)(PQ)_i^{\ \mu}, \tag{2}$$

where N^μ is the dimension of the representation (μ) and G is the order of the permutation group S_γ. The sum is over all standard tableaux of all patterns of S_γ. If a pair of tableaux $\tau_i^{\ \mu}$

and τ_j^μ are related as

$$\tau_i^\mu = S_{ij}\tau_j^\mu, \qquad (3)$$

then

$$(PQ)_i^\mu = S_{ij}(PQ)_j^\mu S_{ji}. \qquad (4)$$

Henceforth, S_{ij} will only be used to relate standard tableaux. A basis of the left ideal generated by $(PQ)_j^\mu$ is given by $\{S_{ij}(PQ)_j^\mu\}$ where i ranges over all standard tableaux. Rather than use this basis, the usual procedure is to develop a basis from modified idempotents (Young's construction[8]) with which the natural representation matrices of the permutation group are more easily obtained. We do not seek such matrices here and will instead use the idempotents of Eq. (1). From Eq. (2) it is clear that a tensor can be expanded as

$$T_{i_1\cdots i_r} = \sum_{\mu, i} \frac{N^\mu}{G}(PQ)_i^\mu T_{i_1\cdots i_r}. \qquad (5)$$

To obtain a basis of representation (μ) we start with any arrangement of indices $i_1\cdots i_r$ and any standard tableau τ_k^μ and select the components

$$\{(N^\mu/G)S_{jk}(PQ)_k^\mu T_{i_1\cdots i_r}\}$$

(j ranges over all standard tableaux). These components all appear in Eq. (5) because $S_{jk} \times T_{i_1\cdots i_r}$ certainly appears as an index arrangement and we can use Eq. (4) in the form

$$S_{jk}(PQ)_k^\mu T_{i_1\cdots i_r} = (PQ)_j^\mu S_{jk} T_{i_1\cdots i_r}.$$

Because the ideals of distinct standard tableaux are independent, a second independent basis is obtained with the components

$$\{(N^\mu/G)S_{ji}(PQ)_i^\mu T_{i_1\cdots i_r}\},$$

where $i \neq k$, index j ranges over all standard tableaux, and the indices $i_1\cdots i_r$ have the same arrangement as before. Proceeding in this way all standard tableaux are exhausted to obtain a complete set of independent bases of representation type (μ). The set is complete because all basic states appear among the components of the Peirce resolved tensor and the number of independent bases obtained is equal to the number of standard tableaux. Independent bases for the equivalent representations are now written down in a form which clearly shows that for all multidimensional representations of the permutation group either interpretation of Weyl's statement fails to yield irreducibly invariant subspaces.

First basis:

$$A_1^\mu \equiv (N^\mu/G)(PQ)_1^\mu T_{i_1\cdots i_r},$$
$$A_2^\mu \equiv (N^\mu/G)(PQ)_2^\mu S_{21} T_{i_1\cdots i_r},$$
$$\vdots$$
$$A_n^\mu \equiv (N^\mu/G)(PQ)_n^\mu S_{n1} T_{i_1\cdots i_r}.$$

Second basis:

$$B_1^\mu \equiv (N^\mu/G)(PQ)_1^\mu S_{12} T_{i_1\cdots i_r},$$
$$B_2^\mu \equiv (N^\mu/G)(PQ)_2^\mu T_{i_1\cdots i_r},$$
$$\vdots$$
$$B_n^\mu \equiv (N^\mu/G)(PQ)_n^\mu S_{n2} T_{i_1\cdots i_r}.$$

Nth basis ($n = N^\mu$):

$$D_1^\mu \equiv (N^\mu/G)(PQ)_1^\mu S_{1n} T_{i_1\cdots i_r},$$
$$D_2^\mu \equiv (N^\mu/G)(PQ)_2^\mu S_{2n} T_{i_1\cdots i_r},$$
$$\vdots$$
$$D_n^\mu \equiv (N^\mu/G)(PQ)_n^\mu T_{i_1\cdots i_r}. \qquad (6)$$

In another communication[9] it is shown how these results can be extended to the unitary representations, and how to use tensor methods to evaluate generator matrix elements, group matrices (in polynomial form), Clebsch-Gordan coefficients, and recoupling coefficients for the classical groups.

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tation mentioned above. It is true that each Young tableau labels an invariant subspace by defining one state of the subspace, and these authors construct such "Weyl" states. Using these initial states and shift operations, "Gelfand" states are obtained to complete the basis and thus use of the erroneous portion of Weyl's theorem is avoided.

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$$\bar{f}_3=\tfrac{1}{2}[u(1)v(3)w(2)-u(2)v(3)w(1)+u(3)v(1)w(2)-u(3)v(2)w(1)],$$

$$\bar{f}_4=\tfrac{1}{2}[u(1)v(3)w(2)-u(2)v(1)w(3)+u(2)v(3)w(1)-u(3)v(1)w(2)].$$

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CHIRAL ALGEBRA, CONFIGURATION MIXING, AND MAGNETIC MOMENTS

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We present a model for baryon states based on the chiral U(3)⊗U(3) algebra at infinite momentum. We shall explore the possibility that the expectation values of the commutators of the time components of vector and axial-vector currents[1] between stable octet baryons are saturated by single-particle states and observed resonances, and shall also consider the magnetic moments of these states. In the infinite-momentum limit the chiral algebra is equivalent to the collinear U(3)⊗U(3) algebra,[2] whose irreducible representations we shall label by $(n,m)_\lambda$ where n and m are the dimensions of the SU(3) representations generated by $V_0{}^i+A_3{}^i$ and $V_0{}^i-A_3{}^i$, respectively; λ is the eigenvalue of the operator $A_3{}^0$ (we shall refer to this as the "quark helicity").

In a first approximation, the octet baryons and the helicity-$\frac{1}{2}$ decuplet states form the representation $(6,3)_{1/2}$ and this classification leads to the familiar static SU(6) results.[1] Since some of these predictions do not agree with experiment, we must allow the baryons to transform reducibly under the algebra. In addition, in order to obtain nonzero magnetic moments, we must introduce an additional degree of freedom associated with an "orbital angular momentum" excitation.[3] To be more precise, we define the "orbital helicity" by $\Lambda_3\equiv J_3-\lambda$ where J_3 is the true helicity. (We take the momentum always along the positive z axis.) Thus we are led to classify physical states as linear combinations of representations, $[(n,m)_\lambda,\Lambda_3]$. We emphasize that in this scheme we do not require the finite linear combinations of states of the chiral algebra to comprise a complete representation of SU(6)$_W$, and for this reason we restrict ourselves to helicities (J_3,λ,Λ_3) rather than the full angular momentum.[4] Nor do we insist that the states we consider are necessarily those predicted on the basis of a simple three-quark model.[5]