The present results show that for light nuclei even the relative spectroscopic factors extracted from direct-reaction theories must be taken with caution until a detailed explanation of the observed effects can be given.

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REFRACTION OF ELECTRON BEAMS BY INTENSE ELECTROMAGNETIC WAVES*

T. W. B. Kibble
Department of Physics, Imperial College, London, England
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It has long been known that electromagnetic waves are refracted by passing through a cloud of electrons. In a previous paper[1] it was suggested that the predicted intensity-dependent frequency shift in high-intensity Compton scattering[2] might be regarded as an example of the complementary effect, the refraction of a beam of electrons passing through an electromagnetic wave.

Here we shall show that this is a very general phenomenon. In particular, in an electromagnetic wave whose intensity is independent of time, the electron has an effective potential energy $\frac{1}{2}m\mu^2c^2$, where μ^2 is the intensity parameter [Eq. (5) below], and so the wave appears like a medium of refractive index $[1-\mu^2c^2/v^2]^{1/2}$.

Both the "field-gradient force" which has been shown by Phillips and Sanderson[3] to lead to significant effects on the experiment proposed to measure the frequency shift, and the $\vec{v}\times\vec{B}$ force which provides the acceleration mechanism responsible for this shift[1] are specific examples of a general force proportional to the spatial or temporal gradient of the field intensity, and leading to secular changes in the electron momentum or energy. (This is quite separate from the effect of radiation pressure,[4] though it might be regarded as due to the pressure gradient, if the electron were assigned an effective volume $r_0\lambda^2/\pi$.)

Let us consider an electron moving in an oscillating electromagnetic field whose amplitude is a slowly varying function of space and time. We wish to investigate the averaged motion of the electron, the analog of the guiding-center drift of a particle in a magnetic field. It is convenient to choose a frame of reference in which near $t=0$ the electron is, on the average, at rest at the origin. Then generalizing the method of Ref. 3, we may take approximately

$$\vec{E}(t,\vec{r}) = \mathrm{Re}\{[\vec{E}_0 + t(\partial\vec{E}_0/\partial t) + \vec{r}\cdot\nabla\vec{E}_0]e^{i\omega t}\}, \qquad (1)$$

where the complex amplitude $\vec{E}_0$ and its derivatives are evaluated at the origin. To the same order we need only the leading term in the magnetic field,

$$\vec{B}(t,\vec{r}) = \mathrm{Re}\{\vec{B}_0 e^{i\omega t}\}. \qquad (2)$$

If we first neglect $\vec{r}$ and $d\vec{r}/dt$ on the right-hand side of the Lorentz force equation, then we obtain the first-order solution

$$\vec{r}_1 = -\frac{e}{m\omega^2}\mathrm{Re}\left\{\left[\vec{E}_0 + \left(t - \frac{2}{i\omega}\right)\frac{\partial\vec{E}_0}{\partial t}\right]e^{i\omega t}\right\}. \qquad (3)$$

Then substituting in the right-hand side of the Lorentz force equation and taking the time average, we obtain

$$\left\langle\frac{d^2\vec{r}}{dt^2}\right\rangle = -\left(\frac{e}{m\omega}\right)^2 \times \tfrac{1}{2}\mathrm{Re}\left[\vec{E}_0{}^*\cdot\nabla\vec{E}_0 - \frac{i\omega}{c}\vec{E}_0{}^*\times\vec{B}_0\right].$$

But from Maxwell's equations,

$$-\frac{i\omega}{c}\vec{B}_0 = \vec{\nabla}\times\vec{E}_0,$$

so that this equation reduces to

$$\langle d^2\vec{r}/dt^2\rangle = -\tfrac{1}{2}\nabla\mu^2c^2, \qquad (4)$$

where

$$\mu^2 = (e/mc\omega)^2 \times \tfrac{1}{2}(\vec{E}_0{}^*\cdot\vec{E}_0) = (e/mc\omega)^2\langle\vec{E}^2\rangle. \qquad (5)$$

This is precisely the intensity parameter defined previously.[1,2]

Similarly, we may find the rate of change of energy of the electron by examining

$$\left\langle\frac{d\vec{r}}{dt}\cdot\frac{d^2\vec{r}}{dt^2}\right\rangle = \left(\frac{e}{m\omega}\right)^2 \times \tfrac{1}{2}\mathrm{Re}\left[\vec{E}_0{}^*\cdot\frac{\partial\vec{E}_0}{\partial t}\right] = \frac{1}{2}\frac{\partial}{\partial t}\mu^2c^2. \qquad (6)$$

Thus we may determine the averaged motion of the electron be regarding it as a particle moving under a conservative force with potential energy function $\frac{1}{2}m\mu^2c^2$. The effect is clearly to push the electron away from regions of high intensity. [Relativistically, as will be shown in detail in a later publication, the electron moves precisely like a particle with the variable rest-mass $m(1+\mu^2)^{1/2}$.] In the case where the intensity is time-independent, we see that the electromagnetic wave behaves towards the electron like a medium of refractive index $[1-\mu^2c^2/v^2]^{1/2}$, where v is the electron velocity. Thus μ plays a role analogous to the plasma frequency. Just as an electromagnetic wave with $\omega < \omega_p$ cannot penetrate a plasma, so an electron with $v/c < \mu$ cannot penetrate the electromagnetic wave.

At the focal spot of a ruby-laser beam, one can achieve intensities of the order of 10^{13} W cm^{-2}, corresponding to $\mu^2 = 10^{-6}$. In the neighborhood of the focal spot the intensity gradients are extremely sharp, so that the effect on an electron passing close to it is considerable. Consider, for example, an electron which crosses the axis of the beam perpendicularly at a distance b from the focus. Let the radius of the focal spot be a and the semiangle of divergence of the beam α. Then one can estimate the angle of deviation of the electron to be

$$\delta\theta = \alpha\frac{a^2}{b^2}\frac{\mu_0{}^2c^2}{v^2}, \qquad (7)$$

where $\mu_0{}^2$ is the value of μ^2 at the focus. This can evidently be quite large for a slow electron passing close to the focus. This refraction should be possible to observe fairly easily. It would certainly be interesting to do so.

It follows that, contrary to hopes expressed earlier,[1] the focusing of a laser beam has a drastic effect on the expected frequency of light scattered from free electrons. Indeed, we may expect Doppler shifts of order μ rather than μ^2. To understand why this is so, it may be helpful to examine the relativistic form of Eqs. (4) and (6); namely,

$$dp_\mu/d\tau = \tfrac{1}{2}m\,\partial_\mu\mu^2, \qquad (8)$$

where now $c=1$. If μ^2 is a function only of $n\cdot x$ then we find the solution

$$P_\mu(\tau) = P_\mu(0) + n_\mu f(n\cdot x), \qquad (9)$$

where f is that root of the quadratic equation

$$n^2f^2 + 2n\cdot p(0)f = m^2[\mu^2(n\cdot x) - \mu^2(0)], \qquad (10)$$

which vanishes when the right-hand side is zero. So long as $n\cdot p(0)$ is not small, we may neglect f^2 and hence we obtain a velocity of order μ^2. (This is exact for the plane-wave case discussed earlier, where $n^2=0$.) On the other hand, we can obtain a velocity of order μ if $n\cdot p(0)=0$, which is the case if the spatial variation of intensity is much more rapid than the temporal variation, as it is near a focus. (The specific condition is that $\partial_0\mu$ should be small compared to $\mu c\nabla\mu$.)

It is interesting to note that this effect may have some connection with the phenomenon observed by Ramsden and Davies.[5] They focused an intense ruby-laser beam in air, and found a Doppler shift in the light scattered from the resulting plasma. This was interpreted[6] as being due to a shock wave expanding outwards from the focus with a velocity of up to 10^7 cm/sec. The electrons in this case are not, of course, entirely free, and the mechanism described above cannot account for the acceleration of the plasma to velocities of this order. Nevertheless, the very sharp intensity gradient in the neighborhood of the focus may well give an acceleration which provides the initial triggering mechanism for the build-up of the shock wave.

Eberly[7] has proposed an alternative experiment designed to observe the intensity-dependent frequency shift, based on the Kapitza-Dirac effect.[8] This is the scattering of an electron at the Bragg angle by standing waves; it has been observed experimentally by Bartell, Thompson, and Roskos.[9] No focusing is involved in this experiment, and so the intensity gradients are much smaller. However, Eberly's calculation is based on the results given in Refs. 1 and 2 which were obtained for the case of plane waves. It was assumed that the electron was initially at rest when it was overtaken by the electromagnetic wave, and accelerated in the manner described above, giving it an average velocity in the propagation direction. In this situation the temporal and spatial gradients are equal in magnitude. But in the actual experiment the situation is rather different and, in particular, the spatial gradients in transverse directions are important. The electron does not enter the beam in this way but from one side. It will therefore suffer a refraction in which its velocity component normal to the boundary will be decreased. The parallel component of momentum is unchanged, and since it is this component which enters the formula for the Bragg angle ($p\sin\frac{1}{2}\theta=k$) there will in fact be no effect.

The essential difference between the two physical situations may be described thus. In each case the electron has to acquire a mass $m(1+\mu^2)^{1/2}$. When it is overtaken by the beam, it does this by increasing its energy (and one component of momentum) by an amount of order $m\mu^2$. But when it enters from the side the energy is substantially unchanged, and the normal component of momentum changes by an amount of order $m\mu$.

The effect discussed here is an inherently interesting one which may well repay experimental study. A more detailed (and fully relativistic) theoretical treatment including, in particular, a discussion of energy and momentum conservation will be given in a subsequent paper.

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