is not a temperature at which a phase transition occurs but rather a mid-point of two regimes which merge smoothly together, it is expected that the behavior expected from an interpolation from the values for $T \gg T_c$ and $T \ll T_c$ given above would obtain.

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# EFFECT OF ELECTRIC FIELD ON THE TRANSVERSE MAGNETORESISTANCE IN $n$-INDIUM ANTIMONIDE AT 1.5°K

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The effect of a strong electric field on the electron energy levels has been observed in $n$-type indium antimonide in a transverse magnetic field at low temperature. This effect was predicted theoretically by Aronov[1] and observed through the study of the Franz-Keldysh effect in a magnetic field.[2] We have also observed such an effect (analogous to the Stark effect) through the study of Shubnikov–de Haas magneto-oscillation in a strong electric field. It has been shown[3] that $n$-type InSb can be treated on the basis of a free-electron model for a degenerate electron gas where ionized impurity scattering is dominant. The energy of an electron in degenerate semiconductors in a transverse magnetic field is given as

$$\epsilon^{(l)}(l, K_z) = (l+\tfrac{1}{2})\hbar\omega_{c0} + \frac{\hbar^2 k_z^2}{2m^*} \pm \tfrac{1}{2}g\mu_B B_0. \qquad (1)$$

The component $\sigma_{xx}$ of the conductivity tensor has sharp maxima corresponding to the quantum condition, and is usually observed in the experimentally measured quantity $\rho_{xx} = \sigma_{xx}/\sigma_{xy}^2$. Here $l$ is the magnetic quantum number, the projection of the quasimomentum in the direction of the magnetic field to $K_z$, $\mu_B$ the Bohr magneton, $\omega_{c0} = eB_0/m^*c$ the cyclotron frequency, and $g$ the spectroscopic splitting factor. $B_0$ is the magnetic field with $E \to 0$.

When an electric field is applied, the energy of an electron becomes

$$\epsilon^{(l)}(l, k_z, k_y) = (l+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*} - chk_y\left(\frac{E_x}{B}\right) - \frac{m^*c^2}{2}\left(\frac{E_x}{B}\right)^2 \pm \tfrac{1}{2}g\mu_B B, \qquad (2)$$

where $\omega_c = eB/m^*c$. $B$ is the magnetic field with crossed electric field. The third term shows the potential energy for individual cyclotron orbit centers, and it is seen that the $K_y$ degeneracy is removed. The fourth term is the energy shift of the magnetic sub-bands

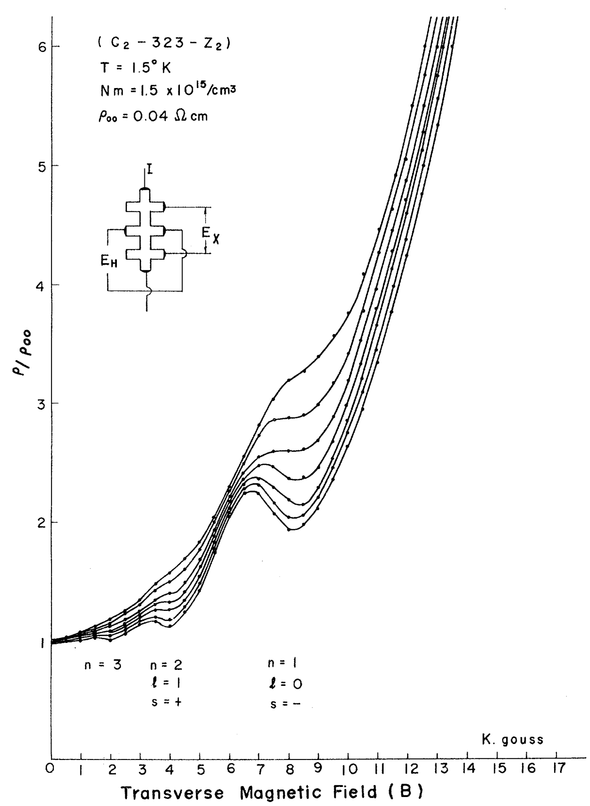


FIG. 1. The transverse magnetoresistance with the applied electric field as parameter. The samples contain an electron concentration of $1.5 \times 10^{15}/cm^3$, and measurements were made at 1.5°K. Corresponding electric fields are, from top to bottom, 140, 120, 80, 60, 30, and 10 mV/cm.

(Landau levels).[4]

In this paper, the results of experiments showing the shift of these sub-bands and the influence of the spin on these sub-bands when a strong electric field is applied will be described.

The electric dipole energy density produced in these magnetic sub-bands by applying an external electric field is $ED=E[2(n+1)hc/eB]^{1/2}$ and is ~$10^{-16}$ erg for a field of 1 V/cm. This is small in comparison with the Fermi energy of the electrons (~$10^{-15}$ erg for an electron density of about $10^{15}/\mathrm{cm}^3$), so that the effect of the electric field on the characteristic energy of the conduction electrons can be assumed to be small in the absence of scattering. Then the following expression for the energy shift of the magnetic sub-bands due to the electric field should hold:

$$\frac{\hbar e}{m^*c}(B-B_0)(l+\tfrac{1}{2}) \cong chk_y\left(\frac{E_x}{B}\right)+\frac{m^*c}{2}\left(\frac{E_x}{B}\right)^2 \pm \tfrac{1}{2}g\mu_B(B-B_0). \quad (3)$$

Therefore,

$$\Delta B=\frac{chk_y(E_x/B)+\frac{1}{2}m^*c^2(E_x/B)^2}{(2l+1\pm\frac{1}{2}gm^*/m)\mu_B^*}. \quad (4)$$

We measured the Shubnikov–de Haas oscillations in the transverse magnetoresistance under a strong electric field. The result is shown in Fig. 1. The sample was cut in the shape shown in the upper left section of Fig. 1. The carrier density in this sample was $1.5\times10^{15}/\mathrm{cm}^3$ at 1.5°K. The shift of the magnetic fields corresponding to the maximum value of the resistivity, $\rho_{xx}$, can be clearly seen.

In Fig. 2 the shift of the energy levels of the magnetic sub-bands caused by the applied electric field are plotted. The levels $n=1$ and $n=2$ refer to the first and second maxima of the Shubnikov–de Haas oscillations, respectively. This effect resembles the well-known Stark effect observed for atoms and molecules.

The electric field for dipole displacement should be the total electric field in which the electron in the magnetic sub-band moves, i.e., $E_t=(E_x^2+E_H^2)^{1/2}$, where $E_H$ is the Hall volt-

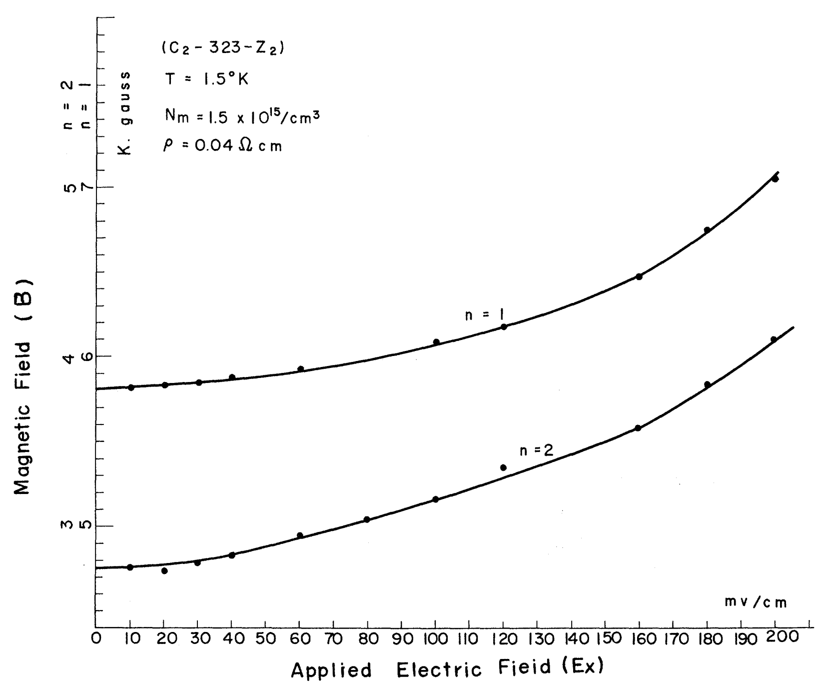


FIG. 2. The shift of the magnetic field for the maxima $\rho_{xx}$ as a function of applied electric field; $n=1$ and $n=2$ are the first and second maxima in $B$, respectively. Vertical scale shows magnetic field corresponding to Fermi energy.

age, rather than just the externally applied electric field, $E_x$. The experimental values are replotted in Fig. 3 as a function of $E_t$.

The vertical bars in Fig. 3 show our estimate of experimental error in our measurements. The solid curves are obtained from Eq. (4), using the value $m^*/m = 0.013$ and $E_t$ in place of $E_x$. The value of $k_y$ was chosen to be 800 $\text{cm}^{-1}$ as maximum value to satisfy the condition $V_d \leq V_{\text{phase}}$ (where $V_d$ means the drift velocity of the center of cyclotron motion of conduction electrons, expressed as $cE/B$, and $V_{\text{phase}}$ means $\hbar k_y/2m^*$). This condition shows that the phonon distribution is in thermal equilibrium.[5] In spite of the maximum value of $k_y$, the first term of Eq. (4) is negligible compared to the second term. The values of $g$ are obtained experimentally by adjusting the experimental points to solid curves in Fig. 3. These values are 50.8 and 50.4 for the $n = 1$ peak and the $n = 2$ peak, respectively.

The experimental points may deviate from the calculated curves when the electric field exceeds a value where the conduction electrons

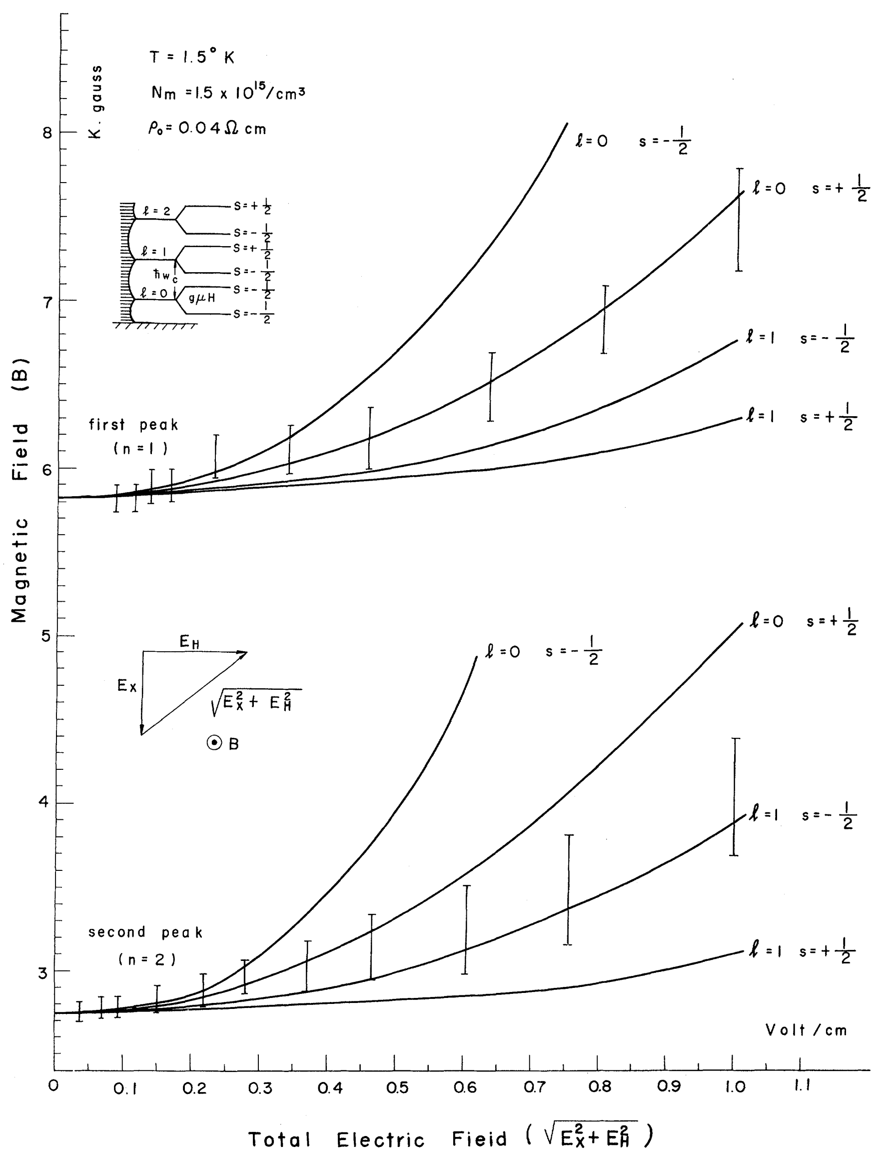


FIG. 3. The shift in energy of the Landau levels as a function of the total electric field. The vertical bars are experimental values with errors, and the solid lines are calculated from Eq. (4).

may become heated. When electrons of one spin direction are heated, their spins can flip via an effect similar to the Overhauser effect; this picture is consistent with the increase in polarized spin, reported by Clark and Feher.[6]

Experiments showing spin Shubnikov–de Haas effects in InAs have been reported by Amirkhanov, Bashirov, and Zakiev[7] and in InSb by Amirkhanov and Bashirov[8] and also by Antcliffe and Stradling.[9] The theory has been discussed by Gurevich and Efros.[10] The $g$ values calculated by using their theory from the maxima magnetic field in the limit of $E \to 0$ are 53.2 and 49.3 corresponding to $n=1$ and $n=2$ peaks, respectively, which coincide with experimental values.

Our experiments have shown the effects of an applied electric field on the energy-band structure in a strong magnetic field. Agreement with an approximate free-electron model theory is quite good. Studies at higher electric fields to investigate "heating effect" are planned.

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# ULTRASONIC ATTENUATION IN PURE STRONG-COUPLING SUPERCONDUCTORS*

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In a recent issue of this Journal, Deaton[1] has reported measurements of the attenuation of longitudinal sound in pure crystals of superconducting lead. In the limit $ql \gg 1$ [$q$ = phonon wave number, $l$ = electronic mean free path], the attenuation is reported to be anomalously small, this feature being considerably more noticeable than the expected dependence of the attenuation on the orientation of the crystal. Deaton has suggested that the anomaly is to be understood by analogy with the thermal conductivity which is also anomalously low in superconducting lead and for which a reasonably rational explanation exists.[2,3] It is the purpose of this note to point out that in the limit $ql \gg 1$ a straightforward calculation of sound attenuation in the strong-coupling model is possible. The calculation does not lend support to the conjecture mentioned above.

In the limit $ql \gg 1$ the sound-attenuation rate is given by the reciprocal of the lifetime of a phonon of wave number $q$, since the complications which ensue from the tendency of the electrons to relax to the local lattice velocity are then irrelevant. Working out the phase space for the phonon to decay into a pair of electronic excitations gives for the relative attenuation $(\alpha_s/\alpha_n)$

$$\frac{\alpha_s(\vec{q})}{\alpha_n(\vec{q})} = \frac{\sum_{\vec{k}}\int_{-\infty}^{\infty} d\omega\, \mathrm{Tr}\{\tau_3 a_{\vec{k}+\vec{q}}^{\,s}(\omega)\tau_3 a_{\vec{k}}^{\,s}(\omega)\}\,\mathrm{sech}^2\tfrac{1}{2}\beta\omega}{\sum_{\vec{k}}\int_{-\infty}^{\infty} d\omega\, \mathrm{Tr}\{\tau_3 a_{\vec{k}+\vec{q}}^{\,n}(\omega)\tau_3 a_{\vec{k}}^{\,n}(\omega)\}\,\mathrm{sech}^2\tfrac{1}{2}\beta\omega}. \tag{1}$$