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QUARK MODELS AND HIGH-ENERGY SCATTERING

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The large amount of precise experimental data now available on total cross sections for meson-nucleon scattering presents a challenge to the theorist.[1] Symmetries[2,3] and quark models[4] have had a certain degree of success in obtaining relations between these cross sections which are in agreement with experiment. However, some predictions from SU(3) symmetry seem to be in disagreement with experiment.[1,5] Furthermore, the most striking regularity of the data has not been predicted by any of these symmetries or quark models, namely the equality of the K^+p and K^+n total cross sections over a wide energy range.[1,6] Many models and theories predict that all meson-baryon cross sections become equal at sufficiently high energy.[4,7] However, experimental data show that some of these are more equal than others, as indicated in Fig. 1. This feature has not been predicted by any of the higher symmetries which include SU(3).

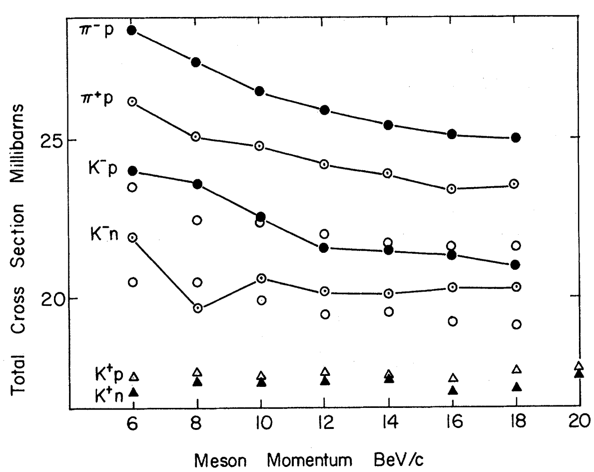

FIG. 1. Meson-baryon cross sections and relations (3a), (5a), and (5b). ○ left- and right-hand sides of Eq. (3a), ● left- and right-hand sides of Eq. (5a), ⊙ left- and right-hand sides of Eq. (5b).

We should like to show that good agreement with experiment is obtained by a slight extension of the quark model[4] along the lines suggested by Kokkedee and Van Hove.[8] In addition to the additivity assumption for the two-body quark scattering amplitudes assumed in the previous treatments,[4] we have certain simplifying assumptions about the two-body quark-quark and quark-antiquark scattering amplitudes. The basic physical idea is that scattering of 6- to 20-BeV/c mesons is sufficiently close to the asymptotic region so that the two-body scattering amplitudes exhibit some, but not all of the asymptotic features. In particular, it is assumed that the quark-quark amplitudes exhibit these asymptotic features, while they are not yet present in the quark-antiquark amplitudes, possibly because of the presence of the annihilation channel in the latter.[8] We consider several different combinations of these assumptions, both with and without SU(3) symmetry.

In order to enable a fair comparison with experiment of different treatments, we express all predicted relations between meson-baryon total cross sections in the following standard form: The expressions on both sides of the equality involve only sums, no differences. They are normalized so that in the limit where all meson-baryon cross sections are equal, the expressions on two sides are just equal to the common meson-baryon cross section. We first list relations which have been previously obtained. We consider only total cross sections, which are related to the forward scattering amplitude by the optical theorem.

The antisymmetric sum rule[4] follows directly from the basic additivity assumption of the quark model, without any symmetry assumptions. In our standard form this becomes

$$\tfrac{1}{3}[\sigma(K^-p)+\sigma(\pi^+p)+\sigma(K^0p)] = \tfrac{1}{3}[\sigma(K^+p)+\sigma(\pi^-p)+\sigma(\bar{K}^0p)]. \quad (1)$$

The additional assumption of SU(3) symmetry for the basic two-body amplitudes[4] leads to the Johnson-Treiman relations[2]

$$\tfrac{1}{3}[\sigma(K^-p)+2\sigma(\pi^+p)]=\tfrac{1}{3}[\sigma(K^+p)+2\sigma(\pi^-p)], \quad (2a)$$

$$\tfrac{1}{3}[\sigma(K^-p)+2\sigma(K^+n)]=\tfrac{1}{3}[\sigma(K^+p)+2\sigma(K^-n)], \quad (2b)$$

and also the symmetric sum rule

$$\tfrac{1}{4}[\sigma(\pi^+p)+\sigma(\pi^-p)+\sigma(K^-n)+\sigma(K^+n)] = \tfrac{1}{2}[\sigma(K^-p)+\sigma(K^+p)]. \quad (3a)$$

Equations (2a) and (3a) can be combined to give the more convenient form

$$\tfrac{1}{2}[\sigma(\pi^+p)+\sigma(K^-n)]=\tfrac{1}{2}[\sigma(\pi^-p)+\sigma(K^+n)] = \tfrac{1}{2}[\sigma(K^-p)+\sigma(K^+p)]. \quad (3b)$$

In our standard form, relations always appear as equalities between weighted means of different meson-baryon cross sections. A consistent measure of the relative agreement with experiment of different relations is given by the absolute value of the discrepancy in millibarns or by the percent deviation.

We now consider the consequences of the following new set of assumptions:

(1) The basic additivity assumptions of Ref. 4, without SU(3) symmetry. This leads immediately to the sum rule (1).

(2) Neglect of the imaginary part of the charge-exchange amplitude for nonstrange quark-quark scattering. This would follow from the physical assumption that the nonstrange quark-quark scattering is already in the asymptotic region, while other amplitudes are not. If the notation $\mathcal{P}\mathcal{N}\lambda$ is used for the three quarks, then this assumption together with isospin implies that $\mathcal{P}\mathcal{P}$, $\mathcal{P}\mathcal{N}$, $\mathcal{N}\mathcal{P}$, and $\mathcal{N}\mathcal{N}$ two-body contributions to the total cross sections are all equal. This leads to one new relation in addition to (1),

$$\sigma(K^+p)=\sigma(K^+n). \quad (4a)$$

Substitution of (4a) into (1) gives the simpler sum rule

$$\tfrac{1}{2}[\sigma(K^-p)+\sigma(\pi^+p)]=\tfrac{1}{2}[\sigma(K^-n)+\sigma(\pi^-p)]. \quad (4b)$$

The result (4a) is just the equality of the K^+p and K^+n cross sections mentioned above as a hitherto unexplained feature of the experimental data.

If the assumption of SU(3) symmetry is added to the two assumptions above, relations (2) and (3) are obtained. These can be simplified by substituting the relation (4a) to give

$$\sigma(\pi^-p)=\sigma(K^-p), \quad (5a)$$

$$\sigma(\pi^+p)=\sigma(K^-n), \quad (5b)$$

$$\sigma(K^-n)=\tfrac{1}{2}[\sigma(K^+p)+\sigma(K^-p)]. \quad (5c)$$

However, we wish to avoid the assumption of SU(3) symmetry at this stage. Instead we consider the addition of the following assumption to the two assumptions leading to the relations (4a) and (4b), without assuming SU(3).

(3) The Pomeranchuk theorem applies to the $(\mathcal{P}\mathcal{N})$ and $(\lambda\mathcal{N})$ quark amplitudes. This, together with isospin and the assumptions above, leads to the following equalities between quark-quark and quark-antiquark amplitudes:

$$(\mathcal{P}\mathcal{N})=(\mathcal{N}\mathcal{P})=(\overline{\mathcal{P}}\mathcal{N})=(\overline{\mathcal{N}}\mathcal{P})=(\mathcal{P}\mathcal{P})=(\mathcal{N}\mathcal{N})\equiv \boldsymbol{P}, \quad (6a)$$

$$(\lambda\mathcal{P})=(\lambda\mathcal{N})=(\overline{\lambda}\mathcal{P})=(\overline{\lambda}\mathcal{N})\equiv \boldsymbol{P}-S, \quad (6b)$$

where $\boldsymbol{P}$ defined by Eq. (6a) denotes the common amplitude for the nonstrange quarks and antiquarks, and S, defined by Eq. (6b), represents the contribution of SU(3) symmetry breaking in the strange-quark scattering. The omission of the $(\overline{\mathcal{P}}\mathcal{P})$ and $(\overline{\mathcal{N}}\mathcal{N})$ amplitudes from this assumption corresponds to the physical picture in which an isosinglet annihilation channel still gives significant contribution and breaks the Pomeranchuk theorem for all amplitudes which can have an isosinglet component. By analogy with the definitions of P and S, we can define the "annihilation contribution" A by the relation

$$(\overline{\mathcal{P}}\mathcal{P})=(\overline{\mathcal{N}}\mathcal{N})\equiv P+A. \quad (6c)$$

The assumptions (6) lead to the Johnson-Treiman relations (2), obtained now without SU(3). However, the symmetric sum rule (3a) is not obtained. Thus we obtain the relations (4) and (2).

A simple physical picture of these results is obtained by writing all the meson-baryon amplitudes in terms of the quantities P, S, and A:

$$(K^+p)=(K^+n)=6\boldsymbol{P}-3S, \quad (7a)$$

$$(K^-n)=6\boldsymbol{P}-3S+A, \quad (7b)$$

$$(\pi^+p)=6P+A, \quad (7c)$$

$$(K^-p)=6\boldsymbol{P}-3S+2A, \quad (7d)$$

$$(\pi^-p)=6P+2A. \quad (7e)$$

The Johnson-Treiman relations thus arise in this picture as a result of the annihilation

Table I. Meson-baryon cross sections.[a]

Momentum (BeV/c)	$\sigma_t(K^+p)$ (mb)	$\sigma_t(K^+n)$ (mb)	$\sigma_t(K^-p)$ (mb)	$\sigma_t(K^-n)$ (mb)	$\sigma_t(\pi^+p)$ (mb)	$\sigma_t(\pi^-p)$ (mb)
6	17.0 ± 0.1	17.5 ± 0.4	24.0 ± 0.3	21.9 ± 0.4	26.2 ± 0.2	28.5 ± 0.3
8	17.3 ± 0.1	17.6 ± 0.4	23.6 ± 0.2	19.7 ± 0.4	25.1 ± 0.2	27.5 ± 0.3
10	17.3 ± 0.1	17.5 ± 0.4	22.5 ± 0.2	20.6 ± 0.4	24.8 ± 0.2	26.5 ± 0.3
12	17.3 ± 0.1	17.6 ± 0.4	21.6 ± 0.2	20.2 ± 0.4	24.2 ± 0.2	25.9 ± 0.3
14	17.4 ± 0.1	17.5 ± 0.4	21.5 ± 0.2	20.1 ± 0.4	23.9 ± 0.2	25.4 ± 0.3
16	17.0 ± 0.1	17.4 ± 0.4	21.3 ± 0.4	20.3 ± 0.6	23.4 ± 0.2	25.1 ± 0.3
18	17.1 ± 0.1	17.6 ± 0.4	21.0 ± 0.8	20.3 ± 1.1	23.5 ± 0.2	25.0 ± 0.3
20	17.5 ± 0.1	17.7 ± 0.4	22.4 ± 4.6	···	23.4 ± 0.2	24.8 ± 0.3

[a]See Ref. 6.

contribution A and are obtained simply by counting the number of quarks in the target proton which are the same as the antiquark in the meson and which can therefore give isosinglet annihilation. The SU(3) symmetry breaking does not affect these relations as the symmetry-breaking term S simply gives a constant difference between pion-nucleon and kaon-nucleon amplitudes which cancels out in the Johnson-Treiman relations.

Let us now compare these various sets of predictions with experiment. The relevant experimental data are given in Table I and plotted in Figs. 1 and 2.

Relations (4a) and (4b) are in excellent agreement with experiment. In both cases the discrepancies are within the experimental errors. However, one can argue that a discrepancy of about 0.3 mb is present in Eq. (4a) by averaging the data over all energies. The next best relations are the Johnson-Treiman relations.[2] The discrepancy there is of the order of $\frac{1}{2}$ mb. The "symmetric sum rule" (3a) has a discrepancy of about $2\frac{1}{2}$ mb, while the relations (5a) and (5b) have discrepancies of about 4 mb.

The worst disagreements are thus of the order of 15-20% and are found in relations (5a) and (5b) whose derivation always involves SU(3). On the other hand, the best agreements are found in relations (4a) and (4b) where the discrepancy is two percent or less. These relations are obtained without assuming SU(3). The two Johnson-Treiman relations give discrepancies of 2-3% while the symmetric sum rule has a 12% discrepancy.

The good agreement with experiment of relations (1), (2a), (2b), and (4a) is graphically shown in Fig. 2, which plots the eight quantities appearing on the left- and right-hand sides of these relations. Since these are all normalized weighted means of different meson-baryon cross sections, they can all be expected a priori to be equal in some asymptotic limit. Figure 2 shows that they are far from equal and divide into four well-separated pairs of nearly equal quantities, namely just those pairs which satisfy the relations. The poorer relations (3a), (5a), and (5b) whose derivation requires SU(3) are indicated on Fig. 1 and can be compared with the difference between the K^+p and K^+n cross sections. That these are qualitatively worse than those of Fig. 2 is immediately evident.

These results are very reasonable in view of the derivations. A discrepancy of 10-20% is not unexpected for reaction predictions which completely neglect SU(3)-symmetry breaking. The best prediction is the antisymmetric sum

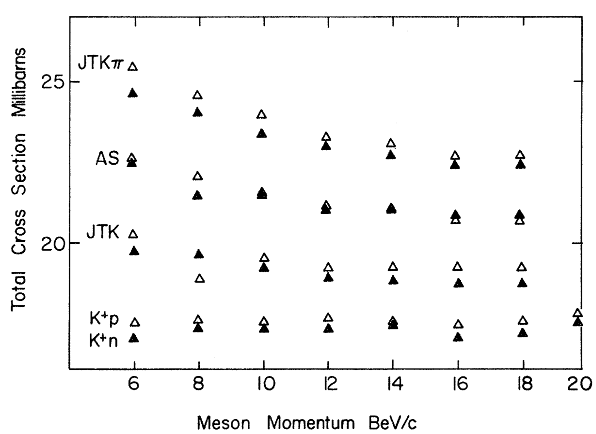

FIG. 2. Experimental tests of relations (1), (2a), (2b), and (4a). △ left-hand side, ▲ right-hand side. JTπK, Johnson-Treiman relation (2a); JTK, Johnson-Treiman relation (2b); AS, antisymmetric sum rule (1).

rule reported previously[2,4] which is obtained only from the quark-model additivity assumption. The next best, still in the 2% range, is obtained by neglecting the charge exchange in nonstrange quark-quark scattering. The two Johnson-Treiman relations require the additional assumption of the Pomeranchuk relation.[6] This is evidently a 2-3% approximation.

The assumption (6) can also be applied to baryon-baryon total cross sections. These give the results

$$\sigma(pp) = \sigma(pn) = 9P, \tag{8a}$$

$$\sigma(\bar{p}p) = 9P + 5A, \tag{8b}$$

$$\sigma(\bar{n}p) = 9P + 4A. \tag{8c}$$

Combining Eqs. (8) and (7) leads to relations between meson-baryon and baryon-baryon scattering, which turn out to be identical to those obtained by Freund,[2]

$$\sigma(\bar{p}p) - \sigma(pp) = 5/4[\sigma(\bar{n}p) - \sigma(np)] = 5[\sigma(\pi^-p) - \sigma(\pi^+p)]. \tag{9}$$

The agreement of these relations with experiment has been discussed,[2] and is quite good in view of the larger experimental errors. However, quark-model relations between meson-baryon and baryon-baryon scattering have been shown[4] to be good only to about 10%, in contrast to the relations for meson-baryon scattering, even without the additional assumption (6). This may be due to breakdown of the basic additivity assumption for the baryon case, or to the effects of binding on the effective quark-quark scattering amplitude[8]; e.g., the "effective mass" of a bound quark may have different values in a baryon and in a meson.

It is not clear whether the success of the relations obtained here should be considered as convincing evidence for the validity of the assumptions used, or whether relations (7) and (8) simply constitute a successful parametrization of the experimental data. In particular, the relation between quark-model derivations without higher symmetries and other symmetry derivations[9] possibly together with universality[2] should be investigated. However, whatever the interpretation, it appears significant that three independent relations can be derived, without the use of SU(3) symmetry, which show an agreement with experiment roughly an order of magnitude better than normally obtained for SU(3) relations between transition amplitudes.

Other derivations[2,4] of the Johnson-Treiman relations which use SU(3) have the difficulty of explaining why they are so good relative to other SU(3) predictions.[5] Note also that the relations (5a) and (5b), which are reasonable SU(3) predictions (20%), do not appear in other derivations. The particular choice of the reactions appearing in (5a) and (5b) seems to be significant. One might just as well expect similar relations involving the charge conjugate mesons; i.e., π^+p and K^+p instead of (5a), or π^-p and K^+n instead of (5b). These are not predicted in this model and are in strong disagreement with experiment; the characteristic discrepancies are about 9 mb or 40-50%.

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[9]It is tempting to try to link these results with a higher symmetry or the algebra of currents, particularly in view of the recent formulations using states at infinite momentum. See R. F. Dashen and M. Gell-Mann, in Proceedings of the Coral Gables Conference, 1966 (to be published). The high-energy scattering states considered here have $v \sim c$ in the center-of-mass system and could be considered to be near the infinite-momentum limit. The equality of $\sigma(K^+p)$ and $\sigma(K^+n)$ is obtainable from the assumption of invariance of the S matrix under a peculiar SU(2)⊗SU(2) group,

where the two SU(2) groups are the isospins of (1) quarks with $p=+\infty$ and antiquarks with $p=-\infty$; (2) quarks with $p=-\infty$ and antiquarks with $p=+\infty$. Unfortunately this group is not a subgroup of the U(12) group generated by the conventional current algebras. The corresponding $SU(2)\otimes SU(2)$ subgroup of U(12) does not give the momentum reversal for quarks and antiquarks and simply gives the trivial result that all forward two-body inelastic processes are negligible in comparison with elastic scattering in the high energy limit. The author is grateful to H. Harari for an elucidation of this point.

POLARIZATION AT FORWARD ANGLES AND AN INVESTIGATION OF STRUCTURE IN πN, KN, AND NN ELASTIC SCATTERING*

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The structure in $\pi^- p$ elastic scattering in the region from 1.6 to 5.3 GeV/c has been recently investigated by Kormanyos et al.[1] by measuring the differential cross section at 180°. Their results show considerable structure associated with the various N^* resonances. The idea first suggested by Ross and Heinz[2] was to investigate the effect of resonances at 180° where the imaginary scattering amplitude, which is dominant at the forward (c.m.) hemisphere, is small.

A similar investigation to that mentioned above can be made by measuring the polarization at forward angles. In this method, spin and parity of resonances such as N^* or Y^* may be determined, and the measurements at forward angles offer an advantage over the backward measurements on the counting rate.

To determine the spin and parity of a resonance, N^*, the partial-wave analysis has been the useful method. In this analysis both the differential-cross-section and polarization data of the wide angular region in the πN elastic scattering were required. Up to an incident-π momentum of 2.5 GeV/c, investigating the existence of resonances by the partial-wave analysis was barely feasible.[3] However, extending the analysis beyond this energy region seems doubtful because the number of parameters in the analysis becomes formidably large; in addition, obtaining the experimental data with reasonable statistical error is difficult especially in the backward hemisphere. The partial-wave analysis also requires the continuity condition, and thus, considerable data at fine energy intervals are needed. If one is interested in a higher resonant state, say at 6 GeV/c, that energy region cannot be investigated without covering the gap up to 6 GeV/c.

The proposed method for measuring the polarization in πN and KN elastic scattering at forward angles,[4] $0<\theta_{\text{c.m.}}<15°$, with fine angular intervals was deduced from the following reasons. It is well known that the product of the differential cross section and polarization can be expanded in terms of the associated Legendre polynomials as

$$\frac{d\sigma(\theta)}{d\Omega}P(\theta)=\lambda^2\sum_{n=1}^{N} b_n P_n'(\theta). \tag{1}$$

If one resonant or a dominant state among several resonant states exists, then

$$\begin{aligned} b_n = {} & \mathrm{Re}(A_{\text{res}})[\Sigma\beta_{n,l_{\text{res}},l'}\mathrm{Im}A_{l'\pm}] \\ & -\mathrm{Im}(A_{\text{res}})[\Sigma\beta_{n,l_{\text{res}},l'}\mathrm{Re}A_{l'\pm}] \\ & +\Sigma\beta_{n,l,l'}[\mathrm{Re}A_{l\pm}\mathrm{Im}A_{l'\pm}-\mathrm{Im}A_{l\pm}\mathrm{Re}A_{l'\pm}], \end{aligned} \tag{2}$$

where the numerical factors, $\beta_{n,l,l'}$, up to $l=3$ are shown in Table I.[5] If the resonant state is consistent with $l+$, then $\beta_{nl'}(\text{res})>0$, and if it is with $l-$, then $\beta_{nl'}(\text{res})<0$, as one can see from Table I. It is reasonable to assume that the third term, which represents the background, in Eq. (2) varies much slower than the first and second terms with respect to the incident-particle energy.

$\mathrm{Im}A_{l'\pm}$ is in general considerably larger than $\mathrm{Re}A_{l'\pm}$. For example, Table II shows the ratio $\mathrm{Re}A_{l'\pm}/\mathrm{Im}A_{l'\pm}$ at the incident-π momentum of 2.5 GeV/c obtained from the partial-wave analysis.[3] The ratio is expected to be further decreased at higher energies. Thus, the first term in Eq. (2) is the most significant term for observing the effect of the resonant state.