MEASUREMENT OF PHOTON BUNCHING IN A THERMAL LIGHT BEAM*

B. L. Morgan
Department of Physics, Imperial College, London, England

and

L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 21 April 1966)

It has been known since the first photon correlation experiments were reported[1-3] that the counts registered by a photoelectric detector illuminated by a light beam from a thermal source do not arrive completely at random. In time intervals of order or less than the coherence time of the light, the probability of counting two pulses is greater than that expected for random events. The "bunching" of photocounts has been most clearly demonstrated in the excess coincidence experiments with two coherently illuminated photodetectors.[2-4] Somewhat less direct evidence for the bunching is also contained in measurements of the fluctuations of counts registered by a single photodetector in a finite time interval.[5-7] Yet one of the most interesting features of the bunching, that the distribution of time intervals between successive counts is closely related to the spectral profile in the case of thermal light, and may be used to determine the spectral profile, has not so far been examined or put to use.[8]

We have measured the distribution of time intervals between successive photon counts of an illuminated photomultiplier in the range 1.4 to 10 nsec, for the light from a low-pressure Hg^{198} discharge lamp. The measurements lead to a spectral width of the blue Hg^{198} line of about 200 Mc/sec. Although this appears to be the first attempt to determine the spectral profile of light from a thermal source in this way, somewhat similar measurements have been reported for light from a "pseudothermal" source,[9,10] which is produced by moving a piece of ground glass in the path of a laser beam.

The experimental setup, which was similar to one that was briefly described by Mandel,[8] is shown in Fig. 1. A light beam from a low-pressure Hg^{198} gas discharge lamp passed through a pin-hole P (diameter 0.54 mm), an optical filter F_1 that isolated the blue 5461-Å line, and a linear polarizer F_2. The beam fell on a 56 AVP photomultiplier through a rectangular aperture S (0.37×0.47 mm^2), whose dimensions were small enough to ensure a degree of coherence of at least 90% across the beam.[11]

The pulses from the photomultiplier were shortened by reflection in a 1-nsec clipping line, and were fed to a specially designed gated pulse counter, which registered an output whenever two pulses appeared at the anode of the photomultiplier with a time separation lying between τ_1 and τ_2. The time τ_1 was determined by the difference of two cable lengths, and could be varied by varying one of the lengths. $\tau_2-\tau_1$ was determined by the width of the gating pulse, and remained constant and equal to about 7.5 nsec as τ_1 was varied. Provision was made for stabilizing the light output of the discharge lamp by feedback from the gating-pulse circuit, as indicated in Fig. 1.

The results of the measurement are shown in Fig. 2(a), which gives the two-pulse counting rate as a function of the time interval τ_1. It will be seen that there is an increase in the counting rate when τ_1 becomes less than 2 or 3 nsec, which is of the order of the coherence time. This illustrates the bunching phenomenon. For comparison, the results of similar measurements carried out with a tungsten lamp as source are shown in Fig. 2(b). The results reveal no significant bunching of photocounts, since the coherence time is unmeasurably short

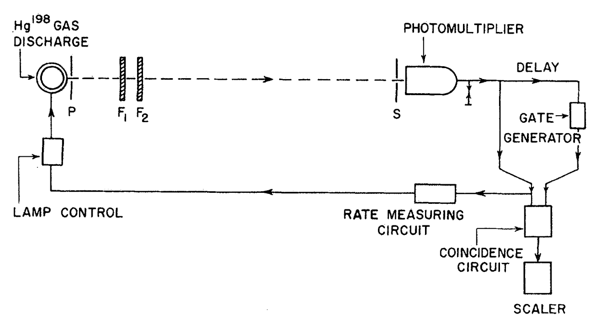

FIG. 1. The experimental setup.

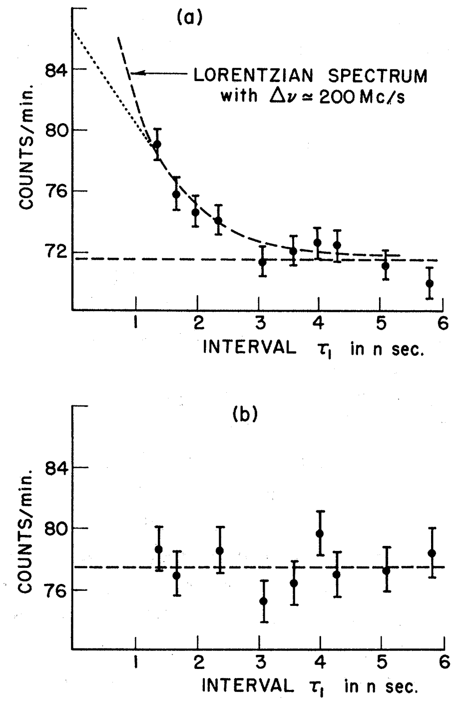

FIG. 2. Experimental results for the two-photon counting rate obtained (a) with the Hg^{198} light source; (b) with the tungsten lamp as light source. The broken horizontal line in each figure corresponds to the "random" pulse rate, which is obtained by taking the average of a number of separate measurements covering the range $\tau_1 = 4$ nsec to $\tau_1 = 8.6$ nsec.

in this case.

The statistical accuracy in these experiments was not high, since the counting rates were limited by the low degeneracy parameter of the source.[12] Each experimental point corresponds to a measurement time of about 50 min. It proved to be impossible to obtain meaningful results for time intervals τ_1 less than 1.4 nsec, since the electronic gate then failed to prevent single pulses from breaking through and being counted as pulse pairs. This problem is avoided in two-channel counting experiments with two photomultipliers, at the cost of a fourfold drop in the delayed-coincidence counting rate.

Let us now briefly consider the explanation of the effect. It can be shown[13,14] that the joint probability $p_2(t_1,t_2)\delta t_1\delta t_2$ of registering two photocounts at times t_1 and t_2 within δt_1 and δt_2, when the photodetector is illuminated normally by a plane light beam, is given by

$$p_2(t_1,t_2)\delta t_1\delta t_2 = \alpha^2\langle :[\vec{A}^\dagger(\vec{x},t_1)\cdot\vec{A}(\vec{x},t_1)] \times[\vec{A}^\dagger(\vec{x},t_2)\cdot\vec{A}(\vec{x},t_2)]:\rangle\delta t_1\delta t_2, \quad (1)$$

where $\vec{A}(\vec{x},t)$ is the configuration-space photon annihilation operator at $(\vec{x},t)$, $\vec{x}$ is any point on the photocathode, and $:O:$ stands for normal ordering of the operator O. α is a constant involving the matrix element for the atomic transition and other constants, and plays the role of a measure of quantum efficiency. For a plane beam of polarized light from a thermal source, the density operator of the field has a well-known form, and the expectation value in (1) may readily be evaluated.[14,15] We find

$$p_2(t_1,t_2)\delta t_1\delta t_2 = r^2[1+|\gamma(t_1,t_2)|^2]\delta t_1\delta t_2, \quad (2)$$

where r is the mean counting rate of the illuminated detector, and $\gamma(t_1,t_2)$ is the normalized second-order autocorrelation function defined by

$$\gamma(t_1,t_2) = \langle\vec{A}^\dagger(\vec{x},t_1)\cdot\vec{A}(\vec{x},t_2)\rangle/\langle\vec{A}^\dagger(\vec{x},t)\cdot\vec{A}(\vec{x},t)\rangle. \quad (3)$$

For a stationary field the averages are of course independent of the origin of time and $\gamma(t_1,t_2)$ is a function only of the difference $\tau = t_2 - t_1$. In Eq. (2) it is assumed that the counts registered by the detector are all due to the absorption of photons. In practice, each photodetector also has a mean residual counting rate b (dark current), which is unrelated to the incident light. This amounted to about 3000 counts/sec in our case, as compared with $r \simeq 10\,000$ counts/sec. Eq. (2) therefore needs to be modified in practice and should read

$$p_2(t_1,t_2)\delta t_1\delta t_2 = [(r+b)^2 + r^2|\gamma(\tau)|^2]\delta t_1\delta t_2. \quad (4)$$

A counting device that accepts pairs of pulses with an interval separation τ in the range $\tau_1 \leq \tau \leq \tau_2$ will therefore count at the rate

$$R = \int_{\tau_1}^{\tau_2} p_2(t_1,t_1+\tau)d\tau = (r+b)^2(\tau_2-\tau_1) + r^2\int_{\tau_1}^{\tau_2}|\gamma(\tau)|^2 d\tau = R(\text{random}) + R(\text{excess}). \quad (5)$$

In this equation the first term represents the "accidental" or random rate of pulse-pair counting which remains constant as long as $\tau_2 - \tau_1$ is constant, while the second term represents the "excess" counting rate due to the bunching phenomenon which varies with τ_1. Since $|\gamma(\tau)|$ becomes very small for values of τ appreciably in excess of the coherence time of the light, it is clear that the second term responsible for the bunching contributes significantly only for values of τ_1 which are of order or less than the coherence time. This is borne out by the results of Fig. 2. For the same reason the upper limit τ_2 in the integral in Eq. (5) may be replaced by ∞ to a good approximation, since $\tau_2-\tau_1 \simeq 7.5$ nsec and τ_2 was never less than about 9 nsec in the experiment.

The broken curve fitted against the data of Fig. 2 is the form of $\int_{\tau_1}^{\infty}|\gamma(\tau)|^2 d\tau$ corresponding to a Lorentzian spectral profile, for which

$$\gamma(\tau)=\exp(-|\tau|/T_c), \qquad (6)$$

and

$$\int_{\tau_1}^{\infty}|\gamma(\tau)|^2 d\tau=\tfrac{1}{2}T_c\exp(-2\tau_1/T_c), \quad \tau_1 \geqslant 0, \qquad (7)$$

where T_c is related to the spectral width $\Delta\nu$ at half-height by $T_c=1/\pi\Delta\nu$. The plotted curve corresponds to a value of $T_c \simeq 1.7$ nsec, or to Lorentzian spectral width $\Delta\nu$ of about 200 Mc/sec.

However, we can show that the spectral profile cannot actually be Lorentzian in the wings. The form of the spectral profile in the wings is largely governed by the behavior of $|\gamma(\tau)|$ in the neighborhood of small τ, and it is interesting that some information about this behavior can actually be deduced from Eq. (5) and from the measurements, even though experimental points were not obtained in this region.

According to Eq. (5), the ratio of the "excess" to the random counting rate extrapolated to zero delay is given by

$$\frac{\text{excess rate } (\tau_1=0)}{\text{random rate}}=\frac{r^2\int_0^{\infty}|\gamma(\tau)|^2 d\tau}{(r+b)^2(\tau_2-\tau_1)}, \qquad (8)$$

and, since R(random) does not vary with τ_1, the slope of the counting rate extrapolated to zero delay has the value

$$(dR/d\tau_1)_{\tau_1=0}=-r^2|\gamma(0)|^2=-r^2 \simeq -10^8 \text{ counts/sec}^2. \qquad (9)$$

This slope is much less than is indicated by the broken curve in Fig. 2(a). Although the value of $\int_0^{\infty}|\gamma(\tau)|^2 d\tau$ is not known, since a portion of the curve is missing in Fig. 2(a), yet it is possible to make an estimate of the value of the integral (say 2 to 3 nsec) merely by inspection of Fig. 2(a). When this value is substituted in Eq. (8), we can compute the excess rate at $\tau_1=0$, and, with the help of (9), sketch in how the plotted curve should be extrapolated. Once the position of the intercept is found, a better estimate of $\int_0^{\infty}|\gamma(\tau)|^2 d\tau$ can be made which, in turn, can be used to correct the value of the intercept. By proceeding iteratively in this way we find that[16]

$$\int_0^{\infty}|\gamma(\tau)|^2 d\tau \simeq 2.8 \text{ nsec}, \qquad (10)$$

that the excess rate at $\tau_1=0$ is about 15.6 counts/min, and that the curve cuts the vertical axis at $R \simeq 87$ counts/min. The curve is therefore expected to continue as shown by the dotted curve in Fig. 2(a).

It appears, then, that the method of pulse-interval analysis can be used to obtain a substantial amount of information about the autocorrelation function and spectral profile of thermal light, even when the results are of limited accuracy and cover only a limited range. The method becomes increasingly attractive as the coherence time of the light increases.

One of us (B.L.M.) wishes to acknowledge financial assistance from the Science Research Council.

---

*This work was supported in part by the U. S. Air Force Cambridge Research Laboratories.

[1]R. Hanbury Brown and R. Q. Twiss, Nature 177, 27 (1956).

[2]R. Q. Twiss, A. G. Little, and R. Hanbury Brown, Nature 180, 324 (1957).

[3]G. A. Rebka and R. V. Pound, Nature 180, 1035 (1957).

[4]E. Brannen, H. I. S. Ferguson, and W. Wehlan, Can. J. Phys. 36, 871 (1958).

[5]C. Freed and H. A. Haus, Phys. Rev. Letters 15, 943 (1965).

[6]F. T. Arecchi, Phys. Rev. Letters 15, 912 (1965).

[7]A. W. Smith and J. A. Armstrong, Phys. Letters 19, 650 (1966).

[8]That spectral profiles might be measured in this way was suggested by a number of authors: L. Mandel, in Electromagnetic Theory and Antennas, edited by E. C. Jordan (The MacMillan Company, New York, 1963), Pt. 2, p. 811; E. Wolf, J. Appl. Phys. (Japan) 4, Suppl. 1, 1 (1965); M. L. Goldberger, W. H. Lewis, and K. M. Watson, Phys. Rev. 142, 25 (1966).

[9]W. Martienssen and E. Spiller, Am. J. Phys. 32, 919 (1964).

[10]F. T. Arecchi, E. Gatti, and A. Sona, Phys. Letters 20, 27 (1966).

[11]See, for example, M. Born and E. Wolf, Principles of Optics (Pergamon Press, Oxford, England, 1966), p. 511, 3rd ed.

[12]L. Mandel, J. Opt. Soc. Am. 51, 797 (1961).

[13]R. J. Glauber, Phys. Rev. 130, 2529 (1963); 131, 2766 (1963).

[14]See, for example, L. Mandel and E. Wolf, Rev. Mod.

Phys. 37, 231 (1965).

[15]L. Mandel, in Progress in Optics, edited by E. Wolf (North-Holland Publishing Company, Amsterdam, 1963), Vol. 2, p. 181.

[16]It has been shown that this integral multiplied by 2 represents a natural measure of coherence time for thermal light: L. Mandel, Proc. Phys. Soc. (London) 74, 233, (1959).

QUARK MODELS AND HIGH-ENERGY SCATTERING

H. J. Lipkin
Department of Physics, Weizmann Institute of Science, Rehovoth, Israel
(Received 20 April 1966)

The large amount of precise experimental data now available on total cross sections for meson-nucleon scattering presents a challenge to the theorist.[1] Symmetries[2,3] and quark models[4] have had a certain degree of success in obtaining relations between these cross sections which are in agreement with experiment. However, some predictions from SU(3) symmetry seem to be in disagreement with experiment.[1,5] Furthermore, the most striking regularity of the data has not been predicted by any of these symmetries or quark models, namely the equality of the K^+p and K^+n total cross sections over a wide energy range.[1,6] Many models and theories predict that all meson-baryon cross sections become equal at sufficiently high energy.[4,7] However, experimental data show that some of these are more equal than others, as indicated in Fig. 1. This feature has not been predicted by any of the higher symmetries which include SU(3).

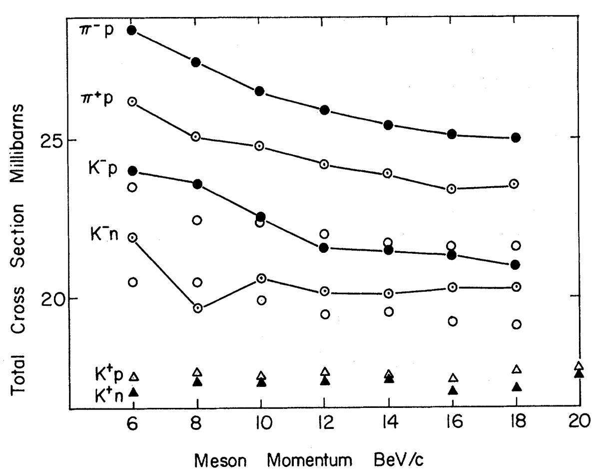

FIG. 1. Meson-baryon cross sections and relations (3a), (5a), and (5b). ○ left- and right-hand sides of Eq. (3a), ● left- and right-hand sides of Eq. (5a), ⊙ left- and right-hand sides of Eq. (5b).

We should like to show that good agreement with experiment is obtained by a slight extension of the quark model[4] along the lines suggested by Kokkedee and Van Hove.[8] In addition to the additivity assumption for the two-body quark scattering amplitudes assumed in the previous treatments,[4] we have certain simplifying assumptions about the two-body quark-quark and quark-antiquark scattering amplitudes. The basic physical idea is that scattering of 6- to 20-BeV/c mesons is sufficiently close to the asymptotic region so that the two-body scattering amplitudes exhibit some, but not all of the asymptotic features. In particular, it is assumed that the quark-quark amplitudes exhibit these asymptotic features, while they are not yet present in the quark-antiquark amplitudes, possibly because of the presence of the annihilation channel in the latter.[8] We consider several different combinations of these assumptions, both with and without SU(3) symmetry.

In order to enable a fair comparison with experiment of different treatments, we express all predicted relations between meson-baryon total cross sections in the following standard form: The expressions on both sides of the equality involve only sums, no differences. They are normalized so that in the limit where all meson-baryon cross sections are equal, the expressions on two sides are just equal to the common meson-baryon cross section. We first list relations which have been previously obtained. We consider only total cross sections, which are related to the forward scattering amplitude by the optical theorem.

The antisymmetric sum rule[4] follows directly from the basic additivity assumption of the quark model, without any symmetry assumptions. In our standard form this becomes

$$\tfrac{1}{3}[\sigma(K^-p)+\sigma(\pi^+p)+\sigma(K^0p)] = \tfrac{1}{3}[\sigma(K^+p)+\sigma(\pi^-p)+\sigma(\bar{K}^0p)]. \quad (1)$$